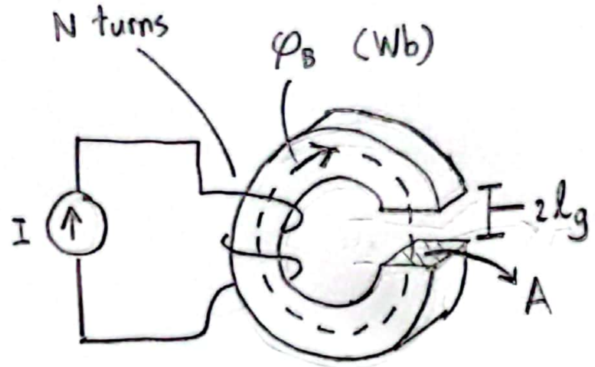
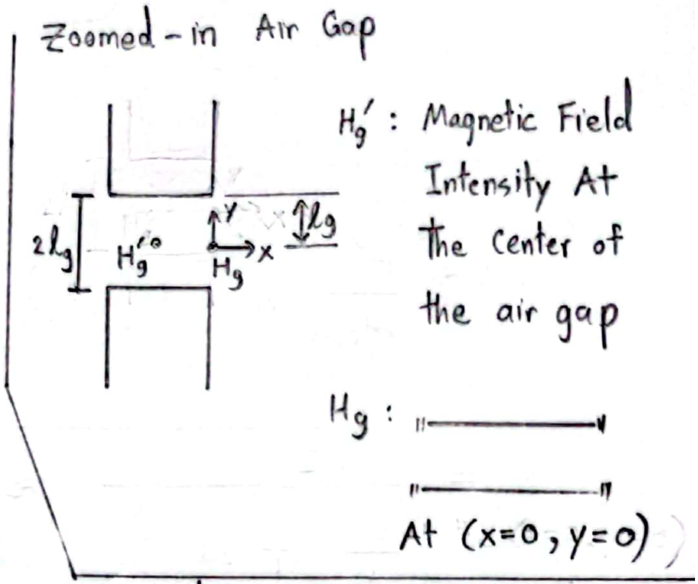
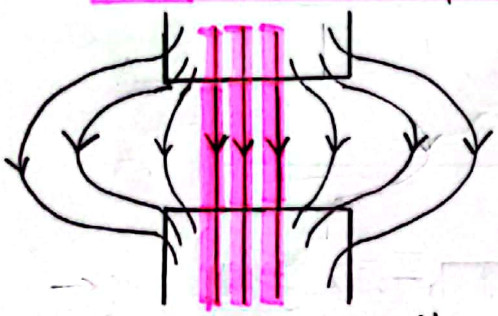
# Fringing Field Formula and Winding Loss Due to Air Gap




## Assumption:

→ Magnetic Flux Density Along the Center of the Air Gap ($B_g'$) is constant and equal to the magnetic flux density along the center of the core ($B_m$)



$$B_g' = B_m$$

## Apply Ampere's Law Along $\varphi_B$

$$NI = \oint_{\text{clockwise}} \vec{H}\cdot d\vec{l}$$

clockwise ↪ direction of $\varphi_B$

$$= \oint_{\text{clockwise}} \left(\vec{H}_m\cdot d\vec{l} + \vec{H}_g'\cdot d\vec{l}\right)$$

$$NI = H_m l_m + H_g'(2l_g) \quad \text{——— ①}$$

$l_m$ → Core's length

From $B_m = B_g'$

$$\mu_0 \mu_{r,m} H_m = \mu_0 \cancel{\mu_{r,g}}^{\,1} H_g'$$

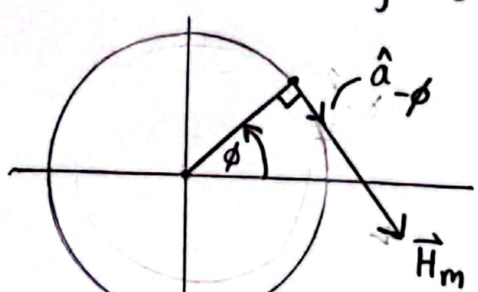
$\mu_{r,m}$: Relative Permeability of the core

