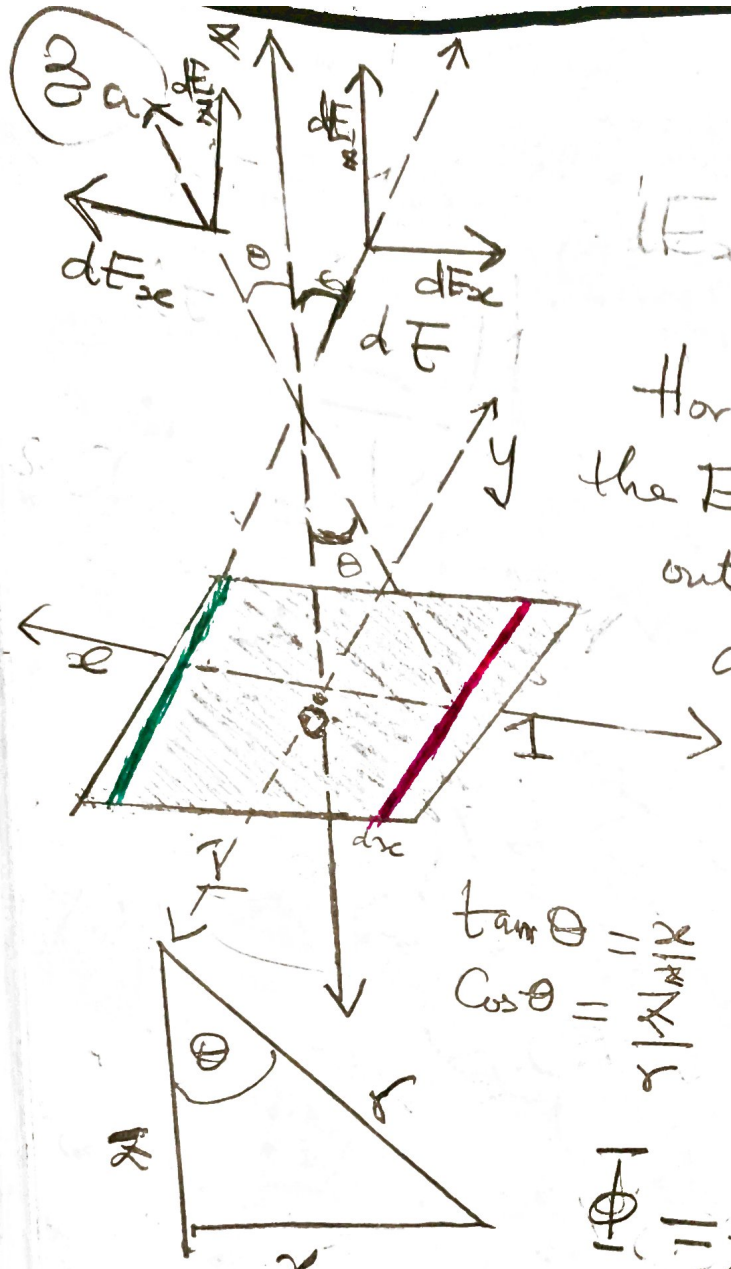

Horizontal components of the E-field cancel each other out. E-field along z will double.

$$dE_z = \cos\theta\, dE$$

$$\frac{z}{r}\, dE$$

$$\tan\theta = \frac{x}{z}$$

$$\cos\theta = \frac{z}{r}$$

$$z = r\cos\theta$$

$$r^2 = z^2 + x^2, \quad r = \sqrt{z^2 + x^2}$$

$$\Phi = \frac{1}{4\pi\varepsilon}\int \frac{\sigma}{r}\, da, \quad \frac{x}{r} = \sin\theta$$

$$x = r\sin\theta$$

$$dq = \sigma\, y\, dx$$

$$\Phi = \left[\frac{1}{4\pi\varepsilon_0}\int_0^{1/2} \frac{\sigma}{\sqrt{z^2+x^2}}\, dx\, y\right] \times 2$$

$$=$$

where $y = 1$, and Φ is doubled to account for both halves of the square.

$$\Phi = \left[\frac{\sigma}{4\pi\varepsilon_0 z}\coth^{-1}\left[\sqrt{1+4z^2}\right]\right]$$