Investigating the relationship between surface area and drag coefficient of a parachute.

Introduction

______The parachute, a safety harness for the airborne, has been used for decades for leisurely activities and warfare. The most commonly used parachutes come in a dome like shape (Figure 1). I will be using the dome parachute for this experiment.

Figure 1. Dome shaped parachute

The drag force, a force opposing the direction of motion generated by the fluid the object is in contact with while moving[1], can be expressed using the equation:

$$D = \frac{C_d \rho V^2 A}{2}$$

where C_d is the drag coefficient, ρ is the air density and V is the velocity of the object. The drag coefficient is described to be a constant depending on the form of the travelling object. At terminal velocity, the velocity at which the drag force (D) is equal to the gravitational force (F_g)/net acceleration = 0 m/s^2, the drag force equation can be rewritten as:

$$D = mg = \frac{C_d \rho V^2 A}{2}$$

[1] Hall, Nancy. "What Is Drag?" NASA, NASA, 13 May 2021, https://www.grc.nasa.gov/www/k-12/airplane/drag1.html.

where m is the mass of the falling object and g is the acceleration due to gravity since $F_g = mg$.

By rearranging the equation one can get:

$$C_d = \frac{2mg}{\rho V^2 A}$$

As entertainment becomes more prominent in modern society, various parachutes are becoming more widely used for sporting purposes. I have become interested in skydiving and other parachute related sports especially in the relationship between the surface area of a parachute and its drag coefficient. Hence, I have designed an experiment calculating the drag coefficient of parachutes with surface areas 0.0025π m^2 (0.05 m radius), 0.01π m^2 (0.1 m radius), 0.0225π m^2 (0.15 m radius), 0.04π m^2 (0.20 m radius) and 0.0625π m^2 (0.25 m radius).

Design

I. Research question

Verifying the relationship between drag coefficient and surface area of a parachute.

II. Hypothesis

If the surface area of a parachute increases, then the drag coefficient of the parachute will decrease because the drag coefficient and the surface area can be seen to have an inverse relationship in the drag coefficient equation.

III. Variables

A. Independent variables:

1. Surface area (m^2)
2. Drag (N)

B. Dependent variables:

1. Drag coefficient

C. Controlled variables:

1. Mass

a) Reason: Parachutes with different masses hanging from them will have different gravitational forces; hence, also different drag forces at terminal velocity.

b) Method: A dime is used as a hanging mass for all different surface areas.

2. Shape and material of the parachute

a) Reason: Different shapes and materials will result in different drag coefficients.

b) Method: The parachutes are made to have the same shape and material when dropping.

3. Wind current

a) Reason: Any movement of air current will create various forces in different directions on the parachute causing error.

b) Method: Any windows and other sources of wind current are closed around the testing area.

IV. Apparatus and materials

A. Scale (±0.05 g)
B. Meter stick (±0.05 m)
C. Recording device
D. Compass
E. Knife
F. Garbage bag
G. Tape
H. String

V. Method

1. Cut circles of radius 0.05 m, 0.1 m, 0.15 m, 0.2 m and 0.25 m using a compass and a knife from a garbage bag.
2. Put a dime in the middle of each cutout circle and connect them using string and tape.
3. Measure the mass of each parachute using the scale.
4. Tape a meter stick perpendicular to the ground.
5. Record the experiment with a recording device (phone).
6. Drop the parachute from 1 m above the ground.
7. Using the video recorded, use a video analysis program (Logger Pro) to find the terminal velocity.
8. Do steps 4-6 five times for each surface area.

Data collection and analysis

Raw data

Table 1. Raw data of various variables used in calculations.

Radius of parachute (±0.0005 m)		0.05 m	0.1 m	0.15 m	0.2 m	0.25 m
Mass (±0.05 g)		3.30g	2.90g	3.65g	4.10g	5.00g
Terminal velocity (m/s)	V_1	2.240	1.512	0.9869	1.065	0.8778
	V_2	2.079	1.533	1.242	0.8971	0.9397
	V_3	2.215	1.393	1.202	1.116	0.8418
	V_4	2.166	1.408	1.128	0.9513	0.9122
	V_5	2.228	1.379	1.120	1.084	0.9685
Surface area (±0.001 m)(m^2)		0.0025π	0.0100π	0.0225π	0.0400π	0.0625π
Acceleration due to gravity (g)(m/s^2)		9.81				
Air density (ρ)(g/m^3)		1225				

Each of the terminal velocities has been found using the video motion analysis function on the *Logger Pro* app (Figure 2).

