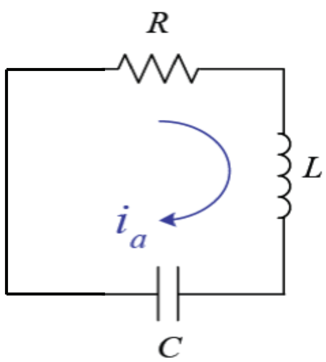

$$v_L + v_R + v_C = v_s$$

$$L\frac{dI}{dt} + RI(t) + v_C(t) = v_s$$

$$LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C(t) = \frac{dv_s}{dt}$$

$$L\frac{d^2v_C}{dt^2} + R\frac{dv_C}{dt} + \frac{v_C(t)}{C} = \frac{1}{C} * \frac{dv_s}{dt} = 0$$

Characteristic Roots

$$L\frac{d^2I_L}{dt^2} + R\frac{dI_L}{dt} + \frac{I_L(t)}{C} = \frac{dv_s}{dt} = 0$$

$$L\lambda^2 + R\lambda + \frac{1}{C} = 0$$

$$\lambda_{1,2} = \frac{-R \pm \sqrt{R - \frac{4L}{C}}}{2L}$$

$$\lambda_1 = -\frac{R}{2L} + \sqrt{\frac{R}{4L^2} - \frac{1}{LC}}$$

$$\lambda_2 = -\frac{R}{2L} - \sqrt{\frac{R}{4L^2} - \frac{1}{LC}}$$

-

Over-damped:

If $\frac{1}{LC} > \frac{R^2}{4L^2}$

$$I(t) = Ae^{(\frac{-R}{2L}+\sqrt{(\frac{R}{2L})^2-\frac{1}{LC}})t} + Be^{(\frac{-R}{2L}-\sqrt{(\frac{R}{2L})^2-\frac{1}{LC}})t}$$

$$\omega_1 = \frac{-R}{2L}, \omega_2 = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$I(t) = Ae^{(\omega_1+\omega_2)t} + Be^{(\omega_1-\omega_2)t}$$

$$\frac{dI(t)}{dt} = A(\omega_1 + \omega_2)e^{(\omega_1+\omega_2)t} + B(\omega_1 - \omega_2)e^{(\omega_1-\omega_2)t}$$

$$I(0) = 0$$

$$I(0) = A + B = 0, A = -B$$

$$\frac{V_c}{L} = \frac{dI(0)}{dt} = A\omega_1 + A\omega_2 + B\omega_1 - B\omega_2 = \frac{V}{L}$$

$$-B\omega_1 - B\omega_2 + B\omega_1 - B\omega_2 = \frac{V}{L}, -2B\omega_2 = \frac{V}{L}$$

$$B = -\frac{V}{2L\omega_2}, A = \frac{V}{2L\omega_2}$$

$$\frac{dI(t)}{dt} = \frac{V}{2L\omega_2}(\omega_1 + \omega_2)e^{(\omega_1+\omega_2)t} - \frac{V}{2L\omega_2}(\omega_1 - \omega_2)e^{(\omega_1-\omega_2)t}$$

$$\frac{V}{2L\omega_2}(\omega_1 + \omega_2)e^{(\omega_1+\omega_2)t} - \frac{V}{2L\omega_2}(\omega_1 - \omega_2)e^{(\omega_1-\omega_2)t} = \frac{kg \cdot m^2 \cdot s^{-3} \cdot A^{-1} \cdot s}{s \cdot kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}} = \frac{A}{s}$$

This is different from what was stated in the paper and the code, but the units check out.

Under-damped:

If
$$\frac{1}{LC} < \frac{R^2}{4L^2}$$

$$\omega_1 = \frac{R}{2L}$$
$$\omega_2 = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$
$$I(t) = e^{-\omega_1 t} A\cos(\omega_2 t) + e^{-\omega_1 t} B\sin(\omega_2 t)$$
$$I(0) = 0 = A$$
$$\frac{dI(t)}{dt} = -\omega_1 e^{-\omega_1 t} B\sin(\omega_2 t) + \omega_2 e^{-\omega_1 t} B\cos(\omega_2 t)$$
$$\frac{dI(0)}{dt} = \omega_2 B = \frac{V}{L}$$
$$B = \frac{V}{L\omega_2}$$
$$\frac{dI(t)}{dt} = -\omega_1 e^{-\omega_1 t} \frac{V}{L\omega_2}\sin(\omega_2 t) + \omega_2 e^{-\omega_1 t} \frac{V}{L\omega_2}\cos(\omega_2 t)$$
$$-\omega_1 e^{-\omega_1 t} \frac{V}{L\omega_2}\sin(\omega_2 t) + \omega_2 e^{-\omega_1 t} \frac{V}{L\omega_2}\cos(\omega_2 t) = \frac{kg \cdot m^2 \cdot s^{-3} \cdot A^{-1} \cdot s}{s \cdot kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}} = \frac{A}{s}$$

References

Pashut T, **Wolfus S**, **Friedman A**, **Lavidor M**, **Bar-Gad I**, **Yeshurun Y**, **Korngreen A**. Mechanisms of magnetic stimulation of central nervous system neurons. *PLoS Comput Biol* 7: e1002022, 2011.

http://ctms.engin.umich.edu/CTMS/Content/Introduction/System/Modeling/figures/RLC.png