

A Dataset & Signal Analysis Interpretation of Quantum Mechanics

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Sunday, May 18th, 2025

Colloquium (by Zoom)

School of Engineering and Physical Science
North South University, Dhaka

A video of this talk is available on YouTube,
<https://youtu.be/j8s2AgFi9uE>

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a Dataset&Signal Analysis Interpretation of QM—outline

Part I: From the Data to Quantum Mechanics

- Quantum Mechanics and Classical Mechanics — as *Dataset* formalisms NOT about ensembles of particles #4
- Classical Mechanics \rightarrow CM_+ , with (1) *more Poisson* and with (2) quantum noise #6
 - The Measurement Problem ↔ noncommutativity #11
- Transition to Part II: Bell inequalities for **Noisy** classical fields #15

Part II: Quantum Field Theory as Signal Analysis

- A first look at Quantum Field Theory \leftarrow with signal analysis in mind #20
 - **Noisy** classical fields, QNDFT, constructed within QFT Tsang&Caves PRX 2012
QM-Free-Subsystems #24

An evolution of ideas in:

"Classical states, quantum field measurement", [Physica Scripta 2019](#) (arXiv)
"An algebraic approach to Koopman classical mechanics", [Annals of Physics 2020](#) (arXiv)
"The collapse of a quantum state as a joint probability construction", [Journal of Physics A 2022](#) (arXiv)
and, ancient history, "Bell inequalities for random fields", [Journal of Physics A 2006](#) (arXiv)

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Part III: (not today)

[arXiv:2109.04412](https://arxiv.org/abs/2109.04412)

- Nonlinear axioms for QFT&QNDFT \leftarrow signal analysis and renormalization
- Towards Quantum&QND Gravity \leftarrow a measurement theoretic path
A more rigorous way to think about interacting QFT&QG as D&SA

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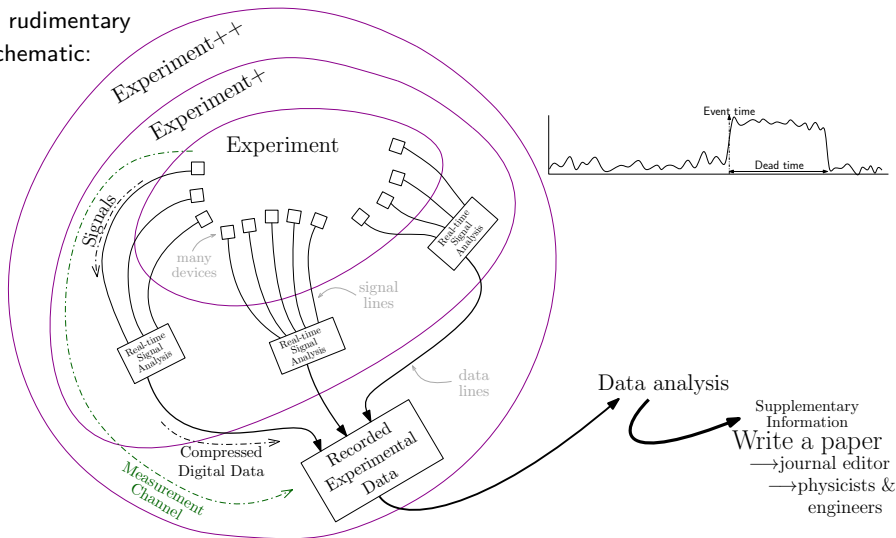
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signal & data analysis — a *dataset* interpretation of QM, *not* an ensemble interpretation

A rudimentary schematic:



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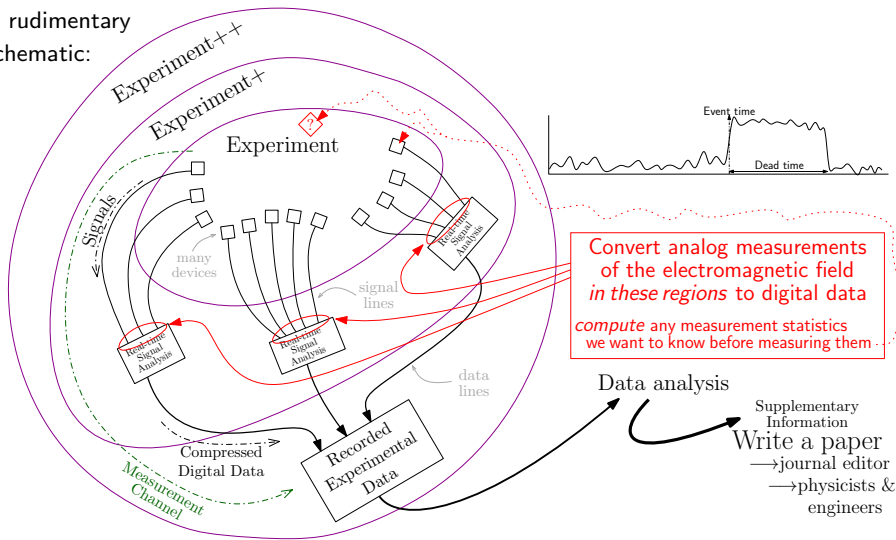
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From an *experiment*, we obtain *datasets*, which contain lists of numbers
→ relative frequencies and other statistics

Something in a theory should generate
expected relative frequencies *et cetera* for future datasets

Anything more ↗ better empirical confirmation,
even though good intuitions about experiments are vital

'Systems' and 'particle properties' can be intuitively both important *and* misleading
In the first instance, Measurements → Datasets \rightsquigarrow particle properties
Particle properties do not belong in a set of axioms

Machine Learning also suggests a focus on datasets

an elementary and concrete matrix model

For a dataset M , with data values m_1, m_2, \dots and relative frequencies p_1, p_2, \dots $\begin{cases} \text{actual} \in \mathbb{Q} \\ \text{expected} \in \mathbb{R} \end{cases}$
the dataset average is $\sum_i p_i m_i$

$$\text{We can write that as } \rho(\hat{M}) = \sum_i p_i m_i = \text{Tr} \left[\begin{pmatrix} p_1 & \cdots & \cdots \\ \vdots & p_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \cdot \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & \ddots \end{pmatrix} \right] = \text{Tr}[\hat{\rho}\hat{M}]$$
$$\hat{\rho} \geq 0, \text{Tr}[\hat{\rho}] = 1 \Rightarrow p_i \geq 0, \sum_i p_i = 1$$

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an elementary and concrete matrix model (of a *Generalized Probability Theory*)

For a dataset M , with data values m_1, m_2, \dots and relative frequencies p_1, p_2, \dots $\begin{cases} \text{actual} \in \mathbb{Q} \\ \text{expected} \in \mathbb{R} \end{cases}$
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$$\hat{\rho} \geq 0, \text{Tr}[\hat{\rho}] = 1 \Rightarrow p_i \geq 0, \sum_i p_i = 1$$

For a different dataset M' , with matrix representation of an operator \hat{M}' diagonal in a different basis, but using the same $\hat{\rho}$,

$$\text{For the average of } M', \rho(\hat{M}') = \sum_i p'_i m'_i = \text{Tr} \left[\begin{pmatrix} p'_1 & \cdots' & \cdots' \\ \vdots & p'_2 & \cdots' \\ \vdots & \vdots & \ddots' \end{pmatrix} \cdot \begin{pmatrix} m'_1 & 0 & 0 \\ 0 & m'_2 & 0 \\ 0 & 0 & \ddots \end{pmatrix} \right] = \text{Tr}[\hat{\rho}\hat{M}']$$
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Multiple probability spaces are contained in a single formal structure

There is *no* physics in this construction — *there is, in particular, no \hbar*

This is just a matrix model of a *Generalized Probability Theory*

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For traditional classical physics, we use different initial conditions for different 'contexts'
Operator algebras give us more tools for modeling *incompatible experiments*

► models of the Kolmogorov axioms appendix

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We can begin “A linear operator \leftrightarrow the data values part of a dataset, $\left(\begin{matrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & \ddots \end{matrix} \right)$ ”,

instead of “A Hilbert space \leftrightarrow a system”, as axioms for QM usually begin

There are *abstract* measurements $\hat{M}_1, \hat{M}_2, \hat{M}_3, \dots, \hat{M}_1 + \hat{M}_2, \dots, \hat{M}_1 \hat{M}_2, \dots$

a dataset data values \leftrightarrow a linear operator eigenvalues \leftrightarrow a random variable outcomes in a sample space, noncommutative \sim quantum or commutative \sim classical, associative, distributive, with unit & scalar \times

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A *state* ρ maps measurement operators to *expected dataset averages when/if we collect data*

$$\rho(\hat{M}_1), \dots, \rho(\hat{M}_1^n), \dots, \rho(\hat{M}_2), \rho(\hat{M}_3), \dots, \rho(\hat{M}_1 + \hat{M}_2), \dots, \rho(\hat{M}_1 \hat{M}_2), \dots,$$

$$\sum_i p_i m_{1i} \quad \sum_i p_i m_{1i}^n$$

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$$\sum_i \rho_i m_{1i} \quad \sum_i \rho_i m_{1i}^n$$

\hookrightarrow a state ρ maps a generating operator $e^{j\lambda \hat{M}_1}$ to a characteristic function $\rho(e^{j\lambda \hat{M}_1})$
 — using the engineer's imaginary, j —
 \hookrightarrow a state ρ maps a Dirac distribution $\delta(\hat{M}_1 - u)$ to a probability distribution $\rho(\delta(\hat{M}_1 - u))$

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Because *relative frequencies* are *positive, normalized, and real-valued*,

$$\rho(\hat{A}^\dagger \hat{A}) \geq 0, \rho(1) = 1, \text{ and } \rho(\hat{A}^\dagger) = \rho(\hat{A})^* \quad \text{where } (\hat{A}\hat{B})^\dagger = \hat{B}^\dagger \hat{A}^\dagger, j^\dagger = -j$$

and because we can *add relative frequencies*,

$$\text{von Neumann linearity: } \rho(\lambda \hat{A} + \mu \hat{B}) = \lambda \rho(\hat{A}) + \mu \rho(\hat{B})$$

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We can also use measurement operators to *modulate* the state ρ to give different expected measurement results,

$$\rho_A(\hat{M}) = \frac{\rho(\hat{A}^\dagger \hat{M} \hat{A})}{\rho(\hat{A}^\dagger \hat{A})}$$

which is the basis of the Gelfand-Naimark-Segal-construction of a Hilbert space

The GNS-construction lets us think of $\rho_\nu(\hat{M})$ as $\langle \nu | \hat{M} | \nu \rangle$ and of $\rho_{A\nu}(\hat{M})$ as $\frac{\langle \nu | \hat{A}^\dagger \hat{M} \hat{A} | \nu \rangle}{\langle \nu | \hat{A}^\dagger \hat{A} | \nu \rangle}$

Take classical mechanics to be an algebra of functions of *position* and *momentum* on phase space that has three binary operations: addition, multiplication, and the Poisson bracket

$$u + v$$

$$u \cdot v$$

$$\{v, u\}$$

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that has *three* binary operations:

addition, multiplication, *and* the Poisson bracket

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We can construct a simpler transformation algebra, with only addition and composition,

by introducing "Multiply by w ", $\hat{Y}_w(u) = w \cdot u$,

and "Poisson by w ", $\hat{Z}_w(u) = \{w, u\}$

$$[\hat{Y}_v, \hat{Y}_w] = 0$$

$$[\hat{Z}_v, \hat{Y}_w] = \hat{Y}_{\{v, w\}} \neq 0$$

$$[\hat{Z}_v, \hat{Z}_w] = \hat{Z}_{\{v, w\}} \neq 0$$

A familiar example: "Poisson by the *Hamiltonian function*" gives

a generator of time evolution, $\hat{Z}_H(u) = \{H, u\}$, the *Liouvillian* operator

We could say that "Poisson by w " gives a generator of w -transformations

Take classical mechanics to be an algebra of functions of *position* and *momentum* on phase space that has three binary operations: addition, multiplication, and the Poisson bracket

$$\begin{aligned} u + v \\ u \cdot v \\ \{v, u\} \end{aligned}$$

We can construct a simpler transformation algebra, with only addition and composition, by introducing "Multiply by w ", $\hat{Y}_w(u) = w \cdot u$, and "Poisson by w ", $\hat{Z}_w(u) = \{w, u\}$

$$\begin{aligned} [\hat{Y}_v, \hat{Y}_w] &= 0 \\ [\hat{Z}_v, \hat{Y}_w] &= \hat{Y}_{\{v, w\}} \neq 0 \\ [\hat{Z}_v, \hat{Z}_w] &= \hat{Z}_{\{v, w\}} \neq 0 \end{aligned}$$

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I suggest:

We can use the \hat{Y} 's and \hat{Z} 's of a more powerful CM_+ without restriction: \hat{Z}_w^n, \dots (see #10)

That gives us an algebraic measurement theory shared with QM, including noncommutativity, measurement incompatibility, ...