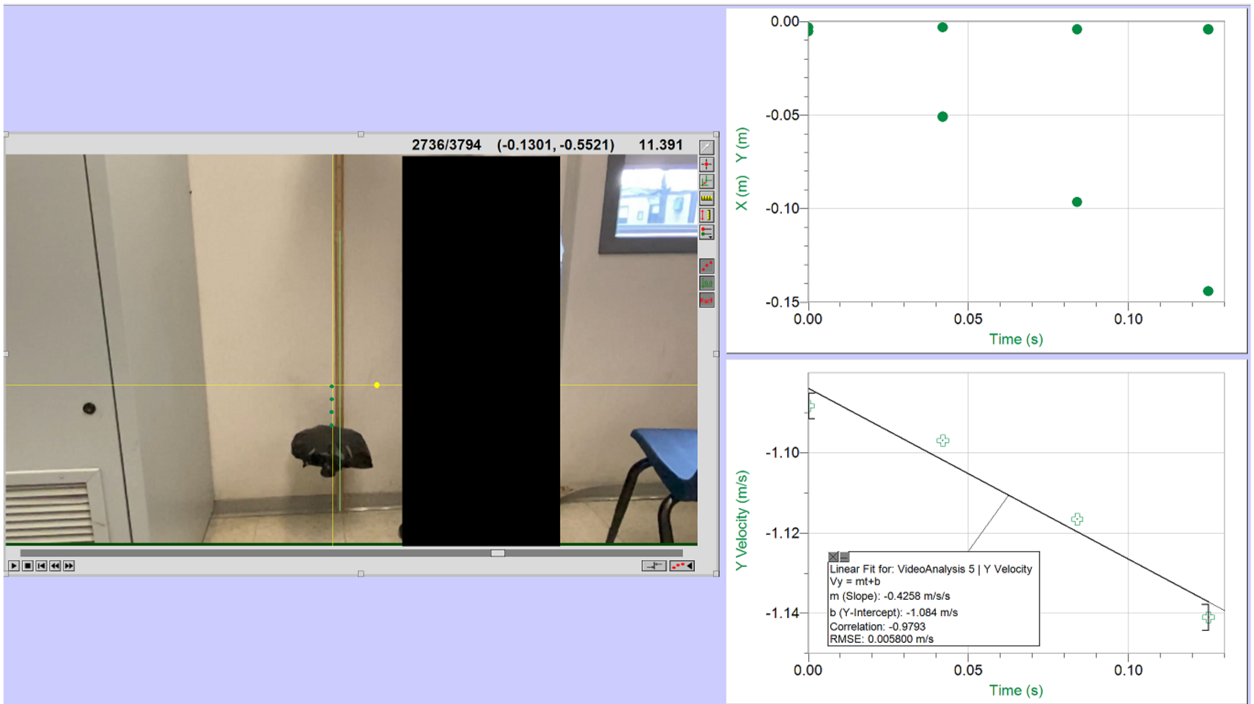


Figure 2. *Logger Pro* analysis of the terminal velocity of the 0.2 m radius parachute

Method of finding terminal velocity in *Logger Pro*:

1. Set a reference length of 1 m according to the meter stick in the video (green line).
2. Find a series of consecutive frames in which the parachute travels at a relatively constant velocity.
3. Track one point on the parachute (tip of the top of the parachute).
4. Set the tracked point of the first frame of the series as the origin (intersection of the yellow lines).
5. Put a point on the tracked point for each of the frames in the series (green dots).
6. Set the video time of the first frame as 0s for the graph time.

7. Graph the y-velocity and set up a line of best fit, the gradient (acceleration of the parachute) should be around to 0 (terminal velocity). The x-velocity is ignored since there is barely any deviation from the line x = 0. The y intercept (b) of the line of best fit is the terminal velocity.

