

**Pure State:**

$$(a) | \text{final} \rangle = \frac{1}{\sqrt{2}} | 45 \rangle \left\{ \frac{1}{\sqrt{2}} [ | A+1 \rangle + | A-1 \rangle ] \right\} + \frac{1}{\sqrt{2}} | 135 \rangle \left\{ \frac{1}{\sqrt{2}} [ | A+1 \rangle - | A-1 \rangle ] \right\}$$

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**Statistical Mixture:**

$$(b) | A+1 \rangle = \frac{1}{\sqrt{2}} | 45, A+1 \rangle + \frac{1}{\sqrt{2}} | 135, A-1 \rangle$$

$$(c) | A-1 \rangle = \frac{1}{\sqrt{2}} | 45, A+1 \rangle - \frac{1}{\sqrt{2}} | 135, A-1 \rangle$$

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Substitute:

$$(d) | A+1 \rangle = \frac{1}{\sqrt{2}} [ | X \rangle + | Y \rangle ]$$

$$(e) | A-1 \rangle = \frac{1}{\sqrt{2}} [ | X \rangle - | Y \rangle ]$$

(d) into (b)

$$| A+1 \rangle = \frac{1}{2} [ | 45, X \rangle + | 45, Y \rangle + | 135, X \rangle + | 135, Y \rangle ]$$

(e) into (c)

$$| A-1 \rangle = \frac{1}{2} [ | 45, X \rangle - | 45, Y \rangle - | 135, X \rangle + | 135, Y \rangle ]$$