



Flow through Convergent-Divergent Nozzles

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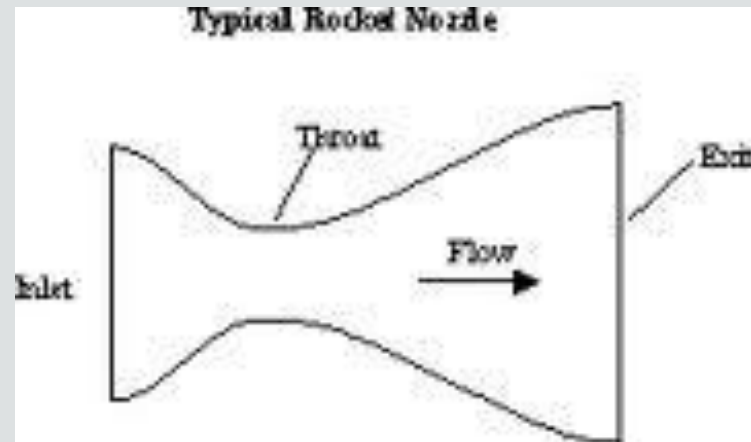


Aims

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The aim of this lecture is to examine Transonic Flow through a Convergent-Divergent Nozzle.



Much of the material for this lecture comes from Mattingly Chapter 2



Learning Outcomes

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By the end of this lecture students should understand:

- The nature of Compressible Flow through a Convergent Divergent Nozzle
- How to calculate the Fluid State at any point through such a Nozzle.



Total Temperature/Pressure vs Mach No.

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For Isentropic Flow of a Perfect Gas:

$$\frac{T}{T_t} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1}$$

$$\frac{P}{P_t} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma - 1}}$$



Total Temperature/Pressure vs Mach No.

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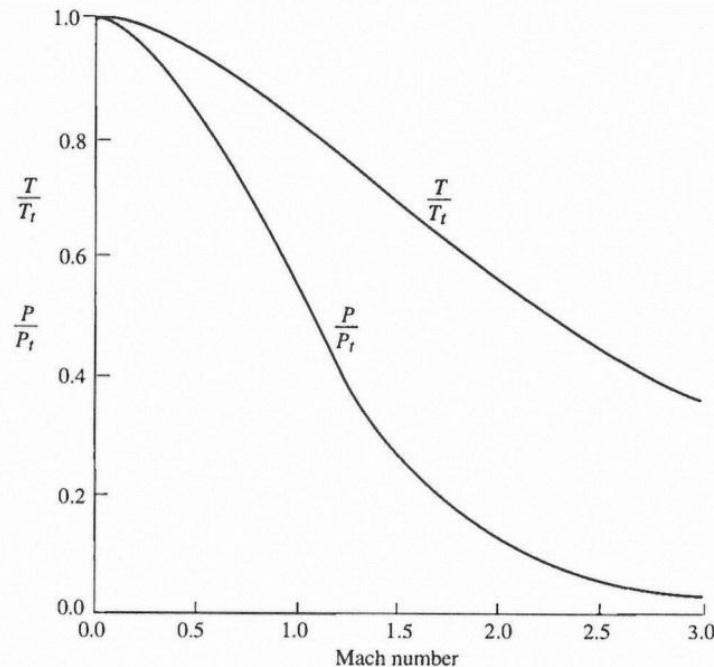


Fig. 2.18 P/P_t and T/T_t vs Mach number ($\gamma = 1.4$).

At $M=1$:

$$\frac{P}{P_t} = 0.528$$

$$\frac{T}{T_t} = 0.833$$



Total Temperature/Pressure vs Mach No.

