

Chapter IV

Simplified Methods Of Furniture Analysis

Introduction

Although the structural analysis of most furniture is a complex process that requires considerable technical skill and involves substantial numbers of tedious computations, assumptions can often be made about the structural behavior of the furniture that significantly reduce the difficulty of the analysis. In the case of frame construction, for example, it is often assumed that slender members that can carry only modest bending loads owing to their geometry may be treated as though they were pinned at each end when, in fact, they may be fixed. This simplifying assumption substantially reduces the amount of time required to analyze a frame and usually has only a minor effect on the accuracy of the solution. Similarly, in analyzing case and carcass construction, it is often assumed that the stiffening effect of the frame supporting the panels is small enough to be neglected. This particular assumption reduces a complex problem to one that can be solved by simple hand computation methods. It must be realized here, however, that such assumptions cannot be made arbitrarily. Instead they must be based on previous experience and sound engineering judgement.

Notation

When carrying out an analysis of a piece of furniture, it is necessary to employ a standard system of notation so that the forces acting on the furniture and its parts along with the resulting deflections may be systematically identified and labeled. Forces and displacements may be graphically represented by arrows that indicate both their path of action and their direction.

From a mathematical point of view, forces and displacements may have either a positive or a negative direction. The positive direction of all six possible independent forces or deflections that can occur at a point are

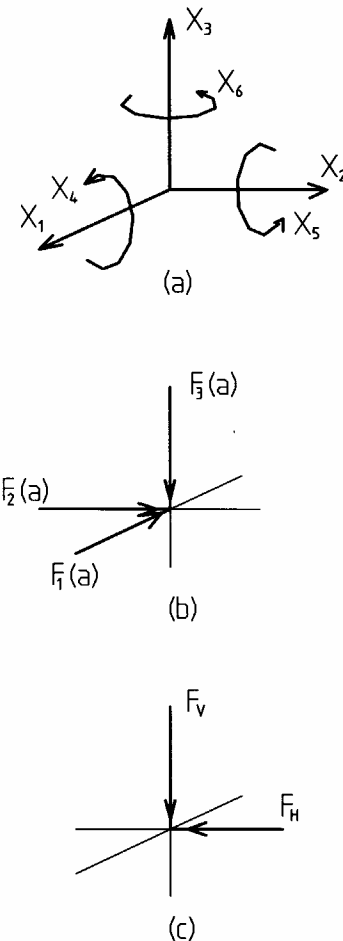


Figure 4-1. [f4-1.dxf]

given in Figure (4-1a) where the symbol "X" denotes either a force or a displacement.

Forces and Displacements

When an external force acts on a joint or a part, it will be denoted by a capital letter "F" as illustrated in Figure (4-1b). The accompanying subscript gives its line of action, corresponding to the appropriate coordinate direction, whereas the letter in parentheses identifies its point of application. Thus, the quantity $F_3(a)$ denotes a vertical force acting at point (a), whereas the quantity $F_2(a)$ denotes a horizontal force acting at the same point.

Often, the local x_2 - and x_3 - coordinates coincide with the horizontal and vertical directions, respectively, so that the symbols F_H and F_V may at times be used instead of F_2 and F_3 , as shown in Figure 4-1c. If a point of application such as point (b) is specified, these symbols will be of the form $F_H(b)$ or $F_V(a)$. Frequently, however, they are used to represent a more general type of force such as someone pushing against the end of a table where the point of application is less specifically defined.

For simplicity, the joints of a piece of furniture should be labeled first. The members, or parts, can then be defined and labeled in terms of the joints. Member (c-d), for example, refers to the member that links joint (c) to joint (d).

Internal forces that act within the members are represented by the small letter "f". The accompanying subscript defines the type of force, and the letters within the parentheses indicate both the point of application and the member on which the force is acting, Figure 4-2. As an example, the symbol $f_4(ef)$ indicates an internal bending moment acting on the e-end of member (e-f). In all cases, the symbol f_4 represents a bending moment, f_3 a shear force, and f_2 an axial force.

Displacements or, deflections, will be represented by the small letter "y". Subscripts and parentheses have the same meaning as when they are used with forces.