### Side Note:

$$\vec{H}_m = H_m\,\hat{a}_{-\phi}$$

$$\vec{H}_g' = H_g'\,\hat{a}_{-\phi}$$

where $H_m$ and $H_g' > 0$




$$\hat{a}_{-\phi} = -\hat{a}_{\phi}$$

$$H_m = \frac{H_g'}{\mu_{r,m}} \quad \text{——— (2')}$$

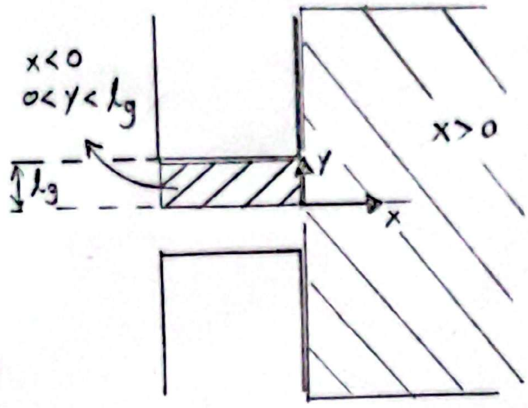
<u>Substitute (2') into (1')</u>

$$NI = \frac{H_g'}{\mu_{r,m}} l_m + H_g'(2l_g)$$

$$\boxed{H_g' = \frac{NI}{\left(\frac{l_m}{\mu_{r,m}} + 2l_g\right)} \approx \frac{NI}{2l_g}} \quad \text{——— (II.2, Roshen)}$$

$\mu_{r,m} \rightarrow$ 1000, 1750, etc.

$$\boxed{H_g \approx 0.9 H_g' = \frac{0.9NI}{2l_g}} \quad \text{——— (II.3, Roshen)}$$




The general form of the scalar potential function is given by Roshen's paper.

$$\vec{H} = -\nabla\Phi(x,y)$$

$$\Phi(x,y) = \begin{cases} \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi y}{l_g}\right) e^{\frac{n\pi x}{l_g}} + \frac{H_g y}{2} \quad --(II.4, Roshen) \\ \text{where } x < 0 \text{ and } 0 < y < l_g \\ \int_0^{\infty} b(p)\sin(py)\, e^{-px} dp \quad ---(II.5, Roshen) \\ \text{where } x > 0 \end{cases}$$

$a_n$, $n$, and $b(p)$ are constants

At $x=0$ and $0<y<l_g$, II.4 = II.5

$$\phi(0,y) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi y}{l_g}\right) + \frac{H_g y}{2} = \int_0^{\infty} b(p)\sin(py)\,dp \quad \text{—— (3)}$$

At $x=0$ and $l_g < y < \infty$,

$$\vec{H}_g = \begin{bmatrix} H_x(x,y) \\ H_y(x,y) \end{bmatrix} = \begin{bmatrix} 0 \\ -H_g \end{bmatrix} = -H_g \hat{a}_y \quad \text{where } H_g > 0$$

From $\vec{H} = -\nabla\Phi(x,y)$

$$\begin{bmatrix} 0 \\ -H_g \end{bmatrix} = \begin{bmatrix} -\frac{\partial}{\partial x}\Phi(x,y) \\ -\frac{\partial}{\partial y}\Phi(x,y) \end{bmatrix}$$

Integrate x - components

$$\int 0 \, dx = \int -\frac{\partial}{\partial x} \Phi(x,y) \, dx$$

$$g(y) = -\Phi(x,y)$$

Function of y

$$\Phi(x,y) = g(y) \quad \text{——— (4)}$$

Integrate y - components

$$\int -H_g \, dy = \int -\frac{\partial}{\partial y} \Phi(x,y) \, dy$$

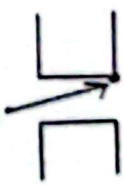
$$\Phi(x,y) = H_g y + f(x) \quad \text{——— (5)}$$

Compare (4) and (5) $\longrightarrow f(x) = 0$ and $g(y) = H_g y$

$$\Phi(x,y) = H_g y$$

At $y = l_g$ and $x = 0$

$\Phi(0, l_g) = H_g l_g \longrightarrow$ Located at the corner of the core



$$\therefore \Phi(0,y) = H_g l_g \quad \text{where } x = 0 \text{ and } \underbrace{l_g < y < \infty}_{\text{Along the edge of the core}} \quad \text{——— (6)}$$

Substitute (II.5) into ⑥ (where $x=0$)

$$\int_0^\infty b(p)\sin(py)\,dp = H_g l_g \quad \text{——} ⑦$$

Grouping ③ and ⑦ together

$$\int_0^\infty b(p)\sin(py)\,dp = \begin{cases} \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi y}{l_g}\right) + \frac{H_g y}{2} & \text{where } x=0,\ 0<y<l_g \\ H_g l_g & \text{where } x=0,\ l_g<y<\infty \end{cases}$$

(II.6, Roshen)

$$= \Phi(x=0, y)$$