Definition 4.1: $${}_pF_q\left(\begin{matrix} a_1,\ldots,a_p \\ b_1,\ldots,b_q \end{matrix};z\right) := \sum_{\lambda=0}^{\infty}\left\{\frac{z^{\lambda}}{\lambda!}\prod_{k=1}^{p}\left[\frac{\Gamma(a_k+\lambda)}{\Gamma(a_k)}\right]\prod_{j=1}^{q}\left[\frac{\Gamma(b_j+\lambda)}{\Gamma(b_j)}\right]\right\}.$$

Theorem 4.2: $\forall a_m, b_l, z \in \mathbb{C} \ni \Re[b_k] > \Re[a_{k+1}] > 0 \wedge \Re[a_1] > 0 \wedge |z| < 1,$

$${}_{n+1}F_n\left(\begin{matrix} a_1,\ldots,a_{n+1} \\ b_1,\ldots,b_n \end{matrix};z\right) = \prod_{k=1}^{n}\left[\frac{\Gamma(b_k)}{\Gamma(a_{k+1})\Gamma(b_k-a_{k+1})}\right]\int_0^1\int_0^1\cdots\int_0^1\prod_{q=1}^{n}\left[t_q^{a_{q+1}-1}(1-t_q)^{b_q-a_{q+1}-1}\right]\left(1-z\prod_{\lambda=1}^{n}t_\lambda\right)^{-a_1} dt_1\ldots dt_{n-1}\,dt_n$$

Proof: Let K denote the coefficient of the integral of Theorem 4.2, and let Ω_n denote the right-hand side of Theorem 4.2, then

$$\begin{aligned}
\Omega_n &= K\int_0^1\int_0^1\cdots\int_0^1\prod_{q=1}^{n}\left[t_q^{a_{q+1}-1}(1-t_q)^{b_q-a_{q+1}-1}\right]\left(1-z\prod_{\lambda=1}^{n}t_\lambda\right)^{-a_1} dt_1\ldots dt_{n-1}\,dt_n \\
&= K\sum_{\lambda=0}^{\infty}\frac{\Gamma(a_1+\lambda)z^{\lambda}}{\Gamma(a_1)\lambda!}\int_0^1\int_0^1\cdots\int_0^1\prod_{q=1}^{n}\left[t_q^{a_{q+1}+\lambda-1}(1-t_q)^{b_q-a_{q+1}-1}\right]dt_1\ldots dt_{n-1}\,dt_n \\
&= \prod_{k=1}^{n}\left[\frac{\Gamma(b_k)}{\Gamma(a_{k+1})\Gamma(b_k-a_{k+1})}\right]\sum_{\lambda=0}^{\infty}\frac{\Gamma(a_1+\lambda)z^{\lambda}}{\Gamma(a_1)\lambda!}\prod_{q=1}^{n}\mathrm{B}(a_{q+1}+\lambda, b_q-a_{q+1}) \\
&= \sum_{\lambda=0}^{\infty}\frac{\Gamma(a_1+\lambda)z^{\lambda}}{\Gamma(a_1)\lambda!}\prod_{k=1}^{n}\left[\frac{\Gamma(b_k)\Gamma(a_{k+1}+\lambda)}{\Gamma(a_{k+1})\Gamma(b_k+\lambda)}\right] = {}_{n+1}F_n\left(\begin{matrix} a_1,\ldots,a_{n+1} \\ b_1,\ldots,b_n \end{matrix};z\right)
\end{aligned}$$

Theorem 4.3: $\forall a_m, b_l, z \in \mathbb{C} \ni \Re[b_k] > \Re[a_k] > 0 \wedge |z| < 1,$

$${}_nF_n\left(\begin{matrix} a_1,\ldots,a_n \\ b_1,\ldots,b_n \end{matrix};z\right) = \prod_{k=1}^{n}\left[\frac{\Gamma(b_k)}{\Gamma(a_k)\Gamma(b_k-a_k)}\right]\int_0^1\int_0^1\cdots\int_0^1\exp\left(z\prod_{\lambda=1}^{n}t_\lambda\right)\prod_{q=1}^{n}\left[t_q^{a_q-1}(1-t_q)^{b_q-a_q-1}\right]dt_1\ldots dt_{n-1}\,dt_n$$

Proof: Let L denote the coefficient of the integral of Theorem 4.3, and let Ξ_n denote the right-hand side of Theorem 4.2, then

$$\begin{aligned}
\Xi_n &= L\int_0^1\int_0^1\cdots\int_0^1\exp\left(z\prod_{\lambda=1}^{n}t_\lambda\right)\prod_{q=1}^{n}\left[t_q^{a_q-1}(1-t_q)^{b_q-a_q-1}\right]dt_1\ldots dt_{n-1}\,dt_n \\
&= L\sum_{\lambda=0}^{\infty}\frac{z^{\lambda}}{\lambda!}\int_0^1\int_0^1\cdots\int_0^1\prod_{q=1}^{n}\left[t_q^{a_q+\lambda-1}(1-t_q)^{b_q-a_q-1}\right]dt_1\ldots dt_{n-1}\,dt_n \\
&= L\sum_{\lambda=0}^{\infty}\frac{z^{\lambda}}{\lambda!}\prod_{q=1}^{n}\mathrm{B}(a_q+\lambda, b_q-a_q) = \sum_{\lambda=0}^{\infty}\frac{z^{\lambda}}{\lambda!}\prod_{q=1}^{n}\frac{\Gamma(a_q+\lambda)\Gamma(b_q)}{\Gamma(a_q)\Gamma(b_q+\lambda)} \\
&= {}_nF_n\left(\begin{matrix} a_1,\ldots,a_n \\ b_1,\ldots,b_n \end{matrix};z\right)
\end{aligned}$$