We can have isomorphisms instead of quantization and the Correspondence Principle

a state for the classical simple harmonic oscillator

The Poisson bracket: $\{v, u\} = \frac{\partial v}{\partial p} \frac{\partial u}{\partial q} - \frac{\partial v}{\partial q} \frac{\partial u}{\partial p}$

$$\hat{Y}_q[u] = q \cdot u, \quad \hat{Z}_p[u] = \{p, u\} = \frac{\partial}{\partial q} u, \quad [\hat{Y}_q, \hat{Z}_p] = -1$$

Two copies of the abstract Weyl algebra

$$\hat{Y}_p[u] = p \cdot u, \quad \hat{Z}_q[u] = \{q, u\} = -\frac{\partial}{\partial p} u, \quad [\hat{Y}_p, \hat{Z}_q] = 1$$

$$\hat{Y}_H[u] = \frac{1}{2}(q^2 + p^2) \cdot u, \quad \hat{Z}_H[u] = \{H, u\} = \left(p \cdot \frac{\partial}{\partial q} - q \cdot \frac{\partial}{\partial p} \right) u$$

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The *Gibbs thermal state* for an SHO at temperature kT is a Normal probability distribution

$$\rho_{\text{Gibbs}}(\delta(\hat{Y}_q - \alpha)\delta(\hat{Y}_p - \beta)) = e^{-(\alpha^2 + \beta^2)/2kT} / 2\pi kT$$

The Fourier transform of that probability distribution, using \hat{Y}_q and \hat{Y}_p and the engineer's imaginary j , is $\rho_{\text{Gibbs}}\left(e^{j\lambda\hat{Y}_q + j\mu\hat{Y}_p}\right) = e^{-kT(\lambda^2 + \mu^2)/2}$

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$$\rho_{\text{Gibbs}}(\delta(\hat{Y}_q - \alpha)\delta(\hat{Y}_p - \beta)) = e^{-(\alpha^2 + \beta^2)/2kT} / 2\pi kT$$

The Fourier transform of that probability distribution, using \hat{Y}_q and \hat{Y}_p and the engineer's imaginary j , is $\rho_{\text{Gibbs}}(e^{j\lambda\hat{Y}_q + j\mu\hat{Y}_p}) = e^{-kT(\lambda^2 + \mu^2)}/2$

This is now the *Weyl-Heisenberg algebra*, so we can use raising and lowering operators,

$$\text{Set } \hat{Y}_q = (a + a^\dagger)\sqrt{kT}, \quad \hat{Z}_p = \frac{(a - a^\dagger)}{2\sqrt{kT}}, \text{ and } [a, a^\dagger] = 1, \text{ so that } [\hat{Y}_q, \hat{Z}_p] = -1, \text{ and set } a|_{\text{vac}} = 0$$

$$\text{and similarly for } \hat{Y}_p \text{ and } \hat{Z}_q, b|_{\text{vac}} = 0, \text{ etc} \quad \longrightarrow \rho_{\text{Gibbs}}(e^{j\alpha(j\hat{Z}_p) + j\beta(j\hat{Z}_q)}) = e^{-(\alpha^2 + \beta^2)/8kT}, \dots$$

a state for the classical simple harmonic oscillator

The Poisson bracket: $\{v, u\} = \frac{\partial v}{\partial p} \frac{\partial u}{\partial q} - \frac{\partial v}{\partial q} \frac{\partial u}{\partial p}$

$$\hat{Y}_q[u] = q \cdot u, \quad \hat{Z}_p[u] = \{p, u\} = \frac{\partial}{\partial q} u, \quad [\hat{Y}_q, \hat{Z}_p] = -1$$

Two copies of the abstract Weyl algebra

$$\hat{Y}_p[u] = p \cdot u, \quad \hat{Z}_q[u] = \{q, u\} = -\frac{\partial}{\partial p} u, \quad [\hat{Y}_p, \hat{Z}_q] = 1$$

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We can construct modulated, non-equilibrium states, $\frac{\langle_{kT} | \hat{A}^\dagger \hat{M} \hat{A} |_{kT} \rangle}{\langle_{kT} | \hat{A}^\dagger \hat{A} |_{kT} \rangle} \longrightarrow$ a thermal Hilbert space

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Instead of trying to map $(q, p) \not\mapsto (\hat{q}, \hat{p})$, as quantization tries to $\left(\begin{array}{l} \text{but fails} \\ \text{Groenewold(1946)} \\ \{u, v\} \not\mapsto [\hat{u}, \hat{v}] \end{array} \right)$

we can map CM_+ to QM, $(q, j\frac{\partial}{\partial q}) \mapsto (\hat{q}_1, \hat{p}_1)$, $(p, j\frac{\partial}{\partial p}) \mapsto (\hat{q}_2, \hat{p}_2)$,

however \underline{kT} is **not** \hbar : thermal noise is **not** quantum noise

What *is* the difference between quantum and thermal noise?

- \hbar has units of action, whereas kT has units of energy
- In QFT, the quantum vacuum is Lorentz invariant, thermal noise is *not* invariant under Lorentz boosts

This difference of symmetry properties *can* be used in CM_+ with \hbar as an amplitude of a Lorentz invariant noise and with kT as an amplitude of a less invariant thermal noise

▶ quantum noise appendix

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▶ quantum noise appendix

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This gives a new reason to think that we must work with field theories, because we can only *define* the Lorentz group in $1+n$ -dimensions

For CM_+ , $\hbar \rightarrow 0$ is *not* a classical approximation, it's a mean-field approximation

unboundedness of the Hermitian generators of time-like evolution

CM_+ can and should include

- (1) noncommutativity
- (2) quantum noise

→ the measurement theory is the same as for QM

CM_+ is, after all, just Hilbert spaces

see Koopman, 1931!

▶ [history appendix](#)

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unboundedness of the Hermitian generators of time-like evolution

CM₊ can and should include

- (1) noncommutativity
- (2) quantum noise

→ the measurement theory is the same as for QM

CM₊ is, after all, just Hilbert spaces

see Koopman, 1931!

▶ history appendix

but there is a more technical difference:

For QM, the Hamiltonian operator is bounded below → analytic properties

For CM₊, in the Gibbs state of the Simple Harmonic Oscillator,

$j\hat{Z}_H$ is the Hermitian generator of evolution,

$$j\hat{Z}_H = j \left(p \cdot \frac{\partial}{\partial q} - q \cdot \frac{\partial}{\partial p} \right) \not\geq 0$$

→ ~~analytic properties~~

(3) **analyticity** is mathematically useful but we do *not* have it for CM₊

We can say that QM is an analytic form of CM₊

like using the analytic signal or the real signal in signal analysis

$\exp(j\omega t)$

$\cos(\omega t + \varphi)$

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how has Classical Mechanics been *straw-manned*?

For a classical SHO, we are allowed to use $\hat{Z}_p = \frac{\partial}{\partial q}$ to generate translations,

$$e^{-\kappa \hat{Z}_p} \cdot \hat{Y}_q \cdot e^{\kappa \hat{Z}_p} = \hat{Y}_q - \kappa = \hat{Y}'_q,$$

but we are not allowed to use \hat{Z}_p^3 to generate a unitary transformation,

$$e^{-\kappa \hat{Z}_p^3} \cdot \hat{Y}_q \cdot e^{\kappa \hat{Z}_p^3} = \hat{Y}_q - 3\kappa \hat{Z}_p^2 = \hat{Y}_q^{\text{very different, not different}}$$

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Quantum mechanics happily uses \hat{Z}_p^3 to transform to a different experiment

\hat{Z}_p^3 *et cetera can* be useful in the same way for classical mechanics

(as for the elementary matrix model on #4)

If we unlock that superpower for classical mechanics,

we can *steel-man*
classical mechanics

the idea of what an initial condition *is* becomes more complicated

An initial condition for CM_+ contains *many* of CM's initial conditions

[so it requires (quantum) state tomography]

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\leftarrow #1 \rightarrow

For a measurement A, with data values $\mathcal{A} = \{\alpha_m\}$, $\hat{A} = \sum_m \alpha_m \hat{P}_m = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \ddots \end{pmatrix}$, and
a measurement B, with data values $\mathcal{B} = \{\beta_n\}$, $\hat{B} = \sum_n \beta_n \hat{Q}_n$,

For a measurement of A, with state vector $|\psi\rangle$,
we obtain the result α_m with probability $p(\alpha_m) = \langle \psi | \hat{P}_m | \psi \rangle$ and similarly for B
for simple cases, \hat{P}_m is a shorthand for $|\alpha_m\rangle\langle\alpha_m| \rightarrow \langle \psi | \alpha_m \rangle \langle \alpha_m | \psi \rangle = |\langle \psi | \alpha_m \rangle|^2$, which is the Born rule

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For two measurements, of A first, to be followed by B,
 we say that the result α_m “collapses” the state vector,

$$|\psi\rangle \rightarrow |\psi_m\rangle = \frac{\hat{P}_m |\psi\rangle}{\sqrt{\langle \psi | \hat{P}_m | \psi \rangle}}, \text{ which is the Lüders rule,}$$

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then we measure B in that state, so we obtain the result α_m followed by β_n
with *conditional* probability

$$p(\beta_n | \alpha_m) = \langle \psi_m | \hat{Q}_n | \psi_m \rangle = \frac{\langle \psi | \hat{P}_m \hat{Q}_n \hat{P}_m | \psi \rangle}{\langle \psi | \hat{P}_m | \psi \rangle},$$

so the *joint* probability is

$$p(\alpha_m \text{ and } \beta_n) = \langle \psi | \hat{P}_m \hat{Q}_n \hat{P}_m | \psi \rangle.$$

We have $p(\alpha_m \text{ and } \beta_n) = \langle \psi | \hat{P}_m \hat{Q}_n \hat{P}_m | \psi \rangle$,
 so the positive operators $\hat{J}_{mn} = \hat{P}_m \hat{Q}_n \hat{P}_m$ generate
 the joint probabilities $\langle \psi | \hat{J}_{mn} | \psi \rangle$ for the results (α_m, β_n) .