Bending Moments and Moment Equations

When an external force or a reaction acts on, or is applied to, some part of a piece of furniture, internal reactive forces are developed in the part that may include axial, shear, and bending forces. Bending forces,

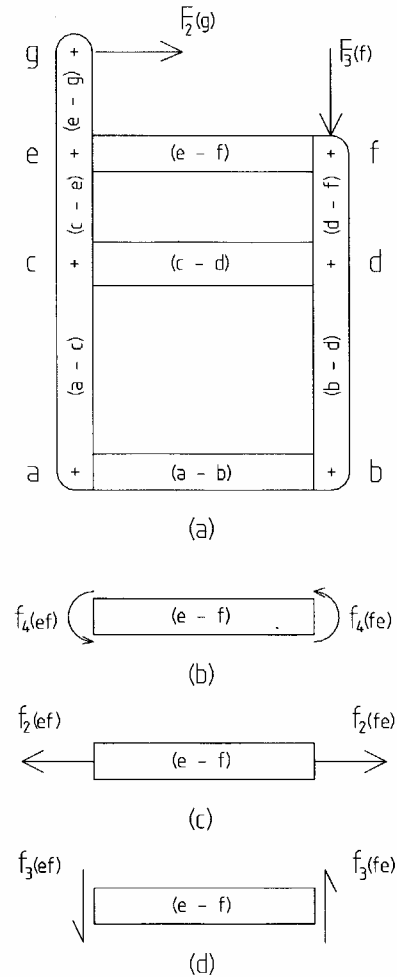


Figure 4-2. [f4-2a.dxf-50]

traditional practice, are of particular concern because they are most often the cause of structural failure in furniture.

In Figure 4-3, a horizontal force, F_H , is shown acting on the backpost of a chair. The backpost of the chair is represented as a free body in Figure 4-3b. The forces developed internally in the "cut" section of the backpost at joint (b) consist of a horizontal shear force and a resisting moment developed by the tensile and compressive forces distributed over the section. The symbol "V" represents the shear force acting on the section at point (b), "C" represents the resultant of the compressive stresses, and "T" represents the resultant of the tensile stresses. The neutral axis of the backpost lies between these two forces. Static equilibrium requires that the sum of the tensile forces on one side of the neutral axis must equal the sum of the compressive forces on the other side. The location of the neutral axis always coincides with the centroid of the cross sectional area of the member.

Referring to the backpost, the product of the external force, F_H , multiplied by the distance from its point of application to joint (b), i.e., h , is termed the external bending force, or, "moment", acting at the point. The moment of the couple formed by forces C and T, i.e.,

$$C \times j/2 + T \times j/2 = (C + T) \times j/2,$$

is called the internal or resisting moment at the section. Equilibrium requires that the internal resisting moment must balance the external moment so that

$$F_H \times h = (C + T) \times j/2$$

In summary, equilibrium requires that the internal resisting moment be equal in magnitude but opposite in direction to the external moment. Further, in keeping with previously discussed sign conventions, moments that tend to produce counterclockwise rotations are termed positive, whereas moments that tend to produce clockwise rotations are termed negative.

In Figure 4-3b, the following moment equation may be written to express the relationship between the external moment, $F_H \times h$, and the internal resisting moment, $f_4(ba)$, acting on the b-end of the backpost, i.e.,

$$f_4(ba) - F_H \times h = 0.$$

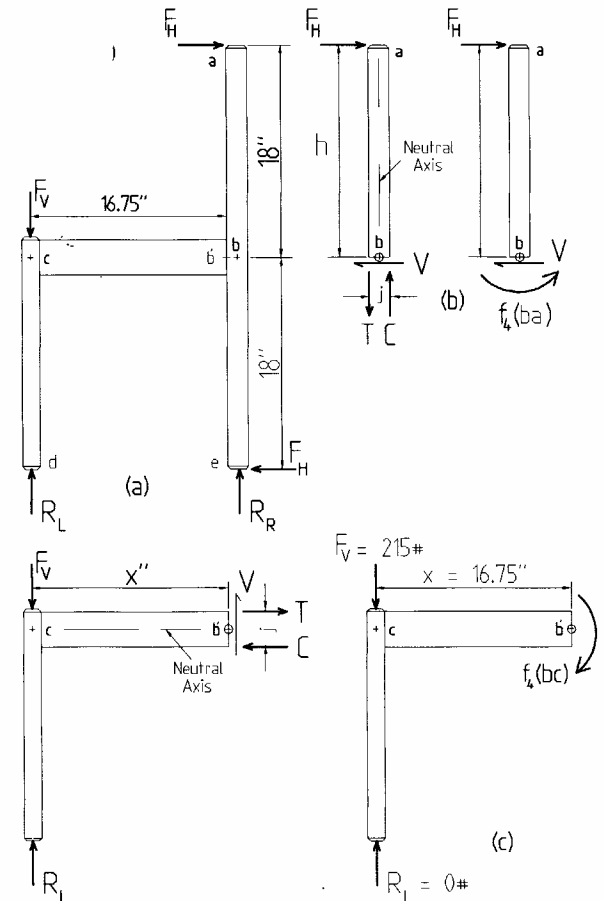


Figure 4-3. [f3-22.dxf]

To demonstrate this relationship, assume that $F_H = 100 \text{ lb}$ and $h = 18 \text{ in}$. For this loading,

$$f_4(ba) = 100 \text{ lb} \times 18 \text{ in} = 1800 \text{ in} \cdot \text{lb}.$$

Notice that this force is positive and therefore acts on the b-end of the backpost in a counterclockwise direction; the assumed direction of action shown in Figure 4-3b, accordingly, is correct.

