Homework

1) DOS in 1D, 2D?

in 1-dimension

$$D(E)_{1D} = \frac{1}{L}\frac{dN}{dE} \qquad N = \frac{2kL}{\pi}, \quad \frac{dN}{dk} = \frac{2L}{\pi}$$

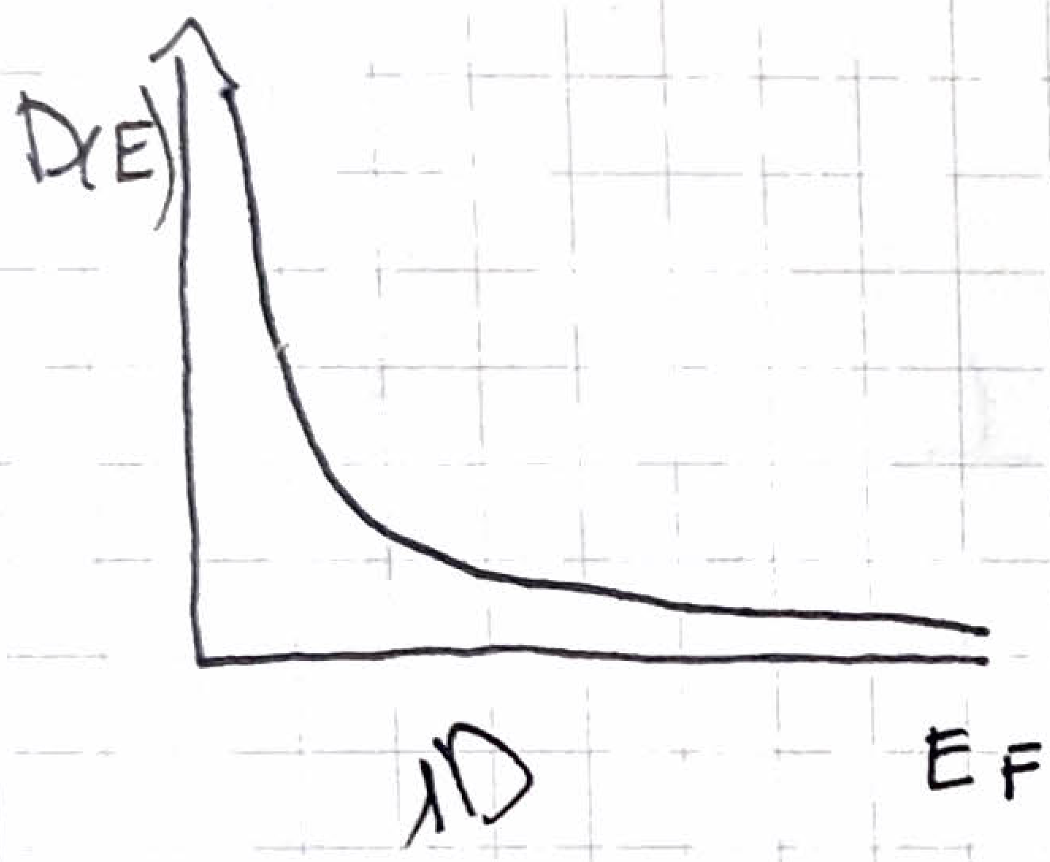
$$E = \frac{\hbar^2 k^2}{2m} \qquad \frac{dE}{dk} = \frac{\hbar^2 k}{m}$$

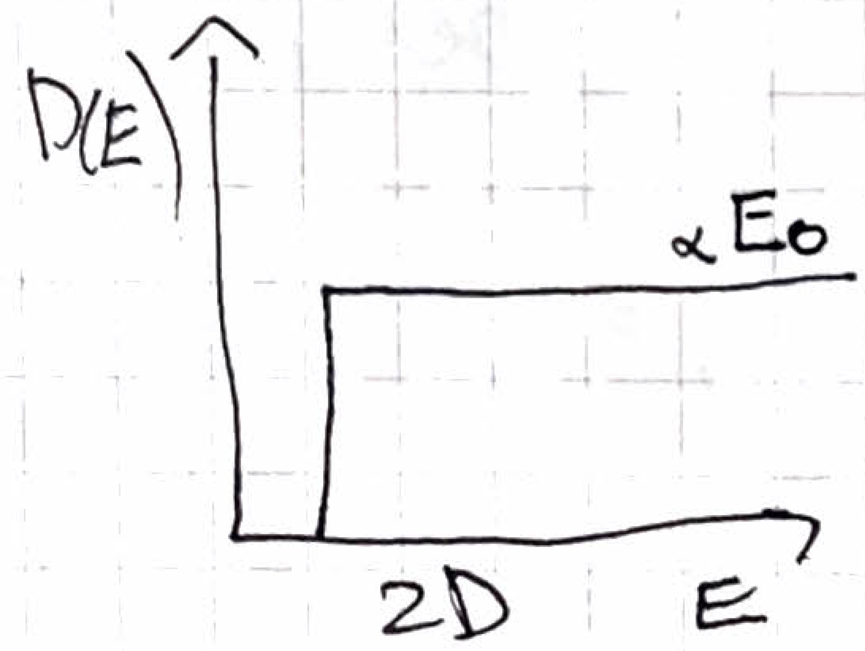
$$D(E)_{1D} = \frac{1}{L}\frac{dN}{dk}\cdot\frac{dk}{dE} =$$