Data processing

With the raw data collected, the average velocity for each of the parachutes can be calculated and used for drag coefficient calculations. To calculate the average velocity the formula

$V_{avg} = \frac{\sum_{i=1}^{5} V_i}{5} = \frac{V_1+V_2+V_3+V_4+V_5}{5}$ will be used. The uncertainty of the average velocity will then be

$\Delta V_{avg} = \frac{V_{max}-V_{min}}{2}$.

Sample calculation using the 0.05 cm radius parachute:

$$V_{avg} = \frac{2.240+2.079+2.215+2.166+2.228}{5} = 2.186\text{m/s}$$

$$\Delta V_{avg} = \frac{2.240-2.079}{2} = 0.080\text{m/s}$$

Table 2. The average velocity (V_{avg}) and its uncertainty (ΔV_{avg}) of parachutes with different radii.

Radius of parachute (±0.0005 m)	0.05 m	0.1 m	0.15 m	0.2 m	0.25 m
Average velocity (V_{avg})(m/s)	2.186	1.445	1.136	1.023	0.908
Uncertainty of average velocity (ΔV_{avg})(m/s)	0.080	0.077	0.130	0.120	0.063

According the the equation $C_d = \frac{2mg}{\rho V_{avg}^2 A}$, all the necessary information has now been found. The drag coefficient will then need to be calculated to find its correlation with surface area.

Sample calculation for 0.05 m radius parachute with uncertainties:

$$C_d = \frac{2\cdot(3.30\pm0.05)\cdot 9.81}{1225\cdot(2.186\pm0.080)^2\cdot(0.0025\cdot\pi\pm0.001)}$$

$$= \frac{64.746\pm2\%}{(9.6211\pm13\%)\cdot(2.186\pm3.7\%)^2}$$

$$= \frac{64.746\pm2\%}{(9.6211\pm13\%)\cdot(2.186^2\pm7.4\%)}$$

$$= \frac{64.746}{45.97534998} \pm 22.4\%$$

$$= 1.408276392 \pm 22.4\%$$

$= 1.4 \pm 0.3$ ← No units since the drag coefficient is unitless

Table 3. The drag coefficient (C_d) and its uncertainty (ΔC_d) of parachutes with different radii.

Radius of parachute (±0.0005 m)	0.05 m	0.1 m	0.15 m	0.2 m	0.25 m
Drag coefficient (C_d)	1.4	0.71	0.6	0.5	0.49
Uncertainty of drag coefficient (ΔC_d)	0.3	0.1	0.2	0.1	0.07

Plotting the drag coefficient (Table 3) on the y-axis and surface area on the x-axis will yield a graph (Figure 3).

