In the Bohr model, $L = nh/2\pi = n\hbar$, where $n = 1, 2, \ldots$. For an $n = 1$ state (a ground state), Eq. (41.30) becomes $\mu = (e/2m)\hbar$. This quantity is a natural unit for magnetic moment; it is called one **Bohr magneton,** denoted by μ_B:

$$\mu_B = \frac{e\hbar}{2m} \quad \text{(definition of the Bohr magneton)} \tag{41.31}$$

(We defined this quantity in Section 28.8.) Evaluating Eq. (41.31) gives

$$\mu_B = 5.788 \times 10^{-5}\ \text{eV/T} = 9.274 \times 10^{-24}\ \text{J/T or A}\cdot\text{m}^2$$

Note that the units J/T and $\text{A}\cdot\text{m}^2$ are equivalent.

While the Bohr model suggests that the orbital motion of an atomic electron gives rise to a magnetic moment, this model does *not* give correct predictions about magnetic interactions. As an example, the Bohr model predicts that an electron in a hydrogen-atom ground state has an orbital magnetic moment of magnitude μ_B. But the Schrödinger picture tells us that such a ground-state electron is in an s state with zero angular momentum, so the orbital magnetic moment must be *zero!* To get the correct results, we must describe the states by using Schrödinger wave functions.

It turns out that in the Schrödinger formulation, electrons have the same ratio of μ to L (gyromagnetic ratio) as in the Bohr model—namely, $e/2m$. Suppose the magnetic field $\vec{B}$ is directed along the $+z$-axis. From Eq. (41.28) the interaction energy U of the atom's magnetic moment with the field is

$$U = -\mu_z B \tag{41.32}$$

where μ_z is the z-component of the vector $\vec{\mu}$.

Now we use Eq. (41.30) to find μ_z, recalling that e is the *magnitude* of the electron charge and that the actual charge is $-e$. Because the electron charge is negative, the orbital angular momentum and magnetic moment vectors are opposite. We find

$$\mu_z = -\frac{e}{2m}L_z \tag{41.33}$$

For the Schrödinger wave functions, $L_z = m_l\hbar$, with $m_l = 0, \pm 1, \pm 2, \ldots, \pm l$, so Eq. (41.33) becomes

CAUTION Again, two uses of the symbol *m* As in Section 41.3, the symbol m is used in two ways in Eq. (41.34). Don't confuse the electron mass m with the magnetic quantum number m_l.

$$\mu_z = -\frac{e}{2m}L_z = -m_l\frac{e\hbar}{2m} \tag{41.34}$$

Finally, using Eq. (41.31) for the Bohr magneton, we can express the interaction energy from Eq. (41.32) as

$$U = -\mu_z B = m_l\frac{e\hbar}{2m}B = m_l\mu_B B \tag{41.35}$$

Orbital magnetic interaction energy → U
Magnetic dipole component in direction of $\vec{B}$ → μ_z
Magnetic-field magnitude → B
Magnitude of electron charge → e
Planck's constant divided by 2π → $\hbar$
Electron mass → m
Bohr magneton → μ_B
Orbital magnetic quantum number $= 0, \pm 1, \pm 2, \ldots, \pm l$ → m_l

The magnetic field shifts the energy of each orbital state by an amount U. The interaction energy U depends on the value of m_l because m_l determines the orientation of the orbital magnetic moment relative to the magnetic field. This dependence is the reason m_l is called the magnetic quantum number.

The values of m_l range from $-l$ to $+l$ in steps of one, so an energy level with a particular value of the orbital quantum number l contains $(2l + 1)$ different orbital states. Without a magnetic field these states all have the same energy; that is, they are degenerate. The magnetic field removes this degeneracy. In the presence of a magnetic field they are split into $2l + 1$ distinct energy levels;

setting $2j + 1$ equal to an even number gives $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$, suggesting a half-integer angular momentum. This can't be understood on the basis of the Bohr model and similar pictures of atomic structure.

