Modelling the Effects of Entanglement

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Abstract

A joint probability distribution of the outcomes of an ideal photonic Einstein-Podolsky-Rosen-Bohm (EPRB) experiment is derived from a naive model governed by Malus law. The same probabilities can be derived from the assertion $P(AB|a,b) = \frac{1}{2}P(A|a)P(B|b,a) + \frac{1}{2}P(B|b)P(A|a,b)$

Entanglement enters the probabilities through the non-local assumption that the entangled pair always have the same polarization angle irrespective of the physical context.

The model makes the same predictions as the quantum theory.

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1. The Experiment

We model an ideal EPRB experiment where symmetrically entangled photons are sent in different directions from a source to beam-splitting polarizers and detectors at A and B (see figure 9.1 in [1]). The recorded results are the polarizer settings and detector clicks from A and B. See [2]. Each station A and B has two detectors - one to catch photons projected to the polarisers main orientation, the other to indicate a projection into the perpendicular mode. In this idealised setup one (and only one) of the two detectors always clicks. This configuration is sometimes called 'two-channel' in reference to the beam-splitting action.

In every run the settings of polarizersA (a, a') and B (b, b') and the order (A, B) in which A's and B's photons interact are randomly set with equal probability for each value. The prepared photons are assumed to have the same polarization orientation on every run.

From the following additional assumptions we can deduce the probabilities $P_{xy}(\alpha, \beta, S)$ where x and y are 0 (perpendicular projection) or 1 (projection) and α and β are the polarizer settings at A and B respectively and $S = (A, B)$ is the order variate.

1. The entangled pair always have the same polarization orientation
2. When either photon passes through a polarizer then they are both projected into the polarizers setting or perpendicular setting the choice being governed by Malus law.

The terms of $P_{xy}(\alpha, \beta, S)$ that follow from this setup and the assumptions are shown in appendix A (4).

The probability of a coincidence is $P_{00}(\alpha\beta) + P_{00}(\alpha\beta)$ which gives (from equations A1 and A4) $P_{co} = \cos{(\alpha - \beta)}^2$. The correlation is $\rho(\alpha\beta) = 2P_{co} - 1 = -\cos(2(\alpha - \beta))$ which is the same as the quantum theoretical correlation - see for instance [1] chapter 9.

2. The Process

The simulation uses the assumption and rules in section 1 and performs a Malus law projection twice in each repetition. The virtual apparatus is assumed to be perfect. There is no doubt that the simulation is reproducing the joint probabilities $P_{xy}(\alpha\beta)$ with remarkable fidelity, without using them explicitly.