Likewise, the internal moment acting on the b-end of the chair side rail, Figure 4-3a, is given by the expression

$$f_4(bc) - F_V \times 16.75 \text{ in} - R_L \times 16.75 \text{ in} = 0$$

Note that for generality, it is assumed that F_V and R_L both act in a positive, i.e., upwards direction.

For the case in which the chair is tilted slightly backwards so that the front leg is lifted off the floor, $R_L = 0$ and $F_V = -215 \text{ lb}$ as shown in Figure 4-3c. The internal moment acting on the b-end of the seat side rail may then be found from the expression

$$f_4(bc) + F_V \times 16.75 \text{ in} = 0.$$

since $R_L = 0$ and F_V is negative because it acts downward, Figure 4-3c, which tends to produce counterclockwise rotation. Solving, gives $f_4(bc) = F_V \times 16.75 \text{ in} = 215 \text{ lb} \times 16.75 \text{ in} = 3600 \text{ in} \cdot \text{lb}$.

Similar expressions can be written for tables. As an example, in Figure 4-4, the internal resisting moment acting on leg (c-d) at joint (c), specifically, $f_4(cd)$, must be equal in magnitude but opposite in sign to the sum of the external moments acting on the joint. Thus, the internal moment acting on the top of the leg at joint (c), that is, $f_4(cd)$ is found by summing the external moment and internal moments acting at this point, i.e.,

$$f_4(cd) + F_H \times h = 0$$

Note that it is assumed in this general expression that F_H acts in a positive direction, i.e., to the right. Substituting the appropriate values, i.e., $F_H = -100 \text{ lb}$, into the above expression gives

$$f_4(cd) - 100 \text{ lb} \times 28 \text{ in} = 0, \text{ or,}$$

$$f_4(cd) = 100 \text{ lb} \times 28 \text{ in} = 2800 \text{ in} \cdot \text{lb}$$

Since the forces must be in equilibrium, the internal moment acting on the c-end of the rail, i.e., $f_4(cb)$, must equal $f_4(cd)$. This moment may also be found by summing moments acting on the c-end of the rail. Summing moments about joint (c) gives

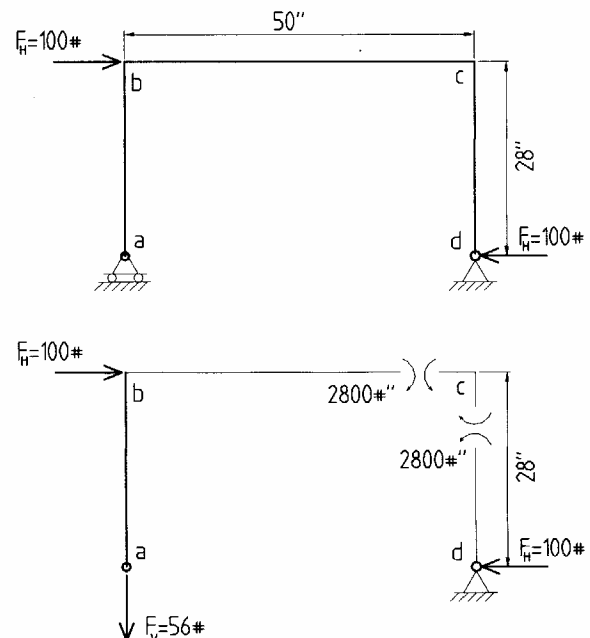


Figure 4-4. [f3-23b.dxf]

$f_4(cb) + 56 \text{ lb} \times 50 \text{ in} = 0$, or,
 $f_4(cb) = -2800 \text{ in} \cdot \text{lb}$. The positive answer obtained indicates that the direction of the internal moment is clockwise as shown in Figure 4-4.

The same techniques may be used to obtain the internal resisting moment acting at any point along the length of any member of a piece of furniture. Consider the table frame shown in Figure 4-5. This table has a vertical 240-lb. load acting downward on the rail. The vertical floor reaction force acting at joint (a), Figure 4-5, is 144 lb. and at joint (d), 96 lb.