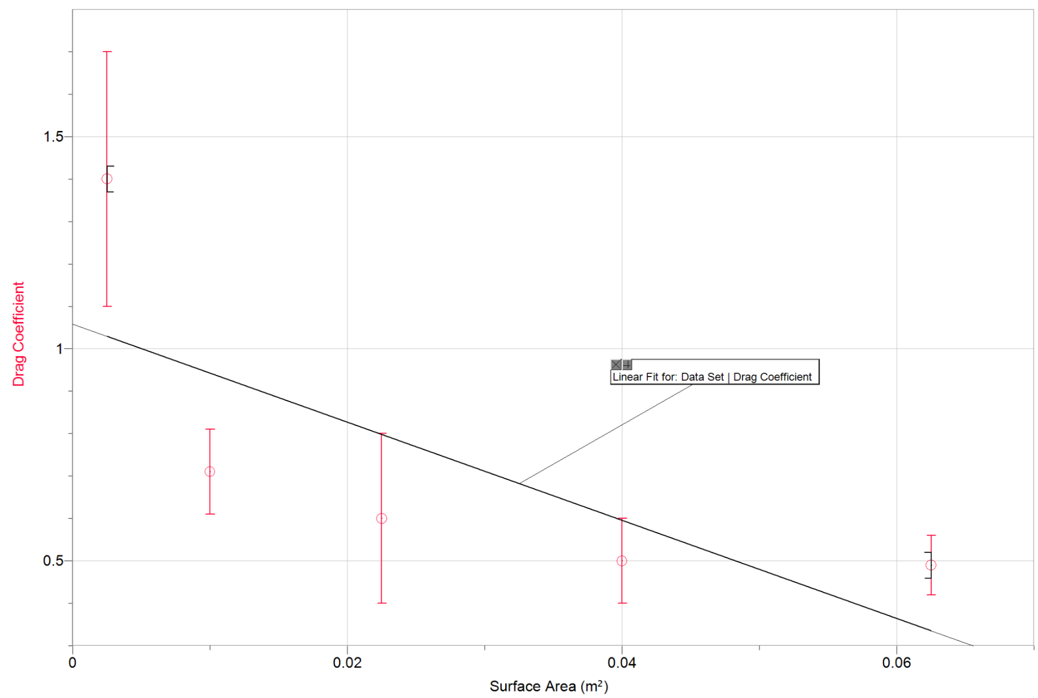


Figure 3. Graph of drag coefficient plotted against surface area.

The equation below will be used to calculate the linear correlation coefficient (r).

$$r = \frac{\sum_{i=1}^{5} (A_i - \overline{A})(C_{d,i} - \overline{C}_d)}{\sqrt{\sum_{i=1}^{5} (A_i - \overline{A})^2 \cdot \sum_{i=1}^{5} (C_{d,i} - \overline{C}_d)^2}}$$

Where A_i is the i[th] surface area, $\overline{A}$ is the average surface area, $C_{d,i}$ is the i[th] drag coefficient and $\overline{C}_d$ is the average drag coefficient. The values of each of these is seen as follows (Table 4).

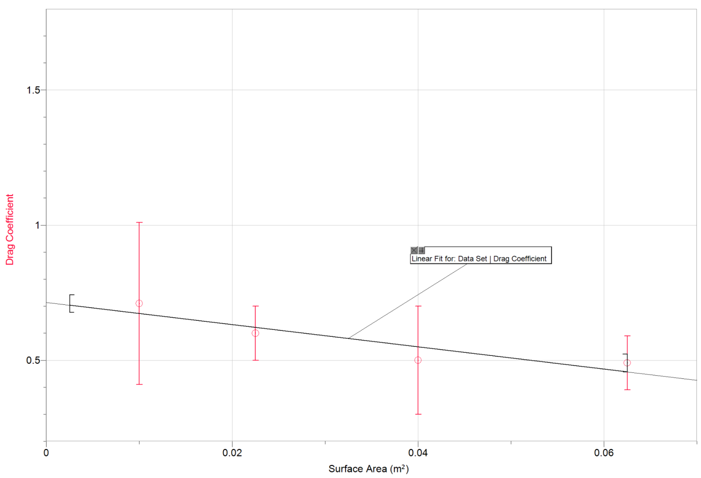
Table 4. Values of needed variables to calculate the linear correlation coefficient.

A_i	A_1	A_2	A_3	A_4	A_5	$\overline{A}$
Value (m^2)	0.0025	0.0100	0.0225	0.0400	0.0625	0.0275
$C_{d,i}$	$C_{d,1}$	$C_{d,2}$	$C_{d,3}$	$C_{d,4}$	$C_{d,5}$	$\overline{C}_d$
Value	1.4	0.71	0.6	0.5	0.49	0.74

The linear correlation coefficient will then be:

$$r = \frac{(0.0025-0.0275)(1.4-0.74)+(0.0100-0.0275)(0.71-0.74)+(0.0225-0.0275)(0.6-0.74)+(0.0400-0.0275)(0.5-0.74)+(0.0625-0.0275)(0.49}{\sqrt{[(0.0025-0.0275)^2+(0.0100-0.0275)^2+(0.0225-0.0275)^2+(0.0400-0.0275)^2+(0.0625-0.0275)^2][(1.4-0.74)^2+(0.71-0.74)^2+(0.6-0.74)^2+(0.5-0.74}$$

$$r = -\ 0.7364 \approx -\ 1$$



As seen in the graph, the drag coefficient for the parachute with a radius of 0.05 m is an outlier of the group. If the outlier is taken out, the graph becomes much more precise (Figure 4).