In 1925 two graduate students in the Netherlands, Samuel Goudsmidt and George Uhlenbeck, proposed that the electron might have some additional motion. Using a semiclassical model, they suggested that the electron might behave like a spinning sphere of charge instead of a particle. If so, it would have an additional *spin* angular momentum and magnetic moment. If these were quantized in much the same way as *orbital* angular momentum and magnetic moment, they might help explain the observed energy-level anomalies.

An Analogy for Electron Spin

To introduce the concept of **electron spin,** let's start with an analogy. The earth travels in a nearly circular orbit around the sun, and at the same time it *rotates* on its axis. Each motion has its associated angular momentum, which we call the *orbital* and *spin* angular momentum, respectively. The total angular momentum of the earth is the vector sum of the two. If we were to model the earth as a single point, it would have no moment of inertia about its spin axis and thus no spin angular momentum. But when our model includes the finite size of the earth, spin angular momentum becomes possible.

In the Bohr model, suppose the electron is not just a point charge but a small spinning sphere that orbits the nucleus. Then the electron has not only orbital angular momentum but also spin angular momentum associated with the rotation of its mass about its axis. The sphere carries an electric charge, so the spinning motion leads to current loops and to a magnetic moment, as we discussed in Section 27.7. In a magnetic field, the *spin* magnetic moment has an interaction energy in addition to that of the *orbital* magnetic moment (the normal Zeeman-effect interaction that we discussed in Section 41.4). We should see additional Zeeman shifts due to the spin magnetic moment.

As we mentioned, such shifts *are* indeed observed in precise spectroscopic analysis. This and a variety of other experimental evidence have shown conclusively that the electron *does* have a spin angular momentum and a spin magnetic moment that do not depend on its orbital motion but are intrinsic to the electron itself. The origin of this spin angular momentum is fundamentally quantum-mechanical, so it's not correct to model the electron as a spinning charged sphere. But just as the Bohr model can be a useful conceptual picture for the motion of an electron in an atom, the spinning-sphere analogy can help you visualize the intrinsic spin angular momentum of an electron.

Spin Quantum Numbers

Like orbital angular momentum, the spin angular momentum of an electron (denoted by $\vec{S}$) is found to be quantized. Suppose we have an apparatus that measures a particular component of $\vec{S}$, say the z-component S_z. We find that the only possible values are

Spin magnetic quantum number $= \pm\frac{1}{2}$

z-component of spin angular momentum of electron → $S_z = m_s\hbar$ ← Planck's constant divided by 2π (41.36)

$$S_z = m_s \hbar \tag{41.36}$$

This relationship is reminiscent of the expression $L_z = m_l\hbar$ for the z-component of orbital angular momentum, except that $|S_z|$ is *one-half* of $\hbar$ instead of an *integer* multiple. In analogy to the orbital magnetic quantum number m_l, we call the quantum number m_s the **spin magnetic quantum number.** Since m_s has only two possible values, $+\frac{1}{2}$ and $-\frac{1}{2}$, it follows that the spin angular momentum vector $\vec{S}$ can have only two orientations in space relative to the z-axis: "*spin up*" with a z-component of $+\frac{1}{2}\hbar$ and "*spin down*" with a z-component of $-\frac{1}{2}\hbar$.

Equation (41.36) also suggests that the magnitude S of the spin angular momentum is given by an expression analogous to Eq. (41.22) with the orbital quantum number l replaced by the **spin quantum number** $s = \frac{1}{2}$:

Magnitude of spin angular momentum of electron →

$$S = \sqrt{\tfrac{1}{2}\left(\tfrac{1}{2} + 1\right)}\hbar = \sqrt{\tfrac{3}{4}}\hbar \qquad (41.37)$$

Maximum value of spin magnetic quantum number $= \frac{1}{2}$

Planck's constant divided by 2π

The electron is often called a "spin-one-half particle" or "spin-$\frac{1}{2}$ particle."

We see that to label the state of the electron in a hydrogen atom completely, we need *four* quantum numbers: n, l, and m_l (described in Section 41.3) to specify the electron's motion relative to the nucleus, plus the spin magnetic quantum number m_s to specify the electron spin orientation.