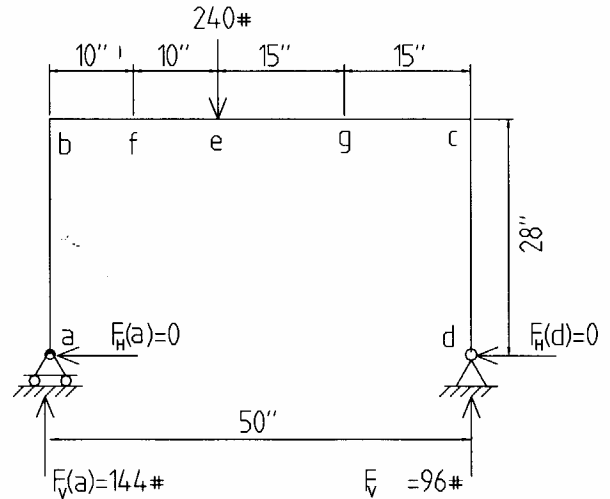


Figure 4-5. [f3-24.dxf]

The internal resisting moment acting at any point along the rail from joint (b) to point (e) is found by summing moments about the desired point. Thus, at point (f), the internal resisting moment is $f_4(f) - 144 \text{ lb} \times 10 \text{ in} = 0$ or,

$$f_4(f) = 144 \text{ lb} \times 10 \text{ in} = 1440 \text{ in} \cdot \text{lb}$$

In like manner, the internal resisting moment at point (e) is

$$f_4(e) - 144 \text{ lb} \times 20 \text{ in} = 0$$

or,

$$f_4(e) = 144 \text{ lb} \times 20 \text{ in} = 2880 \text{ in} \cdot \text{lb}$$

Similarly, the internal resisting moment at point (g) is

$$f_4(g) - 144 \text{ lb} \times 35 \text{ in} + 240 \text{ lb} \times 15 \text{ in} = 0$$

or,

$$f_4(g) = 144 \text{ lb} \times 35 \text{ in} - 240 \text{ lb} \times 15 \text{ in} = 1440 \text{ in} \cdot \text{lb}$$

Finally, the internal resisting moment at joint (c) is

$$f_4(c) - 144 \text{ lb} \times 50 \text{ in} + 240 \text{ lb} \times 30 \text{ in} = 0$$

or,

$$f_4(c) = 7200 \text{ in} \cdot \text{lb} - 7200 \text{ in} \cdot \text{lb} = 0,$$

as it must.

When more than one load acts on the structure as shown in Figure 4-6, the same procedure is followed, i.e., the moments resulting from the various forces acting on the frame are summed about the point in question. Proceeding as before, the vertical floor reaction force, F_v , as shown in Figure 4-6 is 126.7 lb. The internal resisting moment acting at a point, say point (e), for example is then found by summing moments about that point, i.e.,

$$f_4(e) - 126.7 \text{ lb} \times 48 \text{ in} + 100 \text{ lb} \times 28 \text{ in} + 150 \text{ lb} \times 12 \text{ in} = 0,$$

$$\text{or, } f_4(e) = 1482 \text{ in} \cdot \text{lb}.$$

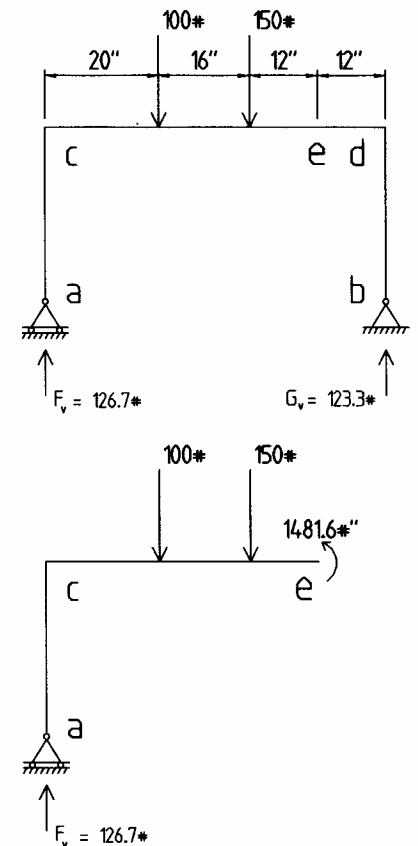


Figure 4-6. [f3-25.dxf]

In the case of line loads, such as the load shown acting on the rail of the table in Figure 4-7, that portion of the load acting on the section of the rail in question is replaced by a resultant force that acts through the centroid of that portion of the load acting on the section.

The total load exerted on the rail by the line load amounts to $5 \text{ lb/in} \times 60 \text{ in} = 300 \text{ lb}$. Owing to symmetry, half of this load is transferred to each reaction point so that $F_V = G_V = 150 \text{ lb}$. The internal resisting moment at point (f), for example, may then be found from the expression

or,

$$f_4(f) - 150 \text{ lb} \times 30 \text{ in} + 150 \text{ lb} \times 15 \text{ in} = 0,$$

Support Reactions

The magnitudes of support reactions are found by applying the laws of equilibrium to the structure in question. As an example, a table side frame is shown in Figure 4-8 that is supported by a pin at one end and by a roller at the other end. The table is 60 inches long; a vertical force acts downward on it at a point 20 inches from its left end. Summing moments about the support at point G gives

$$-F_V \times 60 \text{ in} + 100 \text{ lbs} \times 40 \text{ in} = 0$$

or,

$$F_V = 4000 / 60 = 66.7 \text{ lbs}$$

Similarly, summing moments about the support at point F gives

$$G_V \times 60 \text{ in} - 100 \text{ lbs} \times 20 \text{ in} = 0,$$

or,

$$G_V = 2000 / 60 = 33.3 \text{ lbs}.$$

To check these values, the sum of forces in the vertical x_3 direction is calculated. Thus,

$$\sum F_3 = -100 \text{ lbs} + 66.7 \text{ lbs} + 33.3 \text{ lbs} = 0.$$

It should be noted that since no horizontal forces act on the table, there are no horizontal support reactions.