$$= \frac{1}{L}\cdot\frac{2L}{\pi}\cdot\frac{m}{\hbar\sqrt{2mE}} \qquad \left(k = \frac{\sqrt{2mE}}{\hbar}\right)$$

$$D(E) = \frac{\sqrt{2}}{\pi\hbar} m^{\frac{1}{2}} E^{-\frac{1}{2}} = \frac{2\sqrt{2}}{h} m^{\frac{1}{2}} E^{-\frac{1}{2}} \qquad \left(\hbar = \frac{h}{2\pi}\right)$$

$$D(E) \propto E^{-1/2}$$





in 2-dimensional

$$D(E)_{2D} = \frac{1}{A}\frac{dN}{dE} \qquad A = \left(\frac{\pi}{L}\right)^2 \qquad \text{k-space sphere:} \quad A = \pi k^2$$

number of filled states in a sphere:

$$N = \frac{V_{circle}}{V_{single\,state}} \cdot 2\cdot\left(\frac{1}{2}\cdot\frac{1}{2}\right) = \frac{k^2L^2}{2\pi}$$

$$\frac{dN}{dk} = \frac{L^2 k}{\pi} \qquad E = \frac{\hbar^2 k}{2m} \qquad \frac{dE}{dk} = \frac{\hbar^2 k}{m}$$

$$\frac{dk}{dE} = \frac{m}{\hbar^2 k} \qquad D(E) = \frac{1}{L^2}\cdot\frac{L^2 k}{\pi}\cdot\frac{m}{\hbar^2 k} = \frac{m}{\pi\hbar^2} = \frac{4\pi m}{h^2} \qquad \left(\hbar = \frac{h}{2\pi}\right)$$

$$D(E) \propto E^0$$

2) DOS of phons

- in 1D

$$\varepsilon(k) = 2\hbar\omega_0 |\sin(ka/2)|$$

a - lattice const.

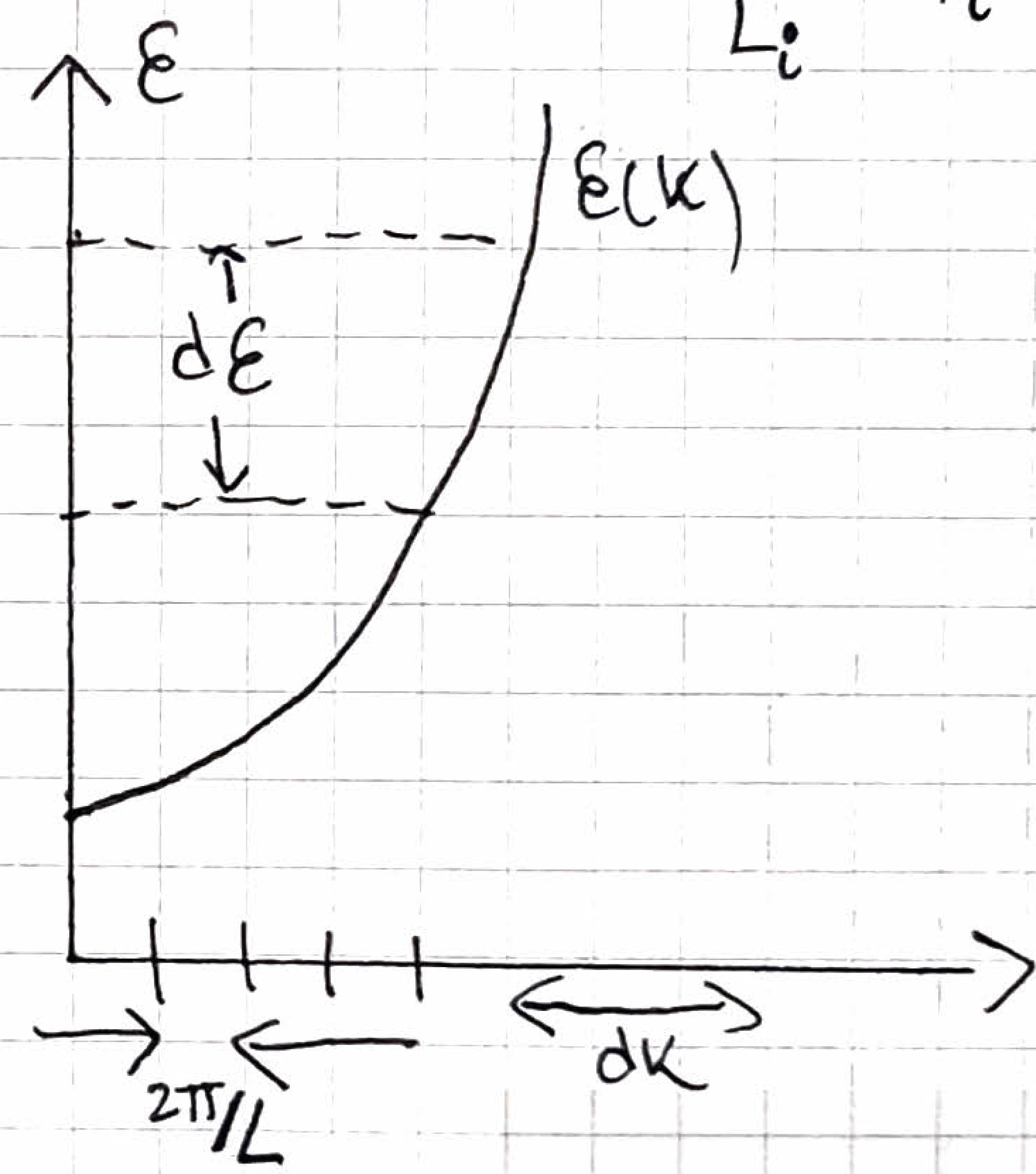
small $k \rightarrow \varepsilon \approx \hbar\omega_0 ka$