To visualize the quantized spin of an electron in a hydrogen atom, think of the electron probability distribution function $|\psi|^2$ as a cloud surrounding the nucleus like those shown in Figs. 41.9 and 41.10. Then imagine many tiny spin arrows distributed throughout the cloud, either all with components in the $+z$-direction or all with components in the $-z$-direction. But don't take this picture too seriously.

Just as the orbital magnetic moment of the electron is proportional to its orbital angular momentum $\vec{L}$ (see Section 41.4), the electron's spin magnetic moment is proportional to its spin angular momentum $\vec{S}$. The z-component of the spin magnetic moment (μ_z) turns out to be related to S_z by

$$\mu_z = -(2.00232)\frac{e}{2m}S_z \qquad (41.38)$$

where $-e$ and m are (as usual) the charge and mass of the electron. When the atom is placed in a magnetic field, the interaction energy $-\vec{\mu} \cdot \vec{B}$ of the spin magnetic dipole moment with the field causes further splittings in energy levels and in the corresponding spectral lines.

Equation (41.38) shows that the gyromagnetic ratio for electron spin is approximately *twice* as great as the value $e/2m$ for *orbital* angular momentum and magnetic dipole moment. This result has no classical analog. But in 1928 Paul Dirac developed a relativistic generalization of the Schrödinger equation for electrons. His equation gave a spin gyromagnetic ratio of exactly $2(e/2m)$. It took another two decades to develop the area of physics called *quantum electrodynamics,* abbreviated QED, which predicts the value we've given to "only" six significant figures as 2.00232. QED now predicts a value that agrees with the currently accepted experimental value of 2.00231930436153(53), making QED the most precise theory in all science.

BIO Application Electron Spins and Dating Human Origins In many atoms, the net spin of all of the electrons is zero (as many electrons are "spin up" as are "spin down"). If these atoms are ionized and lose an electron, however, the net spin of the ion that remains is nonzero. This happens naturally in tooth enamel, where ionization is caused by radioactivity in the environment. The longer a tooth is exposed, the more ions are present. To find the age of fossil teeth, such as those in this skull of *Homo neanderthalensis*, a sample of the enamel is placed in a strong magnetic field. The ion spins align opposite to this field (become "spin down"). The sample is then illuminated with microwave photons of just the right energy to flip the spins to the higher-energy configuration aligned with the field ("spin up"). The amount of microwave energy absorbed in this process (called *electron spin resonance*) indicates the number of ions present and hence the age of the enamel.

EXAMPLE 41.6 ENERGY OF ELECTRON SPIN IN A MAGNETIC FIELD

SOLUTION

Calculate the interaction energy for an electron in an $l = 0$ state in a magnetic field with magnitude 2.00 T.

SOLUTION

IDENTIFY and SET UP: For $l = 0$ the electron has zero orbital angular momentum and zero orbital magnetic moment. Hence the only magnetic interaction is that between the $\vec{B}$ field and the spin magnetic moment $\vec{\mu}$. From Eq. (41.28), the interaction energy is $U = -\vec{\mu} \cdot \vec{B}$. As in Section 41.4, we take $\vec{B}$ to be in the positive z-direction so that $U = -\mu_z B$ [Eq. (41.32)]. Equation (41.38) gives μ_z in terms of S_z, and Eq. (41.36) gives S_z.

EXECUTE: Combining Eqs. (41.36) and (41.38), we have

$$\begin{aligned}\mu_z &= -(2.00232)\left(\frac{e}{2m}\right)\left(\pm\tfrac{1}{2}\hbar\right)\\ &= \mp\tfrac{1}{2}(2.00232)\left(\frac{e\hbar}{2m}\right) = \mp(1.00116)\mu_B\\ &= \mp(1.00116)(9.274 \times 10^{-24}\ \text{J/T})\\ &= \mp 9.285 \times 10^{-24}\ \text{J/T} = \mp 5.795 \times 10^{-5}\ \text{eV/T}\end{aligned}$$

Continued