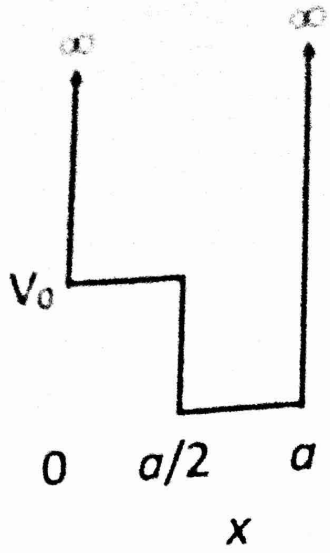

Figure 1: Sketch of an infinite square well potential of width a with a shelf of height V_0 in its left half.

(5) Shelf in a box

A particle of mass m is trapped in an infinite square well potential of width a with a shelf of height V_0 on its left half side, as sketched in Fig. 1. The potential $V(x)$ is thus given by

$$V(x) = \begin{cases} \infty, & x \leq 0, \\ V_0, & 0 < x < a/2, \\ 0, & a/2 \leq x < a, \\ \infty, & x \geq a. \end{cases} \tag{1}$$

Treat the shelf as a perturbation to the particle in a box.

(a) What are the eigenfunctions and eigenvalues for the particle in the box without the shelf?

(b) Treat the shelf as a weak perturbation. What is the first order change $E_n^{(1)}$ to the energy of the particle in the n^{th} eigenstate?

(c) What is the wave function $|\psi_n^{(1)}(x)\rangle$ to first order in the shelf perturbation?