Given below is the joint pmf of two random variables (X, Y), Compute: E(X), E(Y), V(X), V(Y), E(XY), E[X; given Y = 23], V[X; given Y=23], E[Y; given X=-20], V[Y; given X=-20], P[X<3, Y<=23]

X|Y	y = 17	20	23	35	48	p(x)
x = -20	0.02	0.03	0.07	0.02	0.06	0.2
0	0	0.05	0	0.05	0.1	0.2
1	0.05	0.03	0.02	0.07	0.03	0.2
3	0.01	0.02	0.03	0	0.04	0.1
17	0.18	0.04	0.06	0.01	0.01	0.3
p(y)	0.26	0.17	0.18	0.15	0.24	1

$$E(X) = \sum x\, p(x) = -20(0.2) + 1(0.2) + 3(0.1) + 17(0.3) = 1.6$$

$$E(Y) = \sum y\, p(y) = 17(0.26) + 20(0.17) + 23(0.18) + 35(0.15) + 48(0.24) = 28.73$$

$$V(X) = \sum (x - E(X))^2 p(x) =$$

$$= 0.2(-20 - 1.6)^2 + 0.2(0 - 1.6)^2 + 0.2(1 - 1.6)^2 + 0.1(3 - 1.6)^2$$

$$+0.3(17 - 1.6)^2 = 165.24$$

$$V(Y) = \sum (y - E(Y))^2 p(y) =$$

$$= 0.26(17 - 28.73)^2 + 0.17(20 - 28.73)^2 + 0.18(23 - 28.73)^2$$

$$+0.15(35 - 28.73)^2 + 0.24(48 - 28.73)^2 = 149.6571$$

$$E(XY) = \sum_x \sum_y xy p(x, y)$$

$$= (-20)(17)(0.02) + (-20)(20)(0.03) + (-20)(23)(0.07) + (-20)(35)(0.02)$$

$$+(-20)(48)(0.06) + 17(0.05) + 20(0.03) + 35(0.07) + 48(0.03) + 3(17)(0.01)$$

$$+3(20)(0.02) + 3(23)(0.03) + 3(48)(0.04) + 17(17)(0.18) + 17(20)(0.04)$$

$$+17(23)(0.06) + 17(35)(0.01) + 17(48)(0.04)$$

$$= -4.07$$

$$E(X|Y = 23) = \sum x p(x|y = 23) = \sum \frac{xp(x, y = 23)}{p(y = 23)}$$

$$= \frac{1}{0.18}(-20(0.07) + 0.02 + 3(0.03) + 17(0.06)) = -1.5$$

$$V(X|Y = 23) = E\big(X - E(X|Y = 23)\big)^2 = (1.6 + 1.5)^2 = 9.61$$

$$E(Y|X = -20) = \sum yp(y|x = -20) = \sum \frac{yp(y, x = -20)}{p(x = -20)}$$

$$= \frac{1}{0.2}\big(17(0.02) + 20(0.03) + 23(0.07) + 35(0.02) + 48(0.06)\big) = 30.65$$

$$V(Y|X = -20) = E\big(Y - E(Y|X = -20)\big)^2 = (28.73 - 30.65)^2 = 3.6864$$

$$P(X < 3, Y \leq 23) = 0.02 + 0.03 + 0.07 + 0.05 + 0.05 + 0.03 + 0.02 = 0.27$$