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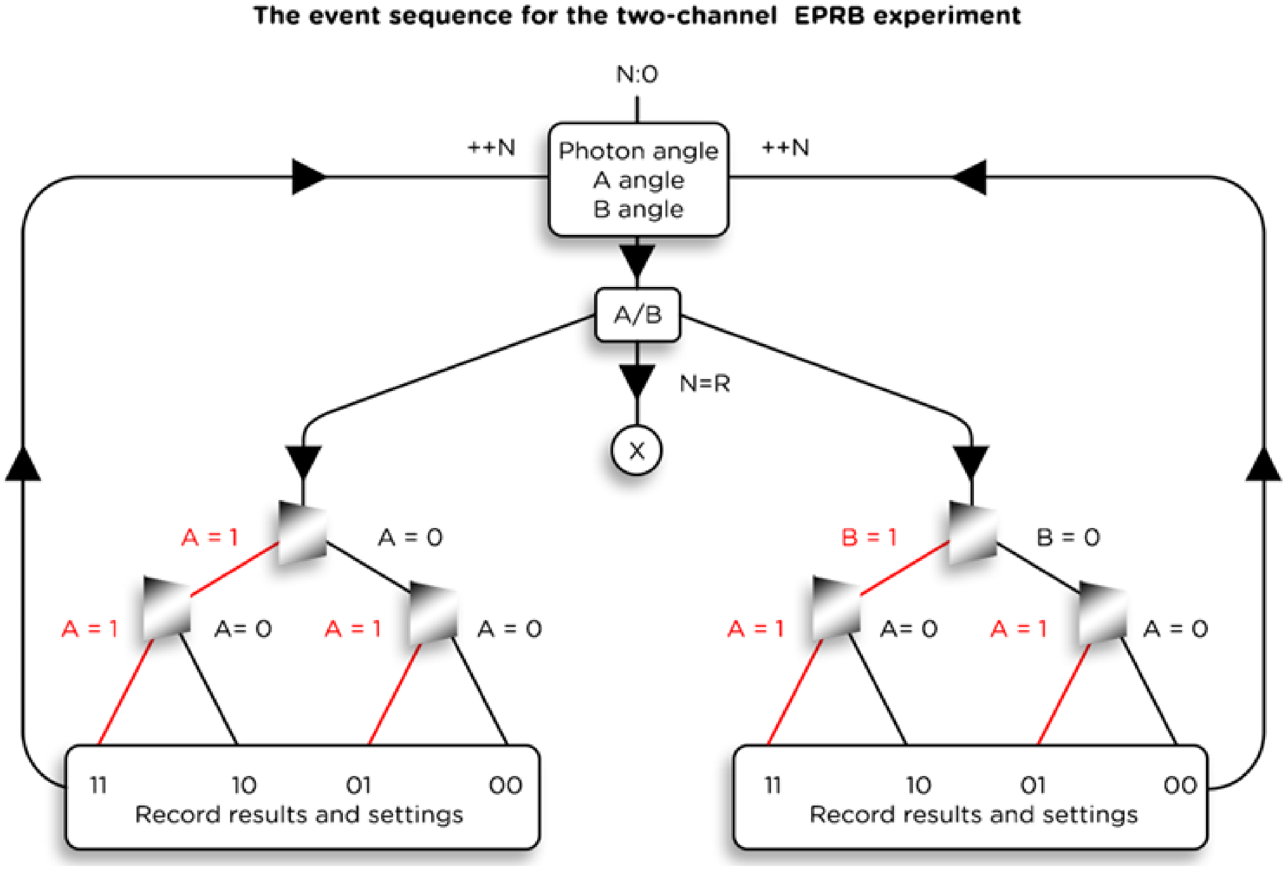
Figure 1: Process decision chart



3. Discussion

It appears that only the assumption that *the entangled pair will behave as a single entity with respect to the entangled property* is required to reproduce the predictions of the quantum mechanical singlet state. This special union can be maintained only if traditional locality and realism is relaxed. Since there are no such assumptions in either model it is not surprising that the ideal experiment can achieve the maximum possible value of the Bell-CHSH statistic.

If a quantum model differs from a corresponding classical model the reason is nearly always quantum interference. This requires the coherent recombination of states which have been separated. There is no opportunity for this to happen in the quantum model of EPRB. Entanglement is a quantum phenomenon which can find classical expression as the condition that the entangled property remains the same (or opposite) for the entangled pair.

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References

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4. Appendix A - the model probabilities

These probabilities are constructed from the product of the first and second decisions in the process tree and also correspond to the expansion of $P(AB|a,b) = \frac{1}{2}P(A|a,\theta_0,S)P(B|b,a) + \frac{1}{2}P(B|b,\theta_0,S)P(A|a,b)$. The parameter θ_0 (the polarization of the prepararred photons) appears only in the first terms and will be integrated out. S is a random variable which determines if the A side or B side is projected first.

If A's photon interacts first this gives (with $\theta_A \in [a,a']$, $\theta_B \in [b,b']$ and θ_0 is constant for all runs)

$$2P_{11}(\alpha,\beta,A) = \cos\left(\theta_A-\theta_0\right)^2\cos\left(\theta_A-\theta_B\right)^2,\ 2P_{10}(\alpha,\beta,A) = \cos\left(\theta_A-\theta_0\right)^2\sin\left(\theta_A-\theta_B\right)^2$$
$$2P_{01}(\alpha,\beta,A) = \sin\left(\theta_A-\theta_0\right)^2\sin\left(\theta_A-\theta_B\right)^2,\ 2P_{00}(\alpha,\beta,A) = \sin\left(\theta_A-\theta_0\right)^2\cos\left(\theta_A-\theta_B\right)^2$$

