18. Solve the differential equation.

$$(5+t)\frac{du}{dt}+u=5+t, \quad t>0$$

19. Solve the initial-value problem.

$$x^2\frac{dy}{dx}+2xy=-\sin x, \quad y(\pi)=0 \qquad \frac{dy}{dx}=\frac{-2xy-\sin x}{x^2}$$

20. A function $y(t)$ satisfies the differential equation $\frac{dy}{dt}=y^4-15y^3+54y^2$.

For what values of y is y decreasing?

$$\int \frac{1}{y^4-15y^3+54y^2}\,dy=\int 1\,dt$$

$$\int \frac{1}{y^2(y^2-15y+54)}=t+C$$

$$\int \frac{1}{y^2(y-9)(y-6)}\,dy=t+C$$

$$\frac{1}{y^2(y-9)(y-6)}=\frac{A}{y-9}+\frac{B}{y-6}+\frac{Cx+D}{y^2}+\frac{E}{y}$$

$$1=A(y-6)(y^3)+B(y-9)(y^3)+(Cy+D)(y)(y-6)(y-9)+E(y-9)(y-6)(y^2)$$

$$1=B(6-9)(6^3) \qquad 1=A(9-6)(9^3)$$

$$B=\frac{1}{-648} \qquad A=\frac{1}{2187}$$