A more complex problem is given in Figure 4-9. In analyzing this chair side frame, it has been decided that the front leg will not carry any horizontal loads; therefore, it has been placed on a roller. It should be noted that this was the analyst's decision and would not necessarily be true for every case. Summing forces in the horizontal direction gives

$$50 \text{ lbs} + G_H = 0,$$

or,

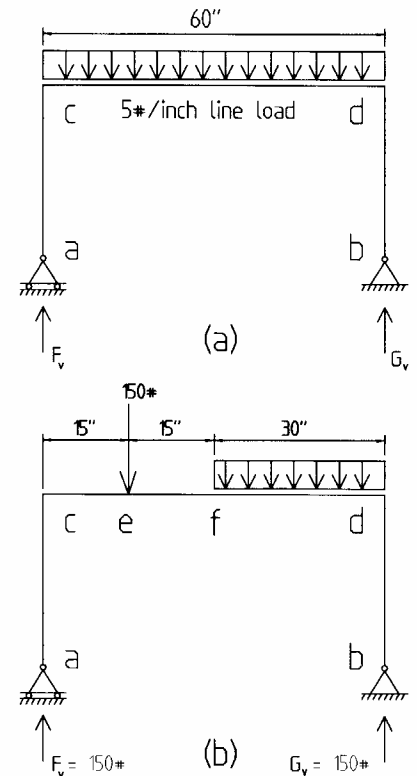


Figure 4-7. [f3-26.dxf]

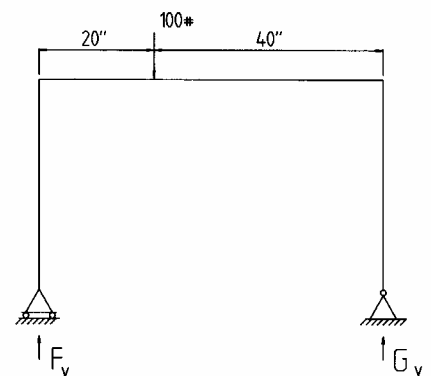


Figure 4-8. [f3-6.dxf]

$$G_H = -50 \text{ lbs.}$$

Thus, G_H acts to the left as shown in Figure 4-9.

F_V is found by summing moments about point G. Carrying out this operation gives

$$-50 \text{ lbs} \times 32 \text{ in} - F_V \times 20 \text{ in} + 180 \text{ lbs} \times 10 \text{ in} = 0,$$

or,

$$F_V = (-1600 + 1800) / 20 = 10 \text{ lbs.}$$

The positive sign of F_V indicates, in this case, that it is operating upward as shown. The remaining vertical force, G_V , is found by summing forces in the vertical direction.

$$G_V + 10 \text{ lbs} - 180 \text{ lbs} = 0$$

or,

$$G_V = 170 \text{ lbs.}$$

To further illustrate the procedures involved in evaluating support forces, let us determine the vertical and horizontal support forces, F_V , G_V , and F_H , G_H , respectively for the chair side frame shown in Figure 4-10.

Since the front leg is supported on a roller, we know that $F_H = 0$. Therefore,

$$G_H - 50 \text{ lbs} + 100 \text{ lbs} = 0,$$

or

$$G_H = -50 \text{ lbs.}$$

applied in the direction shown. Next, summing moments about point G gives

$$-F_V \times 18 + (50 \times 16) + (190 \times 18) + (50 \times 9) + (50 \times 9) - (100 \times 32) = 0 \text{ or}$$

$$F_V = 1920 / 18 = 106.7 \text{ lbs.}$$

Similarly, summing moments about point F gives

$$G_V \times 18 - (100 \times 32) - (50 \times 9) - (50 \times 9) + (50 \times 16) = 0,$$

or,

$$G_V = 3300 / 18 = 183.3 \text{ lbs.}$$

Summing forces in the x_3 - direction gives

$$\sum F_3 = 106.7 + 183.3 - 190 - 50 - 50 = 0 \text{ lbs},$$

which serves as a check on our calculations.