$$D(\varepsilon)d\varepsilon = N(k)dk \cdot \frac{1}{L}$$

$N = \frac{2kL}{\pi}$ $\frac{dN}{dk} = \frac{2L}{\pi}$

$$N(k)dk = \frac{dk}{(2\pi/L)} \cdot 2$$

$k_i = \frac{2\pi}{L_i} m_i$



$$k = \sqrt{\frac{2m\varepsilon}{\hbar^2}} \quad dk = \frac{1}{\hbar}\sqrt{\frac{m}{2\varepsilon}}\, d\varepsilon$$

$$D_{1D}(\varepsilon)d\varepsilon = \frac{l}{\pi}dk$$

$$d\varepsilon = \frac{\hbar^2 k dk}{m} \quad dk = \frac{m d\varepsilon}{\hbar^2 k}$$

$$D_{1D}(\varepsilon)d\varepsilon = \frac{1}{\pi\hbar}\sqrt{\frac{m}{2\varepsilon}}\, d\varepsilon$$

Multiply by 2 for negative k-states

$$D_{1D}(\varepsilon)d\varepsilon = \frac{2}{\pi\hbar}\sqrt{\frac{m}{2E}}\, d\varepsilon$$

$$v = \frac{\hbar k}{m} = \sqrt{2E/m}$$

$$D_{1D}(E)d\varepsilon = \frac{2}{\pi\hbar v} d\varepsilon$$

- in 2D

$$\left(-\frac{\hbar^2}{2m}\nabla^2\right)\psi = \varepsilon\psi$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+k^2\psi=0 \qquad k=\sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi(x,y)=\psi_x(x)\,\psi_y(y)$$

$$\frac{1}{\psi_x}\frac{\partial^2\psi}{\partial x^2}+\frac{1}{\psi_y}\frac{\partial^2\psi}{\partial y^2}+k^2=0 \qquad \text{where } k=\text{const}$$

$$\frac{1}{\psi_x}\frac{\partial^2\psi}{\partial x^2}=-k^2 \qquad \frac{1}{\psi_y}\frac{\partial^2\psi}{\partial y^2}=-k^2 \qquad k^2=k_x^2+k_y^2$$

$$\psi=A\sin(k_x x)+B\cos(k_x x)$$

$$K_x=\frac{n_x\pi}{L} \qquad k_y=\frac{n_y\pi}{L} \quad \text{for } n=\pm 1,2,3$$

$$V_{\text{single-state}}=\frac{\pi}{a}\frac{\pi}{b}=\frac{\pi^2}{V}=\frac{\pi^2}{L^2} \qquad V_{\text{sphere}}=\pi k^2$$

$$k=\sqrt{\frac{2mE}{\hbar^2}}$$

$$N=\frac{V_{\text{circle}}}{V_{\text{single-st}}}\cdot 2\left(\frac{1}{2}\cdot\frac{1}{2}\right)$$ number of filled states in a circle

$$N=\frac{\pi k^2}{\pi^2/L^2}\cdot 2\cdot\frac{1}{4}=\frac{k^2L^2}{2\pi} \qquad N=\frac{k^2L^2}{2\pi}$$

$$N=\frac{\left(\sqrt{\frac{2m\varepsilon}{\hbar^2}}\right)^2L^2}{2\pi}=\frac{mL^2\varepsilon}{\hbar^2\pi}$$

$$\frac{dN}{d\varepsilon}=\frac{dN}{dk}\frac{dk}{d\varepsilon}=\frac{L^2m}{\pi\hbar^2}$$

$$D(\varepsilon)_{2D}=\frac{L^2m/\pi\hbar^2}{L^2}=\frac{m}{\pi\hbar^2}$$

$$D(\varepsilon)_{2D}=\frac{m}{\pi\hbar^2}$$