Instead of collapse affecting a state,

we can take collapse to affect the next measurement, $\hat{P}_m \hat{Q}_n \rightarrow \hat{P}_m \cdot \hat{P}_m \hat{Q}_n \hat{P}_m$

We can use the positive operators \hat{J}_{mn} to construct a “collapse product”,
 $A \blacktriangleright B$, with data values $\{(\alpha_m, \beta_n)\}$, even if $[\hat{A}, \hat{B}] \neq 0$

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The existence of a joint probability is traditionally “classical”: we can indeed
 use commuting operators \hat{A}' and \hat{B}' and a *different* vector state $|\psi'\rangle \in \mathcal{H}'$
 that give the same joint probability, $\langle \psi' | \hat{P}'_m \hat{Q}'_n | \psi' \rangle = \langle \psi | \hat{P}_m \hat{Q}_n \hat{P}_m | \psi \rangle$
 new, ‘classical’ ← original, quantum

Mathematically, this is to use the Naimark Dilation Theorem to construct
 a joint measurement \widehat{AB} that is the same as $A \blacktriangleright B$ (for a larger Hilbert space),
 which can be an *alternative* to “Decoherence”

Naimark's Dilation Theorem[‡] as an *alternative* to Decoherence

[‡] 'Naimark' has an alternate spelling, 'Neumark'

Decoherence takes a *large* Hilbert space model
for *whatever is measured* together with *whatever measures it* and
shows that environmental noise will *very nearly* “collapse” the state
to an after-measurement state

For All Practical Purposes Decoherence works well enough

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Naimark is somewhat the opposite

Naimark takes a *small* Hilbert space model for *whatever is measured* and
constructs a joint probability for consecutive measurements
→ a model for a measurement device in a larger Hilbert space
in the context of earlier and subsequent measurements

See also [The Principle of Deferred Measurement](#)

Naimark constructs a rough model and refines it as needed

*as classical physics
and engineers
always did*

We can use Decoherence or Naimark, whichever seems easier, more appropriate, ...

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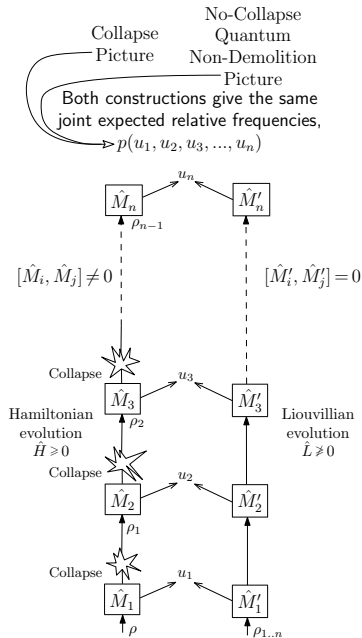
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For signal analysis,
there are *many* measurements
at timelike separation

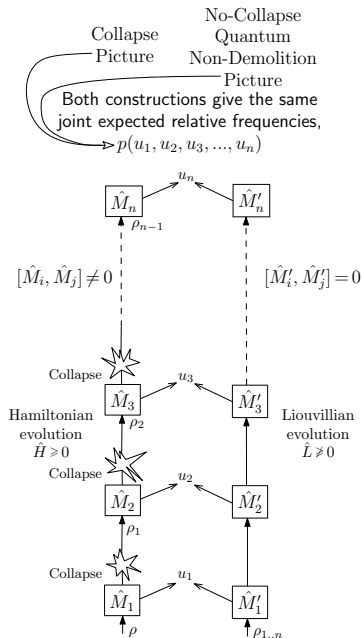
We can use noncommuting $\hat{M}_1, \dots, \hat{M}_{100\dots000}$,
with many collapses,
or we can use $\hat{M}'_1, \dots, \hat{M}'_{100\dots000}$,
which all commute, with no collapses



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there are *many* measurements
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“Collapse” is not
(only or necessarily)
a dynamical process
We can (also) take it to be a
JOINT PROBABILITY
ALGORITHM

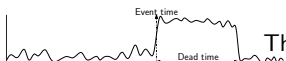
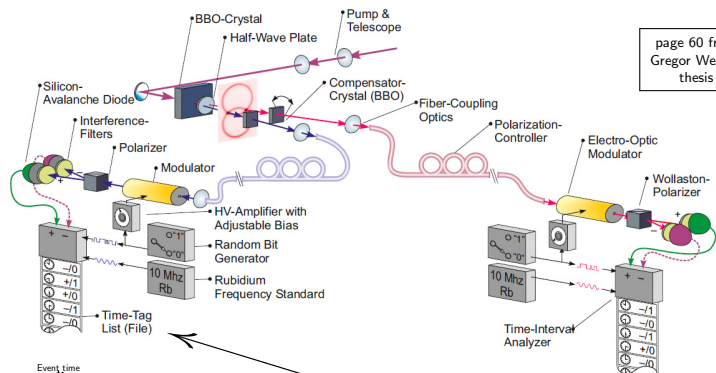


transition to Part II: Bell inequalities — Gregor Weihs's experiment

Think in terms of a **Noisy** classical field and signal analysis, not particles

As for the heat equation, boundary conditions matter differently than they do for particles

A central apparatus **modulates** the ground state; Alice and Bob both have two Avalanche PhotoDiodes, an Electro-Optic Modulator, a Random Bit Generator, and a clock



The time when an APD's signal rises to a higher level is recorded, and which APD it was, and what the EOM setting was: when and 2 bits

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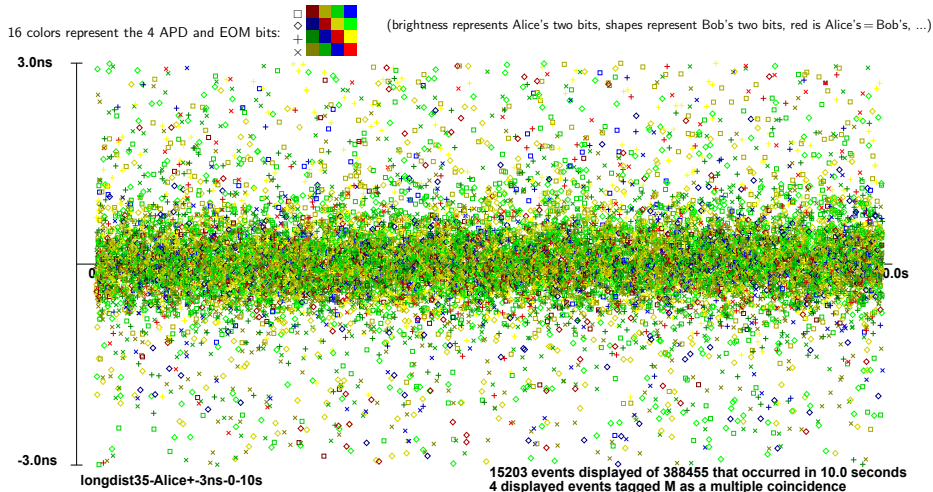
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Gregor gets measurement results I

Alice sees almost 400,000 APD events in 10 seconds
Alice & Bob see over 15,000 coincident event pairs



For over 15,000 of Alice's almost 400,000 events, Bob also records an event within 3 nanoseconds
When Alice and Bob both record an event within 3 nanoseconds, the majority are green or yellow

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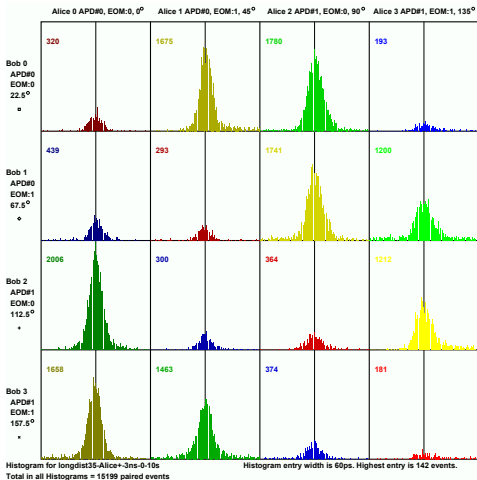
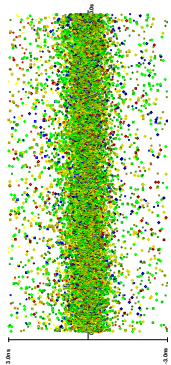
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Gregor gets measurement results II

Alice sees almost 400,000 APD events in 10 seconds
 Alice & Bob see over 15,000 coincident event pairs

16 colors represent the 4 APD and EOM bits:



$$E00 = -0.694$$

$$[320+364, -2006-1780]$$

$$E01 = -0.614$$

$$[439+374, -1658-1741]$$

$$E10 = 0.708$$

$$[1675+1212, -300-193]$$

$$E11 = -0.698$$

$$[293+181, -1463-1200]$$

$$|E00+E01-E10+E11| = 2.714$$

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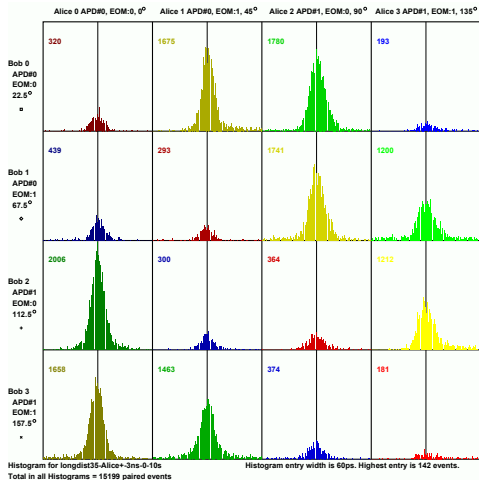
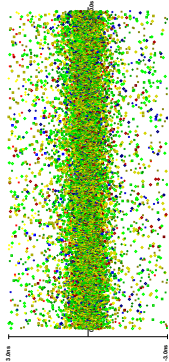
Fermionic fields

#1

Gregor gets measurement results II

Alice sees almost 400,000 APD events in 10 seconds
 Alice & Bob see over 15,000 coincident event pairs

16 colors represent the 4 APD and EOM bits:



$$E00 = -0.694$$

$$[320+364, -2006-1780]$$

$$E01 = -0.614$$

$$[439+374, -1658-1741]$$

$$E10 = 0.708$$

$$[1675+1212, -300-193]$$

$$E11 = -0.698$$

$$[293+181, -1463-1200]$$

$$|E00+E01-E10+E11| = 2.714$$

QM uses noncommuting operators to model Bell-violating statistics

CM has only commuting operators, so it's *computationally incomplete*

But we have added noncommutativity into CM_+ (it's just Hilbert spaces, etc)

[Fine 1982](#) [Landau 1987](#)
 PM, AnnPhys 2020, §7.2

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We engineer the central device so that, when the power is on,
distant *events* (at about ± 400 m, $1.3 \mu\text{s}$ both ways) violate a Bell inequality

