

Let X be an Inner Product Space. If for every closed subspace M , $M^{\perp\perp} = M$, then X is a Hilbert Space (It's complete).

Hint: Use the following map: $T : X \longrightarrow \tilde{X} : T(y) = (x, y) = f(x)$ where (x, y) is the inner product of X .

Relevant equations: S^{\perp} is always closed to every $S \subset X$

Attempt to solution. I don't really know how to solve it, most theorems I have read, have Hilbert Space as a hypothesis. The only idea I had was trying to prove that $\overline{T(M^{\perp})} = M^{\circ}$ (Where M° is the Null Space of M).