Example Problem

For the table frame shown in Figure 4-11,

a. Calculate the vertical floor reaction forces. Calculate the bending force acting at the center of the table top.

$$F_V = 126.67, G_V = 123.33, F_4(30) = 2800 \text{ in-lb.}$$

b. Calculate the vertical and horizontal floor reaction forces. Calculate the bending moment acting at joint d .

$$F_V = -50, G_V = 50, F_4(d) = 3000 \text{ in-lb.}$$

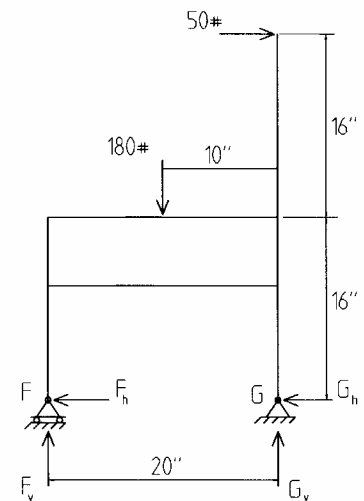


Figure 4-9. [f3-7.dxf-30]

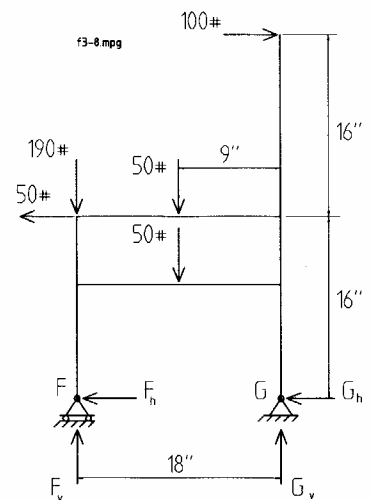


Figure 4-10. [f 3-8.dxf]

- c. Calculate the vertical floor reaction forces and the bending moments acting at the center and at the third points along the length of the table top.
 $F_v = 150$, $G_v = 150$, $F_4(30) = 2250$ in-lb.
 d. Calculate the vertical and horizontal floor reaction forces. Calculate the bending moment acting at joint (d) and at the center of the table top.

Force Analysis of Simple Frames

Chair Side Frame

Consider the simple chair side frame shown in Figure 4-12a. As long as this frame does not include a stretcher, the forces acting on the members and joints can be determined from a consideration of the laws of static equilibrium alone; i.e.,

$$\sum F_2 = 0, \sum F_3 = 0, \sum F_4 = 0.$$

The first step in analyzing this frame is to calculate the floor reactions for the loading shown. To find the floor reaction force, $F_3(a)$, external moments are summed about point (d), i.e.,

$$-F_3(a) \times 20 + 100 \text{ lb} \times 18 \text{ in} - 56.25 \text{ lb} \times 32 \text{ in} = 0$$

or,

$$F_3(a) = (100 \text{ lb} \times 18 \text{ in} - 56.25 \text{ lb} \times 32 \text{ in}) / 20 \text{ in} = 0$$

i.e., the floor reaction force is zero because the chair is essentially "balanced" in position by the external loads applied to it. $F_3(d)$ is found next by summing forces in the x3-direction, i.e.,

$$F_3(d) + 0 - 100 \text{ lb} = 0, \text{ or, } F_3(d) = 100 \text{ lb}.$$

Since $F_3(a) = 0$, i.e., the chair is in a potential "tilt" condition, it is logical to assume that $F_2(a) = 0$. Thus, summing forces in the x_2 -direction, it is seen that

$$F_2(a) + F_2(d) + 56.25 \text{ lb} = 0,$$

or,

$$F_2(d) = -56.25 \text{ lb}.$$

At this point, we now have the information needed to calculate the forces acting on each