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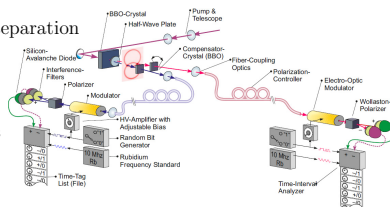
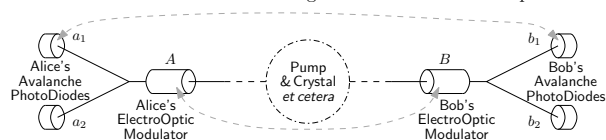
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← #1 →

Events *are* coincident \Rightarrow there *are* significant correlations at space-like separation



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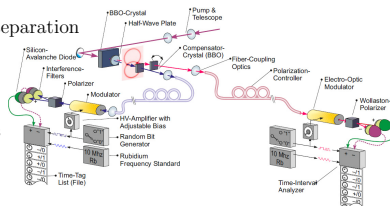
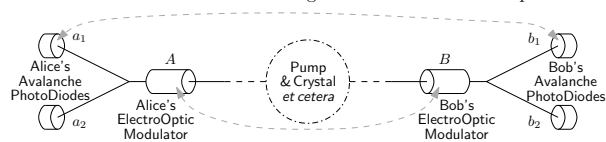
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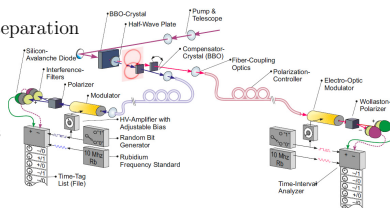
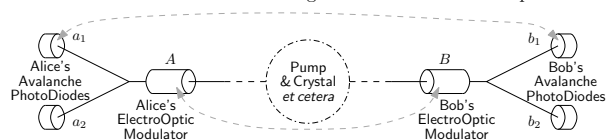
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There *are* correlations between the inner workings of the four APDs

\Rightarrow we should expect correlations between the inner workings of the two EOMs

The EOMs act on the noisy EM field

\Rightarrow the noisy EM field also acts on the inner workings of the two EOMs

With those correlations, Bell inequalities cannot be derived for noisy classical fields

“Bell inequalities for random fields”, PM, [JPhysA 2006](#)

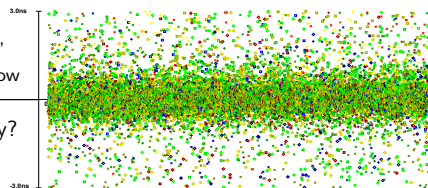
An experiment inspired by a signal analysis perspective

power off
very few
events

lots of
events

lots of
events and
coincidences

lots of events,
coincidences,
and green&yellow



Do events, coincidences, and green&yellow propagate differently?
What changes as we change the distance?
This *probes* nonlocal propagation experimentally

This is technologically important:
after a power failure, how long does it take to be back in operation?

A new interpretation suggests new experiments

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a first look at Quantum Field Theory a nudge towards signal analysis

In QFT textbooks, for a scalar field, $\hat{\phi}(x) = \int [a(k)e^{-jk \cdot x} + a^\dagger(k)e^{jk \cdot x}] \frac{d^4k}{(2\pi)^4}$

$\hat{\phi}(x)$ is *not* a measurement operator, it's an *operator-valued distribution*,

but we construct $\hat{M}_f = \int \hat{\phi}(x)f(x)d^4x$, $f(x)$ is a $\left\{ \begin{array}{c} \text{smearing} \\ \text{test} \\ \text{window} \end{array} \right\}$ function

$f(x)$ tells us where, how big, and what shape the '*f-measurement*' is

The construction is linear, so we can write \hat{M}_f as two components, $\hat{M}_f = a_{f*} + a_f^\dagger$

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For quantum mechanics, we had measurement operators $\hat{M}_1, \dots, \hat{M}_n$

To make contact with experiment, we need *Description*₁, ..., *Description*_n,

Anticipating QFT, we can write $\hat{M}_{\text{Description}_1}, \dots, \hat{M}_{\text{Description}_n}$

For QFT, $\hat{M}_f = \int \hat{\phi}(x)f(x)d^4x$ gives us measurement operators $\hat{M}_{f_1}, \dots, \hat{M}_{f_n}$

The functions f_1, \dots, f_n are formal, idealized *Descriptions*
of how measurements are not point-like

QFT is the same as QM — except that for each measurement
it adds an idealized description of what the measurement does

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For the free quantized Klein-Gordon field, with $\hat{M}_f = a_{f^*} + a_f^\dagger$,

$[a_f, a_g^\dagger] = (f, g)$ is a *pre-inner product*: (f, f) is sometimes zero even if f is non-zero

We can call (f, g) the *overlap* of f and g : $|(f, g)|^2 \leq (f, f)(g, g)$
 $((f, g)$ is given explicitly on #23)

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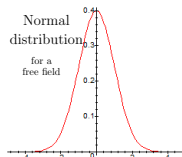
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We can calculate probabilities if all we know is

$$\boxed{\begin{aligned} [a_f, a_g^\dagger] &= (f, g), & [a_f, a_g] &= 0, \\ a_f |v\rangle &= 0, & \hat{M}_f &= a_{f^*} + a_f^\dagger \end{aligned}}$$

Using a Baker-Campbell-Hausdorff identity, $\langle v | e^{j\lambda \hat{M}_f} | v \rangle = e^{-\lambda^2 (f^*, f)/2}$

If $f^* = f$, $\langle v | \delta(\hat{M}_f - u) | v \rangle = \frac{e^{-u^2/2(f, f)}}{\sqrt{2\pi(f, f)}}$ is a Normal probability distribution with variance (f, f)



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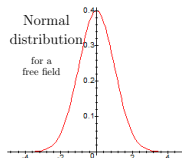
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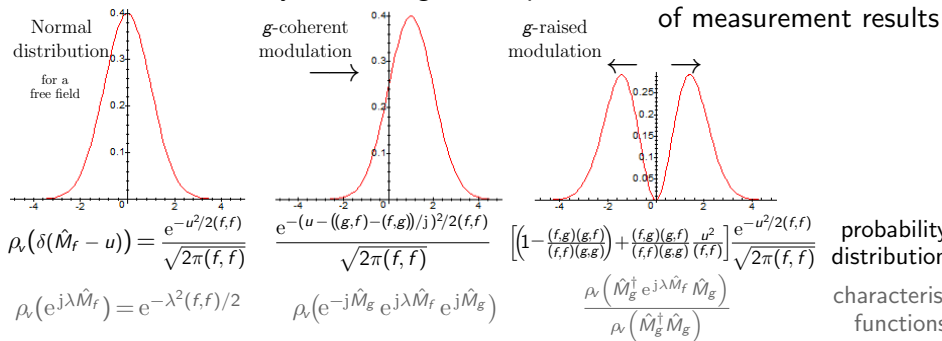
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Modulated states give modulated probability distributions, $\frac{\langle v | \hat{M}_f^\dagger \delta(\hat{M}_f - u) \hat{M}_f | v \rangle}{\langle v | \hat{M}_f^\dagger \hat{M}_f | v \rangle}, \dots$
 and a usefully smaller Hilbert space than the full Fock space
 Quantum Optics is a field theory that uses only a few different window functions

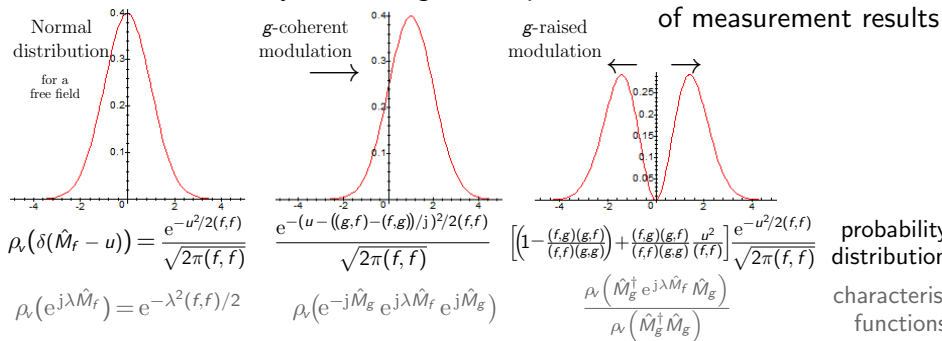
Think of the vacuum state of a quantum field as
a broadband, noisy “carrier signal” for *probabilistic* modulations



$$\hat{M}_f^\dagger = \hat{M}_{f^*} \text{ and } \rho_\nu(\hat{M}_f^\dagger \hat{M}_g) = (f, g)$$

determine the geometric structure of a free quantum field
tell us how a modulation changes measurement results
loosely, g and f describe transmit and receive ‘antennas’
tell us what transmissions the carrier signal supports

Think of the vacuum state of a quantum field as a broadband, noisy “carrier signal” for *probabilistic* modulations



determine the geometric structure of a free quantum field
 tell us how a modulation changes measurement results
loosely, g and f describe transmit and receive ‘antennas’
 tell us what transmissions the carrier signal supports

We can also modulate joint measurements: $\rho_\nu(\hat{A}^\dagger e^{j\lambda_1\hat{M}_{f_1}} e^{j\lambda_2\hat{M}_{f_2}} \dots \hat{A})$
 as Gregor Weihs’s experiment did

Call this a “Wigner-characteristic function”

quantum fields — Gaussian vacuum states

The Wigner-characteristic function gives an effective presentation of the algebraic structure

$$\rho_w(e^{j\lambda_1 \hat{M}_{f_1}} e^{j\lambda_2 \hat{M}_{f_2}} \dots) = \rho_w(e^{\sum_i j\lambda_i \hat{M}_{f_i}}) \exp \left[-\sum_{i < j} \lambda_i \lambda_j [\hat{M}_{f_i}, \hat{M}_{f_j}] / 2 \right]$$

for the Gaussian case

$$= \exp \left[-\sum_{i < j} \lambda_i \lambda_j (f_i^*, f_j) / 2 - \sum_{i < j} \lambda_i \lambda_j [(f_i^*, f_j) - (f_j^*, f_i)] / 2 \right]$$

Gaussian noise term NONCOMMUTATIVITY TERM

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$$= \exp \left[\underbrace{-\sum_{i < j} \lambda_i \lambda_j (f_i^*, f_j) / 2}_{\text{Gaussian noise term}} - \underbrace{\sum_{i < j} \lambda_i \lambda_j [(f_i^*, f_j) - (f_j^*, f_i)] / 2}_{\text{NONCOMMUTATIVITY TERM}} \right]$$

We can fix the geometric & dynamical structure in multiple ways:

$$\text{Klein-Gordon: } (f, g) = \hbar \int \tilde{f}^*(k) \tilde{g}(k) 2\pi \delta(k \cdot k - m^2) \theta(k_0) \frac{d^4 k}{(2\pi)^4}$$

$$\text{Quantum Optics: } (f, g) = -\hbar \int \underbrace{\tilde{f}_{\alpha\mu}^*(k) k^\mu}_{\text{two space-like 4-vectors}} \underbrace{k^\nu \tilde{g}_\nu^\alpha(k)}_{\text{two space-like 4-vectors}} 2\pi \delta(k \cdot k) \theta(k_0) \frac{d^4 k}{(2\pi)^4}$$

both of which are positive-frequency and Lorentz and translation invariant

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In signal analysis, negative frequencies are *not* associated with instability/backward causality
 — Naïvely: for “the real signal” we have $\cos(\omega t + \varphi)$; for “the analytic signal”, $\exp(j\omega t)$