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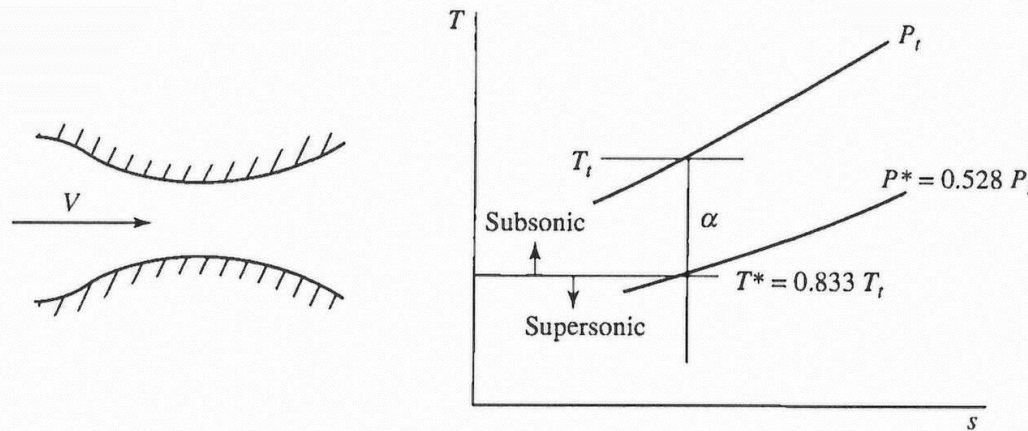


Fig. 2.19 Subsonic and supersonic state points in an isentropic flow.

At $M=1$:

$$\frac{P}{P_t} = 0.528$$

$$\frac{T}{T_t} = 0.833$$

P^* , T^* represent conditions for Mach 1 – sometimes known as the Star State.



Isentropic Area Ratio

Similarly from:

$$\dot{m} = \rho AV = \rho^* A^* V^*$$

We get:
$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

To accelerate subsonic flow, A must get smaller

To accelerate supersonic flow, A must get bigger

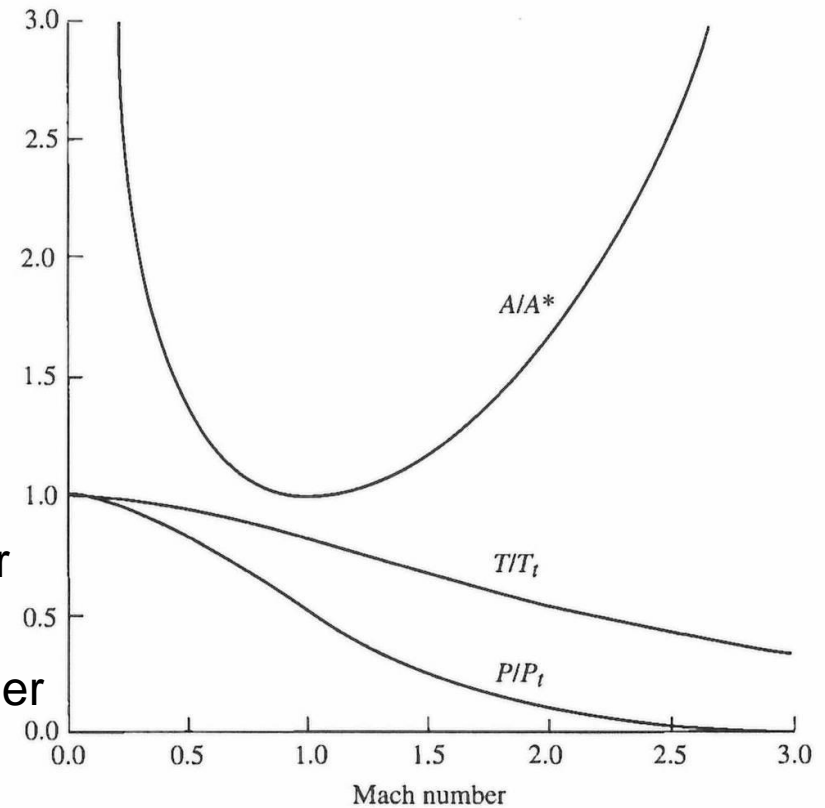


Fig. 2.21 A/A^* , P/P_t , and T/T_t vs Mach number ($\gamma = 1.4$).



Mass Flow Parameter

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How much flow can we get through a given area?