Figure 4. Graph of drag coefficient plotted against surface area after removing the outlier.

Using the linear correlation coefficient equation again using this dataset will yield a linear correlation coefficient of -0.9112 which is a much stronger correlation than the previous -0.7364.

Conclusion and analysis

Conclusion

After processing the data and finding the linear correlation between C_d and A, the conclusion that the drag coefficient and the surface area of a parachute is inversely proportional can be reached since the linear correlation coefficient is seen to be near -1, which means that the relationship is strongly negative. This conclusion supports the hypothesis since an increase in surface area will lead to a decrease in drag coefficient due to the inverse relationship between the two variables.

Although the inverse relationship is proven, the accuracy of the relationship is low. The uncertainty of the correlation is also great since the drag coefficients have a minimum of 14% error (0.1 m radius) and a maximum of 33% error (0.15 m radius) rendering the experiment to be inaccurate. The inaccuracy is further proven in Figure 3, as seen in the large error bars for each data point. On the other hand, the precision of the experiment is relatively high if one is to ignore the outlier of the 0.05 m radius parachute which is seen in Figure 4 since the line of best fit passes relatively close to each of the data points. The outlier could have been the result of various errors which will be discussed below.

Hence, the experiment yields a strong inverse relationship between drag coefficient and surface area; however, the accuracy of the resultant data is low.

Error analysis

The low accuracy and precision stated in the conclusion can be attributed to various systematic and random errors seen in the table below (Table 5).

Legend: -high significance -medium significance

Table 5. Possible errors, their significance and possible improvements

Error	Significance	Improvements
Systematic errors		
Parachute material: the plastic garbage bags used to create the parachutes can be easily deformed.	The deformation of the plastic bags during the fall can change its surface properties and hence the drag and drag coefficients as well.	A more rigid material such as nylon can be used to prevent parachute deformation while falling.

Parachute construction: since the parachutes are constructed by human hands, there are bound to be inaccuracies especially in string length. For the 0.05 m radius parachute, some of the strings connecting the dime to the parachute are longer than the rest.	Uneven tension forces are exerted on the edge of the parachute, causing the parachute to glide sideways. This event affects the overall drag and drag coefficient of the parachute.	More precise construction methods will need to be explored to create better parachutes that will glide straight downwards.
Recording device: the recording device used does not have the best recording quality. When trying to do video motion analysis on *Logger Pro*, the parachute is sometimes blurry.	The blurry image can be detrimental when trying to figure out where the point being tracked on the parachute is.	Use a higher quality camera if available.
Random errors		
Air current: as the room, in which the experiment has taken place, is not heated evenly throughout, there are convection currents of air which will affect the gliding of the parachute. Some miniscule breezes of air can be felt.	These air currents can blow the parachutes in random directions hence causing inconsistencies in the drag which will also affect the drag coefficient.	The experiment can be conducted in a more isolated area which will increase the precision.
Release mechanism: the release mechanism used in the experiment is my hand. The human hand can tremble at times, causing slight fluctuations in the initial dropping conditions.	Although the trembling of the hands may be small, it can undeniably create variances in the gliding path and overall result of a trial. For example, some of the trials are seen to glide sideways and the parachute is observed to tilt more than the rest which could be the result of a tremble of the hand.	Some sort of non-human dropping mechanism can be set up so that each drop will be more precise since there will no longer be random errors stemming from slight tremors.
Air density assumption: the air density (ρ) is assumed to be at room temperature which is 1.225kg/m^3. However, the room used for the experiment may not be at room temperature since there are other experiments also	An assumed air density can be impactful to the final value of the drag coefficient.	Experimentally collect data for the air density during the experiment. Can be done using air density sensors or a balloon, a bucket of water and a mass balance.

taking place that use apparatus to heat up various substances.		

Works cited

Hall, Nancy. “What Is Drag?” NASA, NASA, 13 May 2021, https://www.grc.nasa.gov/www/k-12/airplane/drag1.html.