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so remove the “ $\theta(k_0)$ ”, which gives us

an *everywhere commutative*, **Noisy** classical field theory,
with a Lorentz and translation invariant \hbar -scale noise

For the Gaussian state of Quantum Optics, using the (f, g) on #23

$$\left[\begin{array}{l} [a_f, a_g^\dagger] = (f, g), \quad [a_f, a_g] = 0, \\ a_f |v\rangle = 0, \quad \hat{M}_f = a_{f^*} + a_f^\dagger \end{array} \right] \longrightarrow [\hat{M}_f, \hat{M}_g] = (f^*, g) - (g^*, f) \text{ is sometimes non-zero}$$

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For Quantum Optics, we can find an involution $f \mapsto f^\bullet, f^{\bullet\bullet} = f,$

for which $(f^{*\bullet}, g^{\bullet}) - (g^{*\bullet}, f^{\bullet}) = 0,$ for all test functions f and g

$$\text{For } \hat{M}_f^{\text{QND}} = a_{f^{*\bullet}} + a_{f^{\bullet}}^\dagger \neq \hat{M}_{f^{\bullet\bullet}}, \quad [\hat{M}_f^{\text{QND}}, \hat{M}_g^{\text{QND}}] = 0$$

The \hat{M}_f^{QND} generate an effectively classical QND Optics field:
a commutative algebra of Quantum Non-Demolition measurements
and an isomorphic Hilbert space

For the Gaussian state of Quantum Optics, using the (f, g) on #23

$$\boxed{\begin{aligned} [a_f, a_g^\dagger] &= (f, g), & [a_f, a_g] &= 0, \\ a_f |v\rangle &= 0, & \hat{M}_f &= a_{f^*} + a_f^\dagger \end{aligned}} \longrightarrow [\hat{M}_f, \hat{M}_g] = (f^*, g) - (g^*, f) \text{ is sometimes non-zero}$$

For Quantum Optics, we can find an involution $f \mapsto f^\bullet, f^{\bullet\bullet} = f,$

for which $(f^{*\bullet}, g^{\bullet}) - (g^{*\bullet}, f^{\bullet}) = 0,$

For $\hat{M}_f^{\text{QND}} = a_{f^{\bullet}} + a_{f^{\bullet}}^\dagger \neq \hat{M}_{f^{\bullet}},$

$$[\hat{M}_f^{\text{QND}}, \hat{M}_g^{\text{QND}}] = 0$$

for all test functions f and g

The \hat{M}_f^{QND} generate an effectively classical QND Optics field:

a commutative algebra of Quantum Non-Demolition measurements
and an isomorphic Hilbert space

For quantum optics: $\tilde{f}^\bullet(k) = \frac{1}{2}(1+j\star)\tilde{f}(k) + \frac{1}{2}(1-j\star)\tilde{f}(-k)$

$f \mapsto f^\bullet$ is Lorentz invariant but *not* translation invariant or local,

but both Quantum Optics and QND Optics

are Lorentz invariant and translation invariant

- QFT: \hat{M}_f *sometimes* commutes with \hat{M}_g
- QNDFT: \hat{M}_f^{QND} *always* commutes with \hat{M}_g^{QND}

so the algebras generated by the \hat{M}_f^{QND} and by the \hat{M}_f are *not* isomorphic,
but with an *unrestricted* Poisson bracket —or with $\uparrow\downarrow$ -operators or with $|v\rangle\langle v|$ —
anything we can model with Quantum Optics,
we can also model with QND Optics

QNDFT can use generalized probability, when it's needed — it's 'inside' QFT

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QNDFT can use generalized probability, when it's needed — it's 'inside' QFT
Signal Analysis + probability \rightarrow Stochastic Processes
Stochastic Processes + 1+3-dimensions + algebraic *generalized* probability + quantum noise
like de Broglie-Bohm (for QFT), but linear, Lorentz invariant, and more measurement theoretic
NOT like superdeterminism because it's generalized probability at all scales

If we think CM_+ classically, measurement outcomes are a generalized kind of thermodynamic transitions with two types of noise

As for any experiment, a computer does what it does,
whether we *describe* it using QM or CM_+

From a CM_+ perspective, Quantum Computing uses

- quantum noise as a resource and
 - encoding of many initial conditions in a single structure
- as a kind of hardware acceleration for a specific class of algorithms

Existing Quantum Computing textbooks and programming languages are unchanged
QM \rightarrow programming languages as an abstract layer for noisy signal line models
QFT/QNDFT \rightarrow machine languages for modulating quantum and thermal noise

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Actually recorded measurement results (adopt a 'classical' starting point \sim Copenhagen)
→ Data & Signal Analysis (Metadata, window functions, *Measurement Description*, are vital)
Statistics & Expected Statistics (+classical measurement incompatibility)

Classical Mechanics + noncommutativity + quantum noise →
Quantum and Classical_{+/QND} are types of description, not types of system
There is no such thing as “nonclassicality” if we allow CM₊

For CM₊, measurement events are \sim thermodynamic transitions ^{with two types of noise}
“Quantum” just means Lorentz invariant noise can't be ignored

Part III { We can think of renormalization as a surreptitious introduction of nonlinearity into the Wightman axioms
We can construct a more empiricist QG by focusing on metadata
A more rigorous way to think about interacting QFT&QG as D&SA

Quantum and Classical have been
converging, in numerous ways, for decades

Generalized Probability Theories, phase space methods, contextuality, non-demolition measurement, Koopman CM, time-frequency analysis, stochastic methods, semi-classical methods, superdeterminism, causal modeling, [Cohen 1988](#) on characteristic functions, [Abramsky 2020](#) on Boole's “Conditions of Possible Experience”, ...

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Despite how simple the Wightman axioms look, there are no known well-defined interacting models in 1+3-dimensions, after 60+ years

How can we weaken the Wightman axioms? (or introduce something completely different?)

- A Hilbert space \mathcal{H} supports a unitary action of the Poincaré group
There is a unique lowest energy Poincaré invariant vacuum vector $|v\rangle$
- Quantum fields are measurement operator-valued distributions, linear maps $\hat{M}: f \mapsto \hat{M}_f$ from a space of smearing functions into a $*$ -algebra \mathcal{A} of measurements
- Measurements support an action of the Poincaré group, $U(\Lambda)^\dagger \hat{M}_f U(\Lambda) = \hat{M}_{\Lambda(f)}$
- Microcausality: commutativity at space-like separation
- Completeness: the action of the \hat{M}_f operators on $|v\rangle$ generates all of \mathcal{H}
- Time-slice axiom

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QNFT: Allow commutativity at *all* separations, $[\hat{M}_f^{\text{QND}}, \hat{M}_g^{\text{QND}}] = 0$
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- Quantum fields are ~~measurement operator valued distributions, linear~~ maps $\hat{M}: f \mapsto \hat{M}_f$ from a space of smearing functions into a $*$ -algebra \mathcal{A} of measurements
RENORMALIZATION: Allow quantum fields to be *nonlinear* maps into \mathcal{A}
- Measurements support an action of the Poincaré group, $U(\Lambda)^\dagger \hat{M}_f U(\Lambda) = \hat{M}_{\Lambda(f)}$
- ~~Microcausality: commutativity at space-like separation~~
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There are two linearities implicit in the Wightman axioms:

States are linear, $\rho(\lambda\hat{A} + \mu\hat{B}) = \lambda\rho(\hat{A}) + \mu\rho(\hat{B})$, to ensure a probability interpretation

\hat{M}_f is axiomatically linear in $f(x)$, $\hat{M}_f = \int \hat{M}(x) f(x) d^4x$, $\hat{M}_{\lambda f + \mu g} = \lambda\hat{M}_f + \mu\hat{M}_g$

"A source fragmentation approach to interacting quantum field theory", [arXiv:2109.04412](https://arxiv.org/abs/2109.04412)

In signal analysis, with $f(x)$ and $g(x)$ as *window* or *modulation* functions and (f, g) thought of as a *resonance* or as an *impulse–response pairing*, linearity of the field is not *so* obvious that it should be *axiomatic*
We can use the Wightman axioms in linear *or* nonlinear forms

If we work with a more general construction, $\hat{M}_{Description_1}, \dots, \hat{M}_{Description_n}$, there is no reason to insist on linearity

For the Gaussian case, the matrix $D_{ij} = (Description_i, Description_j)$ just has to be positive semi-definite — a structure known as a *Kernel*

This is not a *strong* argument,
because we might want to say that a *fundamental* field *should* be linear

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In support, nonlinearity appears in real-space renormalization:

Suppose $f(x)$ is a *smearing function* on a square region,
which we split into $N = 9$ fragments, then we apply *majority rule*,

$$\begin{array}{|c|c|c|} \hline -1 & +1 & -1 \\ \hline -1 & +1 & -1 \\ \hline -1 & -1 & +1 \\ \hline \end{array} \longrightarrow -1, \quad \hat{M}_f^{[M]} = \epsilon \left[\sum_{j=1}^N \epsilon \left[\hat{M}_{f_j}^{[M]} \right] \right], \quad f = \sum_{j=1}^N f_j^{[M]}$$

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$$\text{Iterated: } \hat{M}_f^{[N_1, \dots, N_k]} = \epsilon \left[\sum_{j=1}^{N_k} \epsilon \left[\hat{M}_{f_j}^{[N_1, \dots, N_{k-1}]} \right] \right],$$

$\hat{M}_f^{[\dots]}$ is nonlinear in $f(x)$ and different from the inaccessible ‘bare’ \hat{M}_f ,
but the ‘dressed’ $\hat{M}_f^{[\dots]}$ is arguably still a “quantum field”

That’s a slightly better argument that
we should allow and expect \hat{M}_f to be nonlinear in $f(x)$

nonlinearity III — the renormalization group

[Hollowood 2013](#): A measurement F is a function $F(\lambda(\mu); \ell)_\mu = F(\lambda(\mu'); \ell)_{\mu'}$, $\mu, \mu' > \ell$
 $\lambda(\mu)$ are interaction parameters, ℓ is a characteristic length scale, and μ, μ' are cutoffs

ℓ is *unacceptably vague*

In more detail, write experimental results as functionals of test functions, $F[\lambda(\mu); f_1, \dots, f_n]_\mu$

Think of f_1, \dots, f_n as a complete description of an experimental apparatus entered into design software, enough to predict experimental results

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The cutoff scale appropriate for an experiment *also* depends on *details*,

$\mu, \mu' > \ell \rightsquigarrow \mu = \mu[f_1, \dots, f_n]$, which fixes an effective dynamics $\lambda(\mu[f_1, \dots, f_n])$

Hence, $\mathbf{F}[f_1, \dots, f_n] = F[\lambda(\mu[f_1, \dots, f_n]); f_1, \dots, f_n]_{\mu[f_1, \dots, f_n]}$

$\mathbf{F}[f_1, \dots, f_n]$ is now a *more* nonlinear functional of f_1, \dots, f_n

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Renormalization:

If we use different cutoff scales, $\mu' = \mu'[f_1, \dots, f_n]$, the experimental results
 $\mathbf{F}'[f_1, \dots, f_n] = F[\lambda(\mu'[f_1, \dots, f_n]); f_1, \dots, f_n]_{\mu'[f_1, \dots, f_n]}$
must be unchanged, $\mathbf{F}'[f_1, \dots, f_n] = \mathbf{F}[f_1, \dots, f_n]$

That's a more complicated argument, but the end result is the same:
we should allow and expect \hat{M}_f to be nonlinear in $f(x)$

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For algebraic QFT, Strong Additivity is a somewhat weaker form of linearity

For two regions $\mathcal{O}_1, \mathcal{O}_2$, the algebra $\mathcal{A}(\mathcal{O}_1 \cup \mathcal{O}_2)$ is generated by

sums of many products of factors such as $\hat{M}_{f_1} + \hat{M}_{f_2}$, $\hat{M}_{f_1} \in \mathcal{A}(\mathcal{O}_1)$, $\hat{M}_{f_2} \in \mathcal{A}(\mathcal{O}_2)$

This does *not* require ' $f_1 + f_2$ ' to be defined for measurement identifiers f_1, f_2

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This does *not* require ' $f_1 + f_2$ ' to be defined for measurement identifiers f_1, f_2

Laura Ruetsche, in an article “Locality in (Axiomatic) Quantum Field Theory”,
points out that Strong Additivity is

in [The Routledge Companion to Philosophy of Physics](#)

“a precise way of framing in algebraic terms the idea that
the whole is not greater than the sum of its parts.”