Can we express it as a function of Mach Number?

$$\dot{m} = \rho AV$$

$$P = \rho RT$$

$$a = \sqrt{\gamma g_c RT}$$

$$T_t = T + \frac{V^2}{2g_c C_P}$$

$$\frac{T_t}{T} = \left(\frac{P_t}{P}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T}{T_t} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1}$$

$$\frac{P}{P_t} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}}$$



Mass Flow Parameter

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$$MFP = \frac{\dot{m}\sqrt{T_t}}{P_t A} = M\sqrt{\gamma g_c/R} \left\{ 1 + \left[\frac{\gamma - 1}{2} \right] M^2 \right\}^{-\frac{\gamma+1}{2(\gamma-1)}}$$

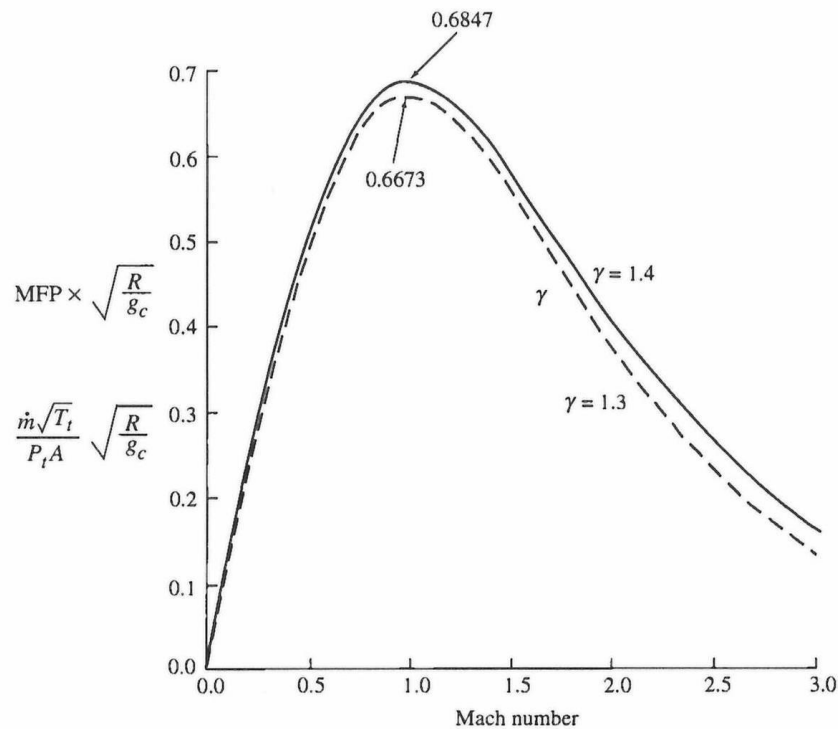


Fig. 2.20 Mass flow parameter vs Mach number ($\gamma = 1.4$ and $\gamma = 1.3$).



Mass Flow



What if we don't know the Mach Number?

$$\dot{m} = \rho AV$$

$$P = \rho RT$$

$$\frac{T_t}{T} = \left(\frac{P_t}{P}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_t = T + \frac{V^2}{2g_c C_p}$$

It can be shown that:

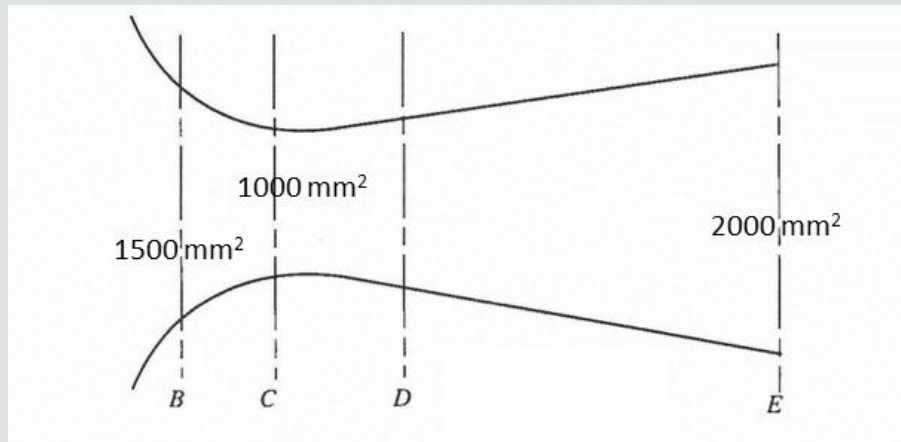
$$\frac{\dot{m}}{A} = \frac{P_t}{\sqrt{T_t}} \sqrt{\frac{2g_c}{R} \frac{\gamma}{\gamma-1} \left[\left(\frac{P}{P_t}\right)^{\frac{2}{\gamma}} - \left(\frac{P}{P_t}\right)^{\frac{\gamma+1}{\gamma}} \right]}$$



