Using the property:

$$\frac{d}{dt}\left(\sum_{n=1}^{\infty} f_n(t)\right) = \sum_{n=1}^{\infty} f_n{}'(t)$$

Leibniz's Rule.

So,

$$\frac{d}{dx}\left(\sum_{n=1}^{\infty} \frac{\sin(nx)}{2^n n}\right) = \sum_{n=1}^{\infty} \frac{n\cos(nx)}{2^n n} = \sum_{n=1}^{\infty} \frac{\cos(nx)}{2^n} = \frac{d}{dx}S(x)$$

So that,

$$\sum_{n=1}^{\infty} \frac{\cos(nx)}{2^n} = \left(\sum_{n=0}^{\infty} \frac{\cos(nx)}{2^n}\right) - \frac{\cos(0*x)}{2^0} = \left(\sum_{n=0}^{\infty} \frac{\cos(nx)}{2^n}\right) - 1$$

So therefore:

$$\begin{aligned}\sum_{n=0}^{\infty} \frac{\cos(nx)}{2^n} &= \sum_{n=0}^{\infty} \frac{\left[\frac{e^{inx}+e^{-inx}}{2}\right]}{2^n} = \sum_{n=0}^{\infty} \frac{\left[\frac{e^{inx}+e^{-inx}}{2}\right]}{2^n} = \sum_{n=0}^{\infty}\left[\frac{e^{inx}+e^{-inx}}{2*2^n}\right] \\ &= \frac{1}{2}\left\{\sum_{n=0}^{\infty}\left[\frac{e^{ix}}{2}\right]^n + \sum_{n=0}^{\infty}\left[\frac{e^{-ix}}{2}\right]^n\right\} = \frac{1}{2}\left\{\frac{1}{1-\frac{e^{ix}}{2}} + \frac{1}{1-\frac{e^{-ix}}{2}}\right\} = \frac{1}{2}\left\{\frac{2}{2-e^{ix}} + \frac{2}{2-e^{-ix}}\right\} \\ &= \frac{1}{2}\left\{\frac{2}{2-\cos(x)-i\sin(x)} + \frac{2}{2-\cos(x)+i\sin(x)}\right\} \\ &= \frac{1}{2}\left\{\frac{2(2-\cos(x)+i\sin(x)) + 2(2-\cos(x)-i\sin(x))}{[2-\cos(x)]^2+\sin^2(x)}\right\} = \frac{1}{2}\left\{\frac{8-4\cos(x)}{5-4\cos(x)}\right\}\end{aligned}$$

So that,

$$S'(x) = \left(\sum_{n=0}^{\infty} \frac{\cos(nx)}{2^n}\right) - 1 = \frac{4-2\cos(x)}{5-4\cos(x)} - 1 = \frac{4-2\cos(x)-5+4\cos(x)}{5-4\cos(x)} = \frac{2\cos(x)-1}{5-4\cos(x)}$$

Therefore,

$$\boxed{S(x) = \int \frac{2\cos(x)-1}{5-4\cos(x)}dx = \tan^{-1}\left(3\tan\left(\frac{x}{2}\right)\right) - \frac{x}{2}}$$

I solved this using mathematica but it seems a complex integral to be solved analytically by hand; though I suspect the professor will ask for an analytical solution.