Note that this is ‘measurement non-holism’ $\not\Rightarrow$ ‘what-is-measured non-holism’, but
neither ‘measurement holism’ nor ‘what-is-measured holism’

should be excluded *axiomatically*,

so, again, we should allow and expect \hat{M}_f to be nonlinear in $f(x)$,

the whole point of which is that $\hat{M}_{\lambda f_1 + \mu f_2}$ is a *new* measurement

that cannot be expressed as a linear combination of \hat{M}_{f_1} and \hat{M}_{f_2}

nonlinearity ∇ — multilinearity

If \hat{M}_f is nonlinear in f , we can construct multi-point operator-valued distributions,

$$\hat{M}(x_1, x_2) = \frac{\delta}{\delta f(x_1)} \frac{\delta}{\delta f(x_2)} \hat{M}_f \Big|_{f(x_1)=f(x_2)=0}$$

see [Polyzou et al](#) for this kind of structure used for bound states

If \hat{M}_f is not expressible as a functional Taylor series (e.g. if we used $\hat{\phi}_{\tanh(f)}$), test functions may wrap around singular points (e.g. at $f(x) = j\frac{\pi}{2}$), potentially resulting in discrete structure

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László&Tercsay, ClassQuantumGrav 2024, [“On the running and the UV limit of Wilsonian renormalization group flows”](#), presents a complicated argument that

page 5: “the space of rescaled correlators” is “similar to that of n -variate distributions” which becomes a surjective map

page 8: “for the special case of bosonic fields” on a “flat spacetime”

Statement (A): over generic spacetime manifolds, the space of rescaled correlators $z(C)^n \mathcal{G}_C^{(n)}$ ($C \in \{\text{coarse-grainings}\}$) of these flows form a topological vector space, which is Hausdorff, locally convex, complete, nuclear, semi-Montel and Schwartz. That is, they form a generalized function space having favorable properties similar to that of n -variate distributions.

The above theorem proves **statement (A)** in section 1. As seen, the topological vector space W_n has rather similar properties to the space of ordinary distributions $\mathcal{D}_n^{\times \ell}$. One may conjecture that $j[\mathcal{D}_n^{\times \ell}] \subset W_n$ saturates W_n . For the generic case, we were unable to construct a proof for this claim. However, for the special case of bosonic fields over affine spaces (flat spacetime), this surjectivity property is proved in the following section.

Given the reasoning on previous slides, let's just use nonlinearity

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Renormalization *suggests* nonlinearity

A conventional Lagrangian deformation $S[\hat{M}]$, from elementary textbooks, is applied to a free Wightman field,

$$\mathcal{Z}[\mathbf{j}] = \langle \nu | \hat{Z}_{\mathbf{j}} | \nu \rangle = \langle \nu | \mathsf{T} \left[e^{j \hat{M}'_{\mathbf{j}}} \right] | \nu \rangle = \frac{\langle \nu | \mathsf{T} \left[e^{j S[\hat{M}] + j \hat{M}_{\mathbf{j}}} \right] | \nu \rangle}{\langle \nu | \mathsf{T} \left[e^{j S[\hat{M}]} \right] | \nu \rangle}$$

A Lorentz invariant generating functional $\mathcal{Z}[\mathbf{j}]$ is not well-defined

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A Lorentz invariant generating functional $\mathcal{Z}[\mathbf{j}]$ is not well-defined

The Reeh-Schlieder theorem asserts that for a free Wightman field *local operators acting on the vacuum vector $|\nu\rangle$ can approximate any vector*

→ for any regularization for which the vector $\hat{Z}_{\mathbf{j}}^{\text{regularized}} |\nu\rangle$ is of finite norm, we can approximate that vector by a vector $\hat{Z}^{\text{local, nonlinear}}[\hat{M}_{F_{\kappa}}[\mathbf{j}]] |\nu\rangle$ that is constructed **using only nonlinear, local functionals of \mathbf{j}**

Renormalization *suggests* nonlinearity

A conventional Lagrangian deformation $S[\hat{M}]$, from elementary textbooks, is applied to a free Wightman field,

$$\mathcal{Z}[\mathbf{j}] = \langle v | \hat{Z}_{\mathbf{j}} | v \rangle = \langle v | \mathsf{T} \left[e^{j \hat{M}'_{\mathbf{j}}} \right] | v \rangle = \frac{\langle v | \mathsf{T} \left[e^{j S[\hat{M}] + j \hat{M}_{\mathbf{j}}} \right] | v \rangle}{\langle v | \mathsf{T} \left[e^{j S[\hat{M}]} \right] | v \rangle}$$

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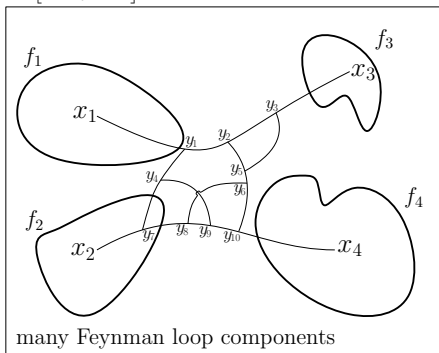
→ for any regularization for which the vector $\hat{Z}_{\mathbf{j}}^{\text{regularized}} |v\rangle$ is of finite norm, we can approximate that vector by a vector $\hat{Z}^{\text{local, nonlinear}}[\hat{M}_{F_{\kappa}}[\mathbf{j}]] |v\rangle$ that is constructed using only nonlinear, local functionals of \mathbf{j}

Renormalization introduces mass/length rescaling, which we can accommodate by using a collection of different fields $\hat{M}_{F_{\kappa}}^{(\kappa)}$ for different functionals $F_{\kappa}[\mathbf{j}]$

a reinvention of interacting QFT II — nonlinearity is *enough*

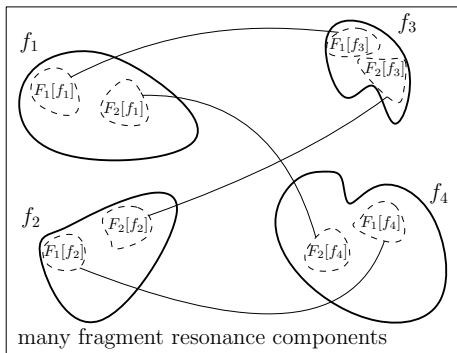
The physics $\approx \hat{Z}_j^{\text{renormalized}}(\mu) |v\rangle$

$\rightarrow \langle v | T[\hat{M}'_4 \hat{M}'_3 \hat{M}'_2 \hat{M}'_1] | v \rangle$, etc as sums of loop integrals



$\approx \hat{Z}^{\text{local, nonlinear}}(\mu) [\hat{M}_{F_\kappa}^{(\kappa)}[j]] |v\rangle$

\rightarrow sums of products of nonlinear overlaps $(F_\kappa[f_i], F_\kappa[f_j])_\kappa$



This is an inverse problem: find local, nonlinear fragment functionals $F_1[\cdot], F_2[\cdot], \dots$

and measurement operators $\hat{M}_{F_1[j]}^{(1)}, \hat{M}_{F_2[j]}^{(2)}, \dots$

that locally approximate what happens in the bulk for path integrals,
which works because of the Reeh-Schlieder theorem

The interplay between local and global is similar to holography

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An analysis of renormalization and of path integrals suggests we *should* introduce nonlinearity and that it's *enough*

$\rho'_\nu(e^{j\lambda_1\hat{M}'_{f_1}}e^{j\lambda_2\hat{M}'_{f_2}}\dots) = \text{some manifestly Poincaré invariant functional of the } \lambda_i \text{ and } f_i^\ddagger$

We want to construct or decompose this functional in ways that give us good intuitions about different experiments

or

We might want to use a mathematics much more elaborate than the Wigner-characteristic function ρ'_ν

\ddagger This can be compared with “[The Nonlinear Program](#)” but using only local functionals

two tentative constructions

- (1) We can construct a nonlinear Gaussian field, as a first step, for which the 2-measurement VEV is nonlinear,

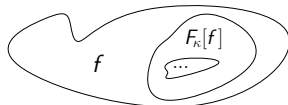
$$\langle v | \hat{M}_f^\dagger \hat{M}_g | v \rangle = (f, g) + \sum_{\kappa} (F_{\kappa}[f], F_{\kappa}[g])_{\kappa},$$

using a sum of many binary relationships
instead of a sum over many paths

where $\text{Supp}[F_{\kappa}[f]] \subseteq \text{Supp}[f]$
for example $f^2, \tanh(f), f \cdot (H \star f), \dots$

- (2) We can construct a recursive “cloud” of measurement operator *fragments*, within the support of f , as an iterated solution of an integral equation such as

$$\hat{M}_f = \hat{\phi}_f - \int [\hat{M}_{F_{\kappa}[f]}]^3 d\mu(\kappa)$$



which integrates over different fragments $F_{\kappa}[\cdot]$ instead of over points in space-time

There are other possibilities, ...

a concise list of the difficulties of QFT

from [Oldofredi&Öttinger 2022](#):

- 1 Divergences (rethinking renormalization eliminates divergences)
- 2 No precise ontological picture (signal analysis & classical measurement incompatibility)
- 3 No particles (nonlinearity & dispersion \rightarrow solitons & [caustics](#)?)
- 4 Haag's theorem[‡] (nonlinearly constructed subalgebras)
- 5 The measurement problem (rethink collapse as a joint probability construction)

[‡] Representations of the free field and of interacting fields are unitarily inequivalent
see [Freeborn, Gilton, and Mitsch, "How Haag-tied is QFT, really?"](#)

towards Quantum Gravity I

Given measurements $\hat{M}_{Description_1}, \dots, \hat{M}_{Description_n}$,
all we need for us to be able to construct a Gaussian state
is one positive semi-definite $n \times n$ overlap matrix $D_{ij} = (Description_i, Description_j)$

