27–4 The ambiguity of the field energy

Before we take up some applications of the Poynting formulas [Eqs. (27.14) and (27.15)], we would like to say that we have not really "proved" them. All we did was to find a *possible* "u" and a *possible* "$\boldsymbol{S}$." How do we know that by juggling the terms around some more we couldn't find another formula for "u" and another formula for "$\boldsymbol{S}$"? The new $\boldsymbol{S}$ and the new u would be different, but they would still satisfy Eq. (27.6). It's possible. It can be done, but the forms that have been found always involve various *derivatives* of the field (and always with second-order terms like a second derivative or the square of a first derivative). There are, in fact, an infinite number of different possibilities for u and $\boldsymbol{S}$, and so far no one has thought of an experimental way to tell which one is right! People have guessed that the simplest one is probably the correct one, but we must say that we do not know for certain what is the actual location in space of the electromagnetic field energy. So we too will take the easy way out and say that the field energy is given by Eq. (27.14). Then the flow vector $\boldsymbol{S}$ must be given by Eq. (27.15).

It is interesting that there seems to be no unique way to resolve the indefiniteness in the location of the field energy. It is sometimes claimed that this problem can be resolved by using the theory of gravitation in the following argument. In the theory of gravity, all energy is the source of gravitational attraction. Therefore the energy density of electricity must be located properly if we are to know in which direction the gravity force acts. As yet, however, no one has done such a delicate experiment that the precise location of the gravitational influence of electromagnetic fields could be determined. That electromagnetic fields alone can be the source of gravitational force is an idea it is hard to do without. It has, in fact, been observed that light is deflected as it passes near the sun—we could say that the sun pulls the light down toward it. Do you not want to allow that the light pulls equally on the sun? Anyway, everyone always accepts the simple expressions we have found for the location of electromagnetic energy and its flow. And although sometimes the results obtained from using them seem strange, noboby has ever found anything wrong with them—that is, no disagreement with experiment. So we will follow the rest of the world—besides, we believe that it is probably perfectly right.

We should make one further remark about the energy formula. In the first place, the energy per unit volume in the field is very simple: It is the electrostatic energy plus the magnetic energy, *if* we write the electrostatic energy in terms of E^2 and the magnetic energy as B^2. We found two such expressions as *possible* expressions for the energy when we were doing static problems. We also found a number of other formulas for the energy in the electrostatic field, such as $\rho\phi$, which is *equal* to the integral of $\boldsymbol{E}\cdot\boldsymbol{E}$ in the electrostatic case However, in an electrodynamic field the equality failed, and there was no obvious choice as to which was the right one. Now we know which is the right one. Similarly, we have found the formula for the magnetic energy that is correct in general The right formula for the energy density of *dynamic* fields is Eq (27.14)

27–5 Examples of energy flow

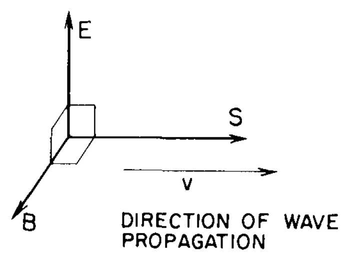

Fig. 27–2. The vectors ***E***, ***B***, and ***S*** for a light wave.

Our formula for the energy flow vector $\boldsymbol{S}$ is something quite new. We want now to see how it works in some special cases and also to see whether it checks out with anything that we knew before. The first example we will take is light. In a light wave we have an $\boldsymbol{E}$ vector and a $\boldsymbol{B}$ vector at right angles to each other and to the direction of the wave propagation. (See Fig 27–2.) In an electromagnetic wave, the magnitude of $\boldsymbol{B}$ is equal to $1/c$ times the magnitude of $\boldsymbol{E}$, and since they are at right angles,

$$|\boldsymbol{E}\times\boldsymbol{B}| = \frac{E^2}{c}$$

Therefore, for light, the flow of energy per unit area per second is

$$S = \epsilon_0 c E^2. \tag{27.16}$$