Example

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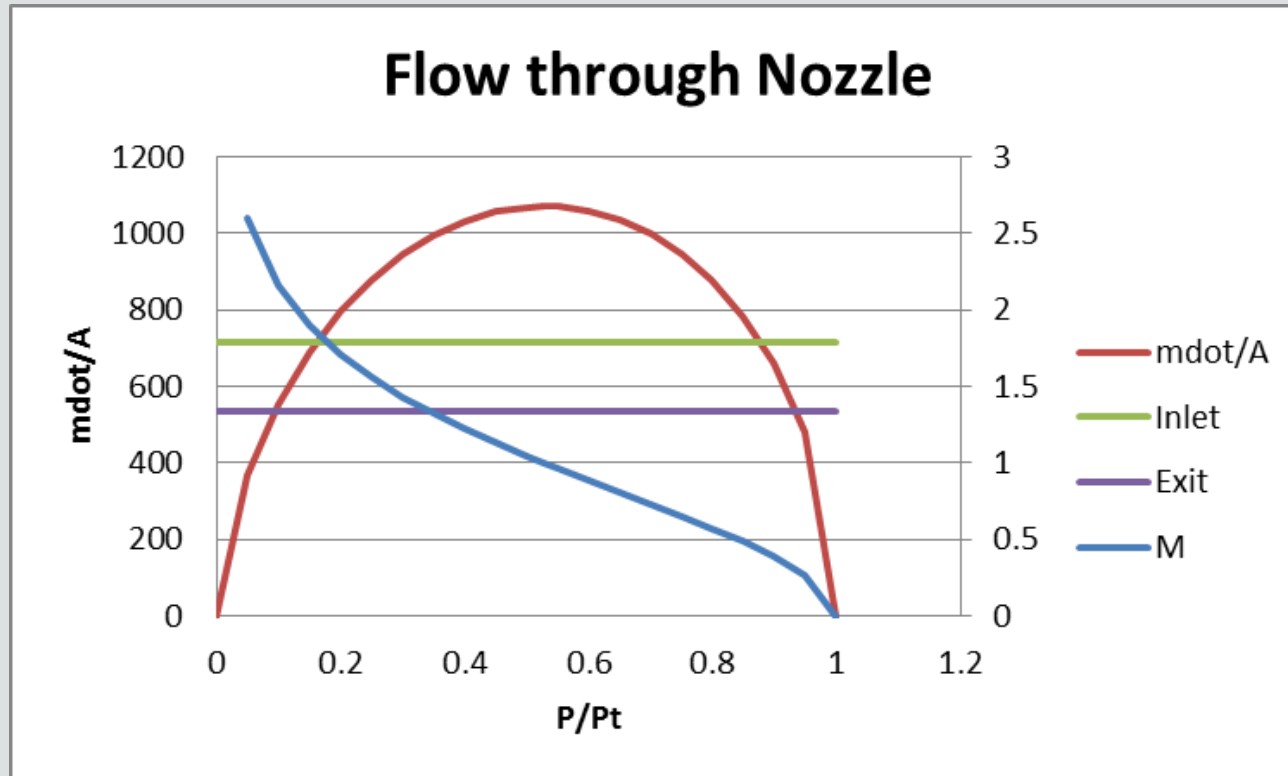


If $P_t = 1.4 \text{ MPa}$ and $T_t = 2800 \text{ K}$ calculate the mass flow rate and the flow Mach Number at the exit E.