Suppose we have a successful deformed Optics, $\rho'_v(e^{j\lambda_1 \hat{M}'_{f_1}} e^{j\lambda_2 \hat{M}'_{f_2}} \dots)$,
which combines many Kernels $(f_i, f_j)_\kappa$ in such a way that ρ'_v is a state

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which combines many Kernels $(f_i, f_j)_\kappa$ in such a way that ρ'_v is a state

For QG, we can introduce a new *description*, $D_i = (f_i, B_i)$, for modulations/measurements,
 $D =$ (a bivector test function, a background we want to impose (metric | torsion | non-metricity))

then wherever the Kernel $(f_i, f_j)_\kappa$ occurs in $\rho'_v(e^{j\lambda_1 \hat{M}'_{f_1}} e^{j\lambda_2 \hat{M}'_{f_2}} \dots)$,
we substitute a new Kernel $(D_i, D_j)_\kappa$,

giving a *candidate* Optics/QG, $\rho_v^{(G)}(e^{j\lambda_1 \hat{M}_{D_1}} e^{j\lambda_2 \hat{M}_{D_2}} \dots)$

Given measurements $\hat{M}_{Description_1}, \dots, \hat{M}_{Description_n}$,
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Suppose we have a successful deformed Optics, $\rho'_v(e^{j\lambda_1 \hat{M}'_{f_1}} e^{j\lambda_2 \hat{M}'_{f_2}} \dots)$,
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For QG, we can introduce a new *description*, $D_i = (f_i, B_i)$, for modulations/measurements,
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This gives us a measurement theoretic approach to gravity
that is not about 'quantizing gravity', 'gravitizing quantum', or 'gravitons',
nor about lattices, loops, strings, Cellular Automata, Causal Dynamical Triangulations, twistors, ...

A very minimal axiom set for Quantum or QND Gravity:

1. Measurement operators $\hat{M}_{g_1}, \hat{M}_{g_2}, \dots, \hat{M}_{g_N}$ are *nonlinear* in the *Measurement Descriptions* g_i
2. A *manifestly Diffeomorphism* invariant state \longrightarrow GNS-construction of a Hilbert space
- 3a. QG: Spectrum Condition and Microcausality: $[\hat{M}_f, \hat{M}_g] = 0$ if f and g are causally separated
- 3b. QNDG: Universal measurement compatibility: $[\hat{M}_f^{\text{QND}}, \hat{M}_g^{\text{QND}}] = 0$ for all f and g
When discussing global properties, transformations, and classical measurement incompatibility, we can use a Poisson bracket, \updownarrow -operators, state projection operators, ...

- 3a. the Spectrum Condition and Microcausality both seem difficult to get right for nontrivial geometry
- 3b. makes \hbar *only* about the amplitude of a Poincaré invariant noise, not also about measurement incompatibility

In these days of LLMs, 'invariance' may be about more than diffeomorphisms

A *Measurement Description* might be in Chinese,
extracted automatically from a journal article

We should take *some* notice of partial, informal, inaccurate, imprecise information

The coordinate systems we use for *Measurement Descriptions* will be continuous
because we can always *imagine* a new measurement *between* previous measurements,
independently of whether the real world is discrete or continuous

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A Dataset & Signal Analysis Interpretation of Quantum Mechanics, Dhaka NSU, May 18th, 2025

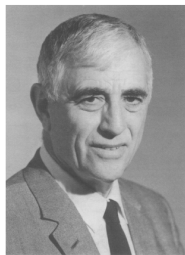
Classical and quantum models are based on the same experimental datasets, but various no-go theorems show that modeling by Classical Mechanics fails for some cases. Elementary signal analysis is less constrained than traditional CM, however, insofar as it does not assume that the data reveals particle properties and it routinely uses Fourier and other integral transforms and Hilbert spaces, making it a helpful intermediary.

This talk will show how we can extend CM to include noncommutativity (spoiler #1: we can do that by using the Poisson bracket), making it as unconstrained as signal analysis. If we also introduce quantum noise as something different from thermal noise (spoiler #2: it's Lorentz invariant; thermal noise isn't), we obtain what I call ' CM_+ '. That lets us rethink the measurement problem as a way to construct joint measurement probabilities and rethink Bell inequalities as about a noisy classical field that is isomorphic to an aspect of Quantum Field Theory, here understood as an analytic noisy signal analysis formalism.

The talk is based on articles in *Physica Scripta* (2019), "Classical states, quantum field measurement"; *Annals of Physics* (2020), "An Algebraic Approach to Koopman Classical Mechanics"; and *Journal of Physics A* (2022), "The collapse of a quantum state as a joint probability construction".



Columbia archives



Bernard Osgood Koopman

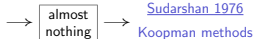
Vol. 17, 1931 *MATHEMATICS: B. O. KOOPMAN* 315
HAMILTONIAN SYSTEMS AND TRANSFORMATIONS IN HILBERT SPACE
 By B. O. KOOPMAN
 DEPARTMENT OF MATHEMATICS, COLUMBIA UNIVERSITY
 Communicated March 23, 1931

In recent years the **theory of Hilbert space** and its linear transformations has come into prominence.¹ It has been recognized to an increasing extent that many of the most important departments of mathematical physics can be subsumed under this theory. In classical physics, for example in those phenomena which are governed by linear conditions—linear differential or integral equations and the like, in those relating to harmonic analysis, and in many phenomena due to the operation of the laws of chance, the essential rôle is played by certain linear transformations in Hilbert space. And the importance of the theory in quantum mechanics is known to all. It is the object of this note to outline certain investigations of our own in which the domain of **this theory has been extended in such a way as to include classical Hamiltonian mechanics**, or, more generally, systems defining a steady n -dimensional flow of a fluid of positive density.

“Questions of ergodicity had been in the foreground for many years and had attracted the attention of powerful mathematicians. Koopman was well versed in this domain and had discussed it with both Birkhoff and von Neumann. In March of 1931, Koopman published a note in the National Academy Proceedings, transforming the problem into one dealing with one parameter unitary groups in Hilbert space.”

Von Neumann quickly proved the ergodic theorem in a Hilbert space sense and Birkhoff established point-wise convergence almost everywhere.

“In Memoriam: Bernard Osgood Koopman, 1900-1981”, [Operations Research 1982](#).



Earlier, Carl Eckart, “Operator Calculus and the solution of the equations of quantum dynamics”, [Heilbron&Rovelli 2023](#), [Phys. Rev. 1926](#)

Three ways to compare QM and CM: Wigner functions, Hilbert spaces, or algebraic
Koopman Eckart

models of the Kolmogorov axioms

From a Data Analysis perspective, *for a dataset, not* for an ensemble of ‘systems’,

A Relative Frequency is an integer ratio $\frac{n}{N(C)}$, $0 \leq n \leq N(C)$ in a context C

Adding RFs in the same context with no double counting \rightarrow a new RF

Adding RFs in the same context with no double counting and no omissions $\rightarrow 1$

We use sample spaces & σ -algebras to keep track of double counting and omission

Historically, the Kolmogorov axioms followed the mathematics of relative frequencies,

RFs give us a model of the Kolmogorov axioms over \mathbb{Q}

Probabilities give us a model of the Kolmogorov axioms over \mathbb{R}

We can use algebraic models of the Kolmogorov axioms over \mathbb{R}

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models of the Kolmogorov axioms

From a Data Analysis perspective, *for a dataset, not* for an ensemble of ‘systems’,

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We can use algebraic models of the Kolmogorov axioms over \mathbb{R}

Generalized Probability gives us ways to interpolate between different contexts

From $\frac{n_1(C_1)}{N(C_1)}$, $\frac{n_2(C_1)}{N(C_1)}$, ..., $\frac{n_1(C_2)}{N(C_2)}$, ..., $\frac{n_1(C_K)}{N(C_K)}$, ... and by *placing* C_{K+1}

relative to earlier contexts, we can infer expected values for new RFs $\frac{n_i(C_{K+1})}{N(C_{K+1})}$

Generalized Probability Theory gives us more tools

[▶ elementary matrix model page](#)

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In QFT, the quantum vacuum is Poincaré invariant, thermal noise is not

$$\langle v | \hat{\phi}(x) \hat{\phi}(y) | v \rangle = \hbar \int \frac{d^4 k}{(2\pi)^4} 2\pi \delta(k \cdot k - m^2) \theta(k_0) e^{-jk \cdot (x-y)} \quad [\text{real K-G QFT}]$$

$$\rho_{kT}(\hat{\phi}(x) \hat{\phi}(y)) = \hbar \int \coth\left(\frac{\hbar k_0}{2kT}\right) 2\pi \delta(k \cdot k - m^2) \theta(k_0) e^{-jk \cdot (x-y)} \frac{d^4 k}{(2\pi)^4}$$

A thermal state for a qSHO is just *extra noise* because \hat{H} is the number operator,

$$[a, a^\dagger] = 1, \quad \hat{H} = a^\dagger a, \quad \hat{M} = a + a^\dagger, \quad \hat{M}' = j(a^\dagger - a)/2, \quad [\hat{M}, \hat{M}'] = j$$

$$\rho_v(e^{j\lambda \hat{M}}) = e^{-\lambda^2/2}$$

$$\rho_{kT}(e^{j\lambda \hat{M}}) = \frac{\text{Tr}[e^{j\lambda \hat{M}} e^{-a^\dagger a/kT}]}{\text{Tr}[e^{-a^\dagger a/kT}]} = \frac{\text{Tr}[e^{j\lambda a^\dagger} e^{j\lambda a} e^{-a^\dagger a/kT}]}{\text{Tr}[e^{-a^\dagger a/kT}]} e^{-\lambda^2/2} = \dots \quad \text{see this URL}$$

$$= \exp\left[-\lambda^2/2 - \lambda^2 \sum_{n=1}^{\infty} e^{-n/kT}\right] = \exp\left[-\lambda^2/2 - \frac{\lambda^2 e^{-1/kT}}{1 - e^{-1/kT}}\right]$$

$$= e^{-\coth(1/2kT) \lambda^2/2}$$

▶ quantum noise page

measurement incompatibility and quantum noise as a cause

If \hat{M}_f and \hat{M}_g are ^{space-like}_{causally} separated, they are *compatible* and $\frac{\rho(\hat{M}_f \hat{M}_g)}{\sqrt{\rho(\hat{M}_f^2)\rho(\hat{M}_g^2)}}$ is a correlation,
otherwise they are *incompatible* and $\frac{\rho(\hat{M}_f \hat{M}_g)}{\sqrt{\rho(\hat{M}_f^2)\rho(\hat{M}_g^2)}}$ has an imaginary component
The imaginary part is non-zero only at time-like separation

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measurement incompatibility and quantum noise as a cause

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The imaginary part is non-zero only at time-like separation

We use *collapse* to construct correlations at time-like separation, *but* we can also construct a **QNDFT** that is empirically equivalent, for which \hat{M}_f^{QND} and \hat{M}_g^{QND} are *always* compatible (then we add transformations to obtain noncommutativity)

QFT folds into the formalism *only* the quantum noise aspect of causality, with amplitude \hbar , leaving other aspects of causality as separate concerns

For QG, causality may be dynamic, so a QNDFT *convention* may be better

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the two-slit experiment

For the two-slit experiment, we use a number of devices that are *designed* so that events happen in them from time to time, randomly, so we report statistics of the events in those devices

