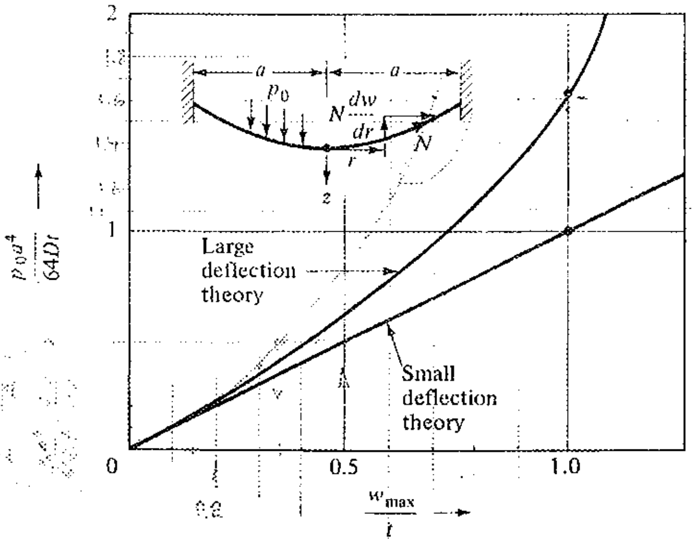

Figure 8.2

employ a simple *approximate approach* which is in good agreement with the exact theory for the circular plate. According to this method, the bending solution and the *membrane or very thin plate* solution (Sec. 10.1) are treated separately. The partial loads carried by the membrane and the bending actions in the plate are then added and equated to the actual plate loading.

As an example, consider the case of a clamped-edge circular plate subjected to a uniform load of intensity p_0 (Fig. 8.2). The *bending solution* for the maximum deflection occurring in the center is, from Eq. (2.14):

$$w_{\max} = \frac{p_0 a^4}{64D} \quad \text{or} \quad p_0 = \frac{64D}{a^4} w_{\max} \qquad (a)$$

To derive the *membrane solution*, refer to Fig. 8.2, where N denotes the *constant* tensile force per unit length. The static equilibrium of vertical forces is expressed by $2\pi r N(dw/dr) = p_0 \pi r^2$, from which $dw/dr = p_0 r/2N$. Integration of the latter expression for the slope at $r = a$ leads to:

$$w_{\max} = \frac{p_0 a^2}{4N} \qquad (b)$$

Determination of the value of N in Eq. (*b*) requires consideration of the midplane deformations.

The radial elongation produced by the deflection w is found from Eq. (7.12*a*) as follows

$$\frac{1}{2}\int_0^a \left(\frac{dw}{dr}\right)^2 dr = \frac{1}{2}\int_0^a \left(\frac{p_0 r}{2N}\right)^2 dr = \frac{p_0^2 a^3}{24N^2}$$