Hint: Use MFP to calculate \dot{m}



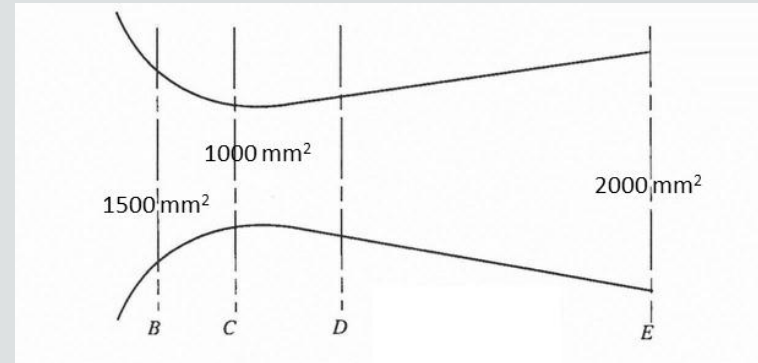
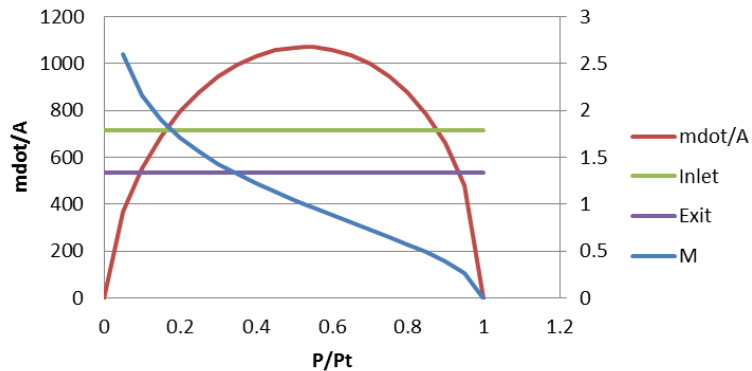
Example





Example

Flow through Nozzle



The nozzle only wants to have exit conditions at e or e'

If these don't match the atmospheric conditions, we get a Shock Wave

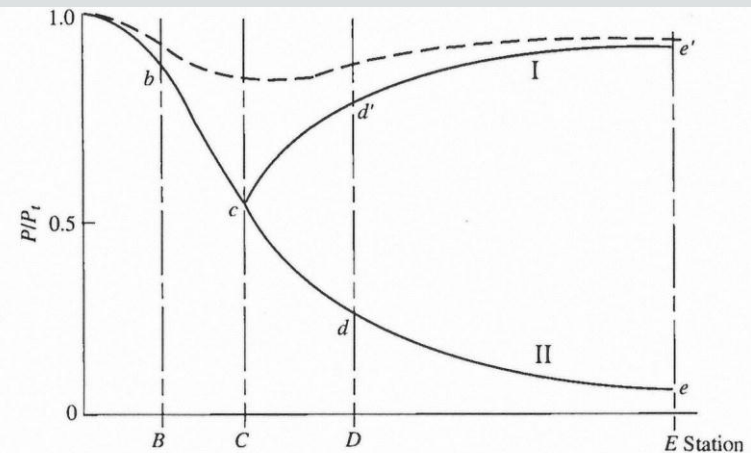


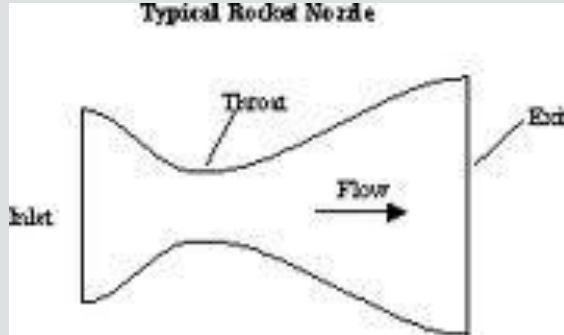
Fig. 2.36 Nozzle pressure distribution.



Summary

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- To accelerate subsonic flow, the cross sectional area must get smaller
- To accelerate supersonic flow, the cross sectional area must get bigger
- The Maximum Mass Flow Rate occurs when the Mach Number at the Throat = 1