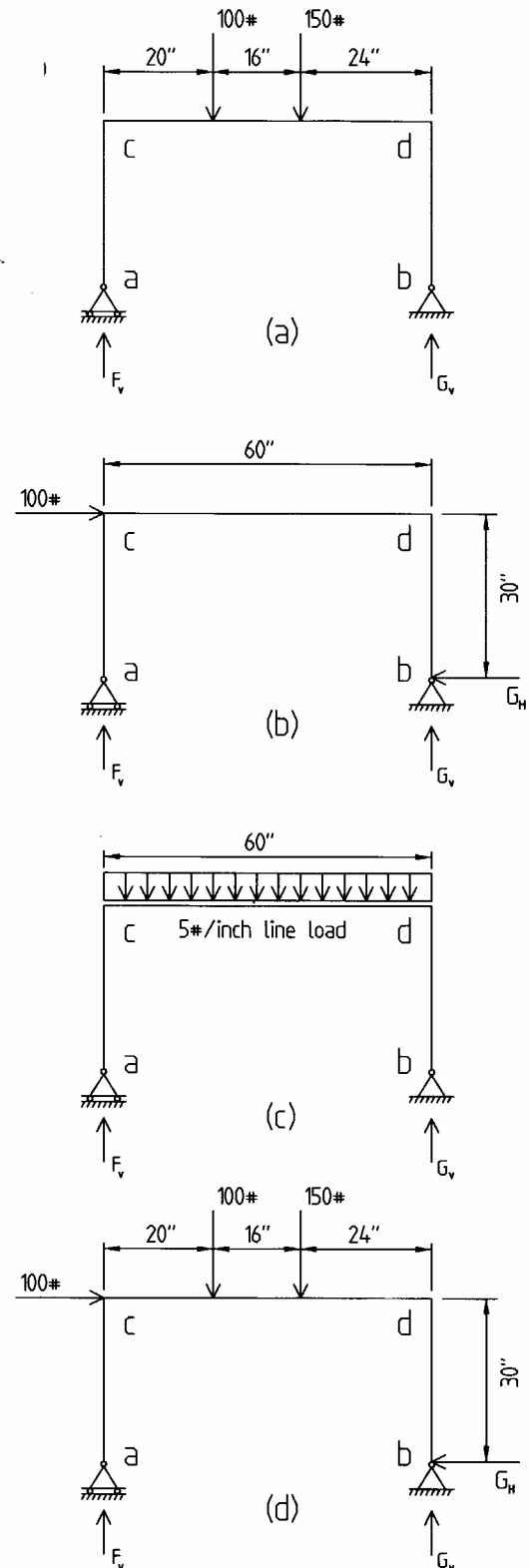


Figure 4-11.

member and joint. The internal moment acting on leg (c-d) at joint (c), for example, is

$$f_4(cd) + 100 \text{ lb} \times 2 \text{ in} - 56.25 \text{ lb} \times 16 \text{ in} = 0,$$

or,

$$f_4(cd) = 56.25 \text{ lb} \times 16 \text{ in} - 100 \text{ lb} \times 2 \text{ in} = 700 \text{ in-lb}$$

Similarly, the internal moment acting on the backpost, member (c-e), at joint (c) is

$$f_4(ce) - 56.25 \text{ lb} \times 16 \text{ in} = 0,$$

or,

$$f_4(ce) = 56.25 \text{ lb} \times 16 \text{ in} = 900 \text{ in-lb}.$$

Likewise, the moment acting on the seat side rail, member (b-c), at joint (c) is

$$f_4(cb) + 100 \text{ lb} \times 16 \text{ in} = 0,$$

or,

$$f_4(cb) = -100 \text{ lb} \times 16 \text{ in} = -1600 \text{ in-lb}.$$

These internal resisting moments are shown in Figure 4-12b along with the external loads and floor reaction forces acting on the frame. It is these forces that the joints and members must be designed to resist. Joint (c) of the seat side rail, for example, must be designed to resist a moment of at least 1600 in-lb. Methods of designing the members and joints in order to safely resist these forces will be treated in subsequent chapters. In this chapter we will concern ourselves solely with calculating the magnitude of the forces.

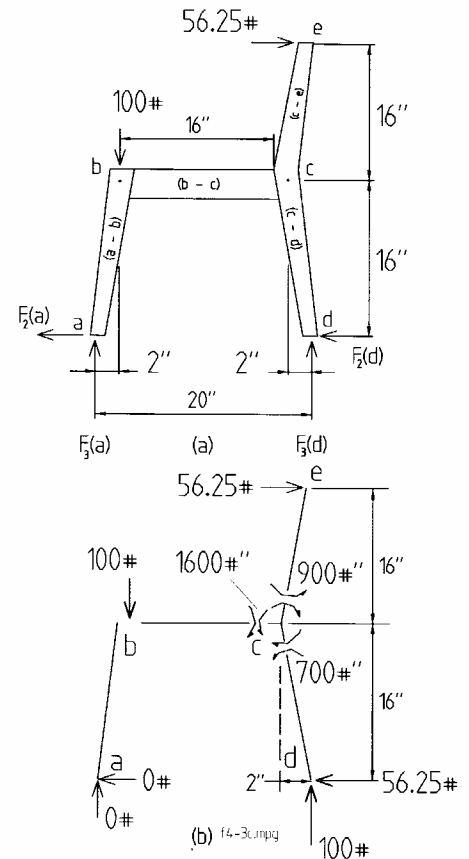


Figure 4-12. [f4-3.dxf]

Table Side Frames

Simple table frames may be analyzed in a manner similar to that used for chair frames. The first step in analyzing such frames is again to determine the floor reaction forces.

