$$if\ h(x)\ is\ continious, integrable\ on [a,b], h(a)=h(b)=0$$

$$prove:\ for\ any\ h(x)$$

$$if\ \ I=\int_a^b M(x)h(x)$$

$$if\ \ I=0\ Then\ M(x)=0$$

$$solution:$$

$$M(x)h(x)=\frac{((M(x)+h(x))^2-M(x)^2-h(x)^2)}{2}$$

$$therefore: \int_a^b (M(x)+h(x))^2\,dx=\int_a^b (M(x)^2+h(x)^2)\,dx$$

$$differentiation\ of\ both\ with\ respect\ \ x$$

$$\int_a^b (M(x)+h(x))(M^{\cdot}(x)+h^{\cdot}(x))\,dx=\int_a^b (M(x)M^{\cdot}(x)+h(x)h^{\cdot}(x))\,dx$$

$$\int_a^b (M(x)M^{\cdot}(x)+M(x)h^{\cdot}(x)+M^{\cdot}(x)h(x)+h(x)h^{\cdot}(x))\,dx=\int_a^b (M(x)M^{\cdot}(x)+h(x)h^{\cdot}(x))\,dx$$

$$Since\ The\ integration\ is\ taken\ on\ [a,b]$$

$$M(x)h^{\cdot}(x)+M^{\cdot}(x)h(x)=0$$

$$\frac{d}{dx}(M(x)h(x))=0$$

$$therefore\ M(x)h(x)=c$$

$$Since\ h(a)=h(b)=0$$

$$therefore\ c=0$$

$$M(x)=0\ because\ we\ can\ find\ function\ h(x)\ different\ \text{from}\ h(x)=0$$