If we move other apparatus to change the surroundings of each device — such as a sheet that has two slits cut through it — the statistics of the events in each device will change because the **Noisy** field will ‘bounce around’ everything differently

There do not have to be any ‘particles’ causing the events in our devices, we engineer different modulations of the **Noisy** field near each device

Nonetheless, thinking about particles and particle properties can sometimes be helpful

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the super-Heisenberg picture

	Apply unitary evolution to the state	Apply collapse to the state	Apply unitary evolution to measurements	Apply collapse to measurements
Schrödinger picture	×	×		
Heisenberg picture		×	×	
super-Heisenberg picture			×	×

This is not a unitary equivalence, but it is an empirical equivalence because we can model the same expected joint statistics

I find this *somewhat* satisfying as a unification of *sorts*, insofar as we can also call it the “*Bohr picture*”, because it’s rather classical and, for Bohr, measurements affect other measurements[†] or the “*QND picture*” or the “*Consistent Histories picture*”, because it’s commutative or the “*Everett picture*”, because it’s no-collapse (but branching is not *necessary*) or the “*Einstein picture*”, because it’s rather classical (but with a Poincaré invariant noise)

[†][Howard 2004](#)

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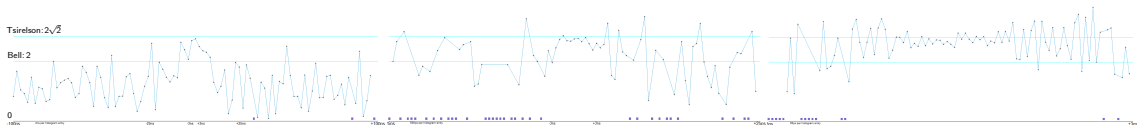
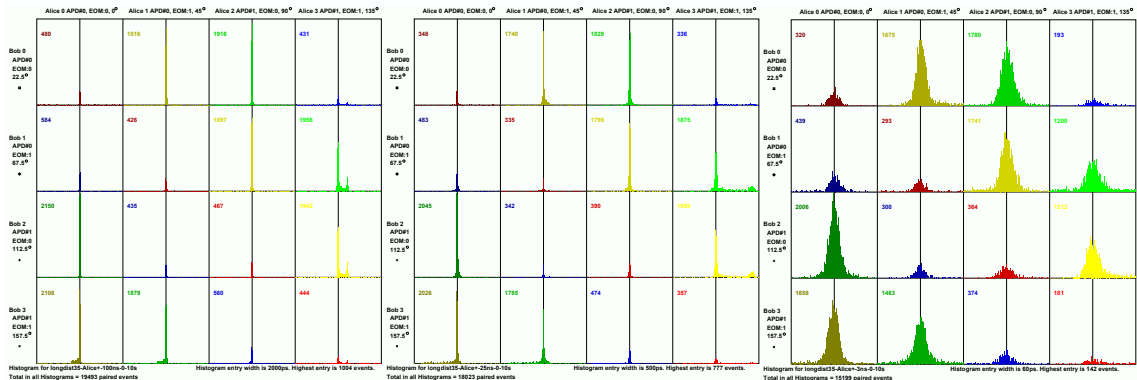
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Gregor's measurement results: more analysis



To the right: ± 3 nanosecond histograms ; below it: the Bell-CHSH inequality calculated for every bar of the 16 histograms
 At center: ± 25 nanosecond histograms — with a subsidiary peak at about ± 20 nanoseconds
 To the left: ± 100 nanosecond histograms to know why we would have to have the apparatus available, to *debug* it

▶ whole apparatus conditioning page

◀ #1 ▶

QND Optics derivations

We use the Hodge dual, $\star, \star^2 = -1$, to construct a pair of helicity projection operators

$$H_{\pm} = \frac{1}{2}(1 \pm j\star), \quad H_{\pm}^2 = H_{\pm}, \quad H_+H_- = H_-H_+ = 0, \quad H_{\pm}^* = H_{\mp}$$

The Quantum Optics inner products for positive and negative frequencies, $(f, g)_{\pm}$, are diagonal in the helicity components, $(f, g)_{\pm} = (H_+f, H_+g)_{\pm} + (H_-f, H_-g)_{\pm}$

For the complex conjugate, $(f^*, g^*)_{\pm} = (g, f)_{\mp}$

We write the *bullet involution* as $f^{\bullet} = H_+f + H_-f^-$

$$\begin{aligned} \tilde{f}^-(k) &= \tilde{f}(-k) \\ (f^-, g^-)_{\pm} &= (f, g)_{\mp} \end{aligned}$$

$$\begin{aligned} \text{Hence, } \langle v | \hat{M}_f^{\text{QND}} \dagger \hat{M}_g^{\text{QND}} | v \rangle &= \langle v | a_{f^{\bullet}} a_{g^{\bullet}}^{\dagger} | v \rangle = (f^{\bullet}, g^{\bullet})_+ = (H_+f^{\bullet}, H_+g^{\bullet})_+ + (H_-f^{\bullet}, H_-g^{\bullet})_+ \\ &= (H_+f, H_+g)_+ + (H_-f^-, H_-g^-)_+ \\ &= (H_+f, H_+g)_+ + (H_-f, H_-g)_-, \end{aligned}$$

which is diagonal in positive frequency, positive helicity and
negative frequency, negative helicity components

$$\begin{aligned} \text{Similarly, } (f^{*\bullet}, g^{\bullet})_{\pm} &= (H_+f^*, H_+g)_{\pm} + (H_-f^*, H_-g)_{\mp} \\ &= (H_+f^*, H_+g)_{\pm} + ((H_+f)^*, H_-g)_{\mp} \\ &= (H_+f^*, H_+g)_{\pm} + ((H_-g)^*, H_+f)_{\pm} \\ &= (H_+f^*, H_+g)_{\pm} + (H_+g^*, H_+f)_{\pm} = (g^{*\bullet}, f^{\bullet})_{\pm}, \text{ so that } [\hat{M}_f^{\text{QND}}, \hat{M}_g^{\text{QND}}] = 0 \end{aligned}$$

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Test function space

Many engineering and physics applications do not need or want the mathematical complexity of a representation of the Poincaré group

Using only a small number of test functions is a commonplace in Quantum Optics
An axiomatic approach should not *exclude* a standard engineering practice
Manifest Poincaré invariance can be enough

Use a finite set of test functions as a starting point,
 $\mathcal{A}(\{f_1, f_2, \dots, f_N\})$, generated by $\hat{M}_{f_1}, \hat{M}_{f_2}, \dots, \hat{M}_{f_N}$, where $f_i \in \mathcal{S}(\mathcal{M})$,
making QFT the same as algebraic QM unless we take an $N \rightarrow \infty$ limit

If we need a representation of the Poincaré group,
we can use a ^{direct or} _{inductive} limit over Schwartz space

[▶ nonlinear axioms page](#)

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Bounded operators

For a finite number of test functions, $\mathcal{A}(\{f_1, f_2, \dots, f_N\})$, the mathematics is just QM, so we can use [our Rigged Hilbert space experience](#) for unbounded operators

The vacuum state typically has tails that diminish rapidly enough that the representation of an unbounded algebra is mathematically well-controlled

$$\text{For the free field vacuum state for } \mathcal{A}(\{f\}), \rho_\nu(e^{j\lambda\hat{M}_f}) = e^{-\lambda^2(f^*,f)/2},$$

where $(f, g) = \rho_\nu(\hat{M}_f^\dagger \hat{M}_g)$

We allow unbounded operators $\hat{M}_{f_1}, \hat{M}_{f_2}, \dots, \hat{M}_{f_N}$, as for the Wightman axioms, but we *can* restrict to using only $f(x) \in \mathbb{R}$ and bounded generating operators $e^{j\lambda\hat{M}_f}$

▶ [nonlinear axioms page](#)

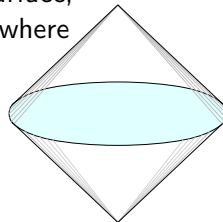
Time-slice axiom

If we know all expected measurement results on a Cauchy surface, that determines all expected measurement results everywhere within forward and backward light-cones

For engineering, we never know anything like that much

Given what we know for measurements $\hat{M}_{f_1}, \hat{M}_{f_2}, \dots, \hat{M}_{f_N}$, and knowing the relationships between $\hat{M}_{f_1}, \hat{M}_{f_2}, \dots, \hat{M}_{f_N}$, and $\hat{M}_{f_{N+1}}$ we want a manifestly invariant vacuum state of a theory to give conditional probabilities for another measurement $\hat{M}_{f_{N+1}}$,

This construction is neutral about reductionism because f_1, f_2, \dots, f_N and f_{N+1} can all be of arbitrary scale cf [Adlam 2024](#)



► nonlinear axioms page

If a theory tells us *everything* about a deformed ElectroMagnetism, at *all* length scales, that tells us *everything* about the currents

For an interacting EM field, $\hat{M}'_f \neq \hat{M}_f$, $\rho'_\nu(e^{j\lambda_1 \hat{M}'_{f_1}} e^{j\lambda_2 \hat{M}'_{f_2}} \dots)$ ^{interacting} $\neq \rho_\nu(e^{j\lambda_1 \hat{M}_{f_1}} e^{j\lambda_2 \hat{M}_{f_2}} \dots)$, ^{free}
we might say that \hat{M}'_f introduces fermionic and other “clouds”, but,
 $\rho'_\nu(e^{j\lambda_1 \hat{M}'_{f_1}} e^{j\lambda_2 \hat{M}'_{f_2}} \dots)$ still must be manifestly Poincaré invariant
in terms of *just* the λ 's and f 's

Electron, Proton, and other nontrivial modulations of the EM field are *useful*, but they may be not *necessary*

► fermionic measurements appendix

We can construct measurement operators using a Dirac fermion test function,

$$\hat{M}_U = \hat{\psi}_U^\dagger \hat{\psi}_U, \text{ where } \{\hat{\psi}_U^\dagger, \hat{\psi}_V\} = (U, V), \{\hat{\psi}_U, \hat{\psi}_V\} = 0$$

$$\frac{\hat{M}_U}{(U, U)} \text{ is a projection operator, } \hat{M}_U^n = (U, U)^{n-1} \hat{M}_U$$

$$\rho_\lambda(\hat{M}_U) = (U, U)_+ \leq (U, U)$$

$$\rho_\lambda(e^{j\lambda \hat{M}_U}) = \left[1 - \frac{(U, U)_+}{(U, U)} \right] + \frac{(U, U)_+}{(U, U)} e^{j\lambda (U, U)}$$

$$\rho_\lambda(\delta(\hat{M}_U - \alpha)) = \left[1 - \frac{(U, U)_+}{(U, U)} \right] \delta(\alpha) + \frac{(U, U)_+}{(U, U)} \delta(\alpha - (U, U))$$

\hat{M}_U is a nonlinear majority rule-type construction

The general case, $\rho_\lambda(e^{j\lambda_1 \hat{M}_{U_1}} e^{j\lambda_2 \hat{M}_{U_2}} \dots)$, is not straightforward

We *could* introduce $\hat{M}_{(f,U)} = \hat{M}_f + \hat{M}_U$, with EM and Dirac components

► fermionic and ElectroMagnetic measurements appendix