If B's photon interacts first

$$2P_{11}(\alpha,\beta,B) = \cos\left(\theta_A-\theta_B\right)^2\cos\left(\theta_B-\theta_0\right)^2,\ 2P_{10}(\alpha,\beta,B) = \sin\left(\theta_A-\theta_B\right)^2\sin\left(\theta_B-\theta_0\right)^2$$
$$2P_{01}(\alpha,\beta,B) = \cos\left(\theta_B-\theta_0\right)^2\sin\left(\theta_B-\theta_A\right)^2,\ 2P_{00}(\alpha,\beta,B) = \sin\left(\theta_B-\theta_0\right)^2\cos\left(\theta_B-\theta_A\right)^2$$

Summing over S gives $P_{xy}(\alpha,\beta) = \frac{1}{2}\left(P_{xy}(\alpha,\beta,A) + P_{xy}(\alpha,\beta,B)\right)$ The full distribution is

$$2P_{11}(\alpha,\beta) = \left(\cos\left(\theta_B-\theta_0\right)^2 + \cos\left(\theta_A-\theta_0\right)^2\right)\cos\left(\theta_B-\theta_A\right)^2 \tag{A1}$$
$$2P_{10}(\alpha,\beta) = \left(\cos\left(\theta_B-\theta_0\right)^2 + \sin\left(\theta_A-\theta_0\right)^2\right)\sin\left(\theta_B-\theta_A\right)^2 \tag{A2}$$
$$2P_{01}(\alpha,\beta) = \left(\sin\left(\theta_B-\theta_0\right)^2 + \cos\left(\theta_A-\theta_0\right)^2\right)\sin\left(\theta_B-\theta_A\right)^2 \tag{A3}$$
$$2P_{00}(\alpha,\beta) = \left(\sin\left(\theta_B-\theta_0\right)^2 + \sin\left(\theta_A-\theta_0\right)^2\right)\cos\left(\theta_B-\theta_A\right)^2 \tag{A4}$$

If $\theta_0 \in [0..\pi/2]$ is integrated out of all the probabilities then $\cos\left(\frac{\theta_X-\theta_0}{2}\right)^2 \to \sin\left(\frac{\theta_X-\theta_0}{2}\right)^2 \to 1/2$ making the two wings equal and bringing the individual probabilities into agreement with the quantum theory. Because the wings are equal we can drop the random variable S.

Finally $P_{11}(\alpha,\beta) = P_{00}(\alpha,\beta) = \cos\left(\theta_B-\theta_A\right)^2,\ P_{10}(\alpha,\beta) = P_{01}(\alpha,\beta) = \sin\left(\theta_B-\theta_A\right)^2$,

5. Appendix B : The CHSH Test

The expected value of the CHSH statistic can be found by finding the correlation from the probability of a coincidence. The probability of a coincidence is $P_c = P_{11}(\alpha,\beta) + P_{00}(\alpha,\beta)$ and the correlation is $C_{\alpha\beta} = 2P_c - 1$. We want $B = C_{ab} + C_{ab'} + C_{ab'} - C_{a'b'}$. The settings are $\alpha = (0,\pi/2)$ and $\beta = (\pi/4,-\pi/4)$ and $\theta_0 = 0$. The tables show probabilities calculated from equations A1-A4.

ab	B_0	B_1	a'b	B_0	B_1	ab'	B_0	B_1	a'b'	B_0	B_1
A_0	0.0625	0.0625	A_0	0.0625	0.0625	A_0	0.2759	0.0991	A_0	0.04733	0.5777
A_1	0.0839	0.7911	A_1	0.0839	0.7911	A_1	0.0473	0.5777	A_1	0.2759	0.0991

From which $C_{ab} = 2(0.0625 + 0.7911) - 1 = 0.7072$, $C_{ab'} = 0.7072$, $C_{ab'} = 0.7072$, $C_{a'b'} = -0.7072$ and $B = 2.828$. A simulation of 50 repetitions of 25000 trials gives a mean 2.8292 with standard deviation 0.017.