For the frame illustrated in Figure 4-13a with the loading shown, the vertical floor reaction force at point (a) is found by summing moments about joint (d), i.e.,

$$-F_3(a) \times 60 \text{ in} - 100 \text{ lb} \times 28 \text{ in} + 200 \text{ lb} \times 30 \text{ in} = 0,$$

or,

$$F_3(a) = (-100 \text{ lb} \times 28 \text{ in} + 200 \text{ lb} \times 30 \text{ in}) / 60 \text{ in} = 53.33 \text{ lb}$$

Similarly, the floor reaction force at point (d) is found by summing external moments about point (a); i.e.,

$$F_3(d) \times 60 \text{ in} - 100 \text{ lb} \times 28 \text{ in} - 200 \text{ lb} \times 30 \text{ in} = 0,$$

or,

$$F_3(d) = (100 \text{ lb} \times 28 \text{ in} + 200 \text{ lb} \times 30 \text{ in}) / 60 \text{ in} = 146.7 \text{ lb}$$

Let us assume here that $F_2(a) = 0$ so that the entire side thrust force is carried by one leg, i.e., $F_2(d) = -100 \text{ lbs}$. This assumption was made deliberately so that one leg could be designed to carry the entire side thrust force. The corresponding internal moment acting on leg (c-d) at joint (c) as a result of this side thrust force is found by summing forces about joint (c); i.e.,

$$f_4(cd) - 100 \text{ lb} \times 28 \text{ in} = 0,$$

or,

$$f_4(cd) = 2800 \text{ in-lb}.$$

For the table shown, the internal moment acting on the c-end of member (c-b) must be equal but opposite in sign to this value, i.e.,

$$f_4(cb) + 200 \text{ lb} \times 30 \text{ in} - 53.3 \text{ lb} \times 60 \text{ in} = 0$$

or,

$$f_4(cb) = -2800 \text{ in-lb}.$$

Finally, because $F_2(a) = 0$, the bending moments at joint (b), $f_4(bc)$ and $f_4(ba)$, are both 0.

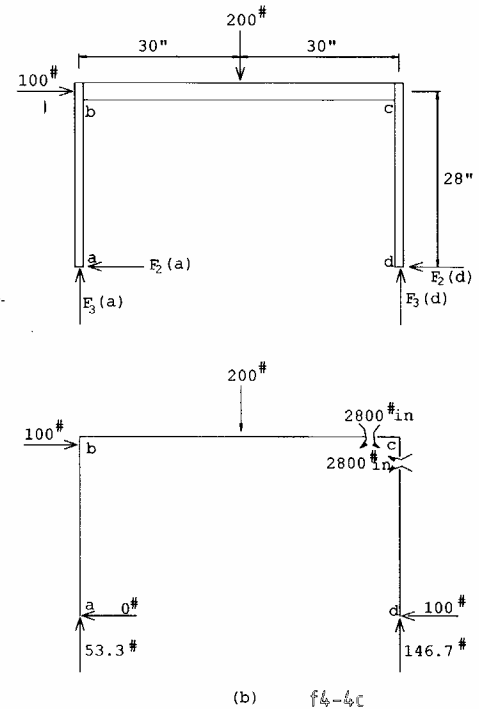


Figure 4-13. [f4-4.dxf]

Folding Chair - Pinned Joints

Folding chairs of the type shown in Figure 4-14a may be analyzed using the laws of static equilibrium alone since they utilize "pinned" joints in their construction. The vertical floor reaction force, $F_3(a)$ may be found by summing external moments about joint (e), i.e.,

$$-F_3(a) \times 18 \text{ in} + 100 \text{ lb} \times 14 \text{ in} = 0,$$

or,

$$F_3(a) = 100 \text{ lb} \times 14 / 18 = 77.78 \text{ lbs}.$$

Similarly, summing moments about joint (a) yields the vertical floor reaction force acting on joint (e), i.e.,

$$F_3(e) \times 18 \text{ in} - 100 \text{ lb} \times 4 \text{ in} = 0,$$

or,

$$F_3(e) = 100 \text{ lb} \times 4 \text{ in} / 18 \text{ in} = 22.2 \text{ lb}.$$

Since there are no external horizontal forces acting on the chair, $F_2(a) = F_2(e) = 0$. The internal moment acting on leg (d-e) at joint (d), therefore, is found by summing moments about joint (d)

$$f_4(de) + F_3(e) \times 4 \text{ in} = 0,$$

or,

$$f_4(de) = -22.2 \text{ lb} \times 4 \text{ in} = -88.8 \text{ in-lb}.$$

In like manner, the moment acting on leg (b-a) at joint (b) is found by summing moments about point (b), i.e.,

$$f_4(ba) - F_3(a) \times 8 \text{ in} = 0,$$

$$f_4(ba) = 77.8 \text{ lb} \times 8 \text{ in} = 622.4 \text{ in-lb}$$

Finally, the moment acting on member (b-f) at joint (b) is found by summing moments about point (b), i.e.,

$$f_4(bf) + 100 \text{ lb} \times 4 \text{ in} = 0$$

or,

$$f_4(bf) = -100 \text{ lb} \times 4 \text{ in} = -400 \text{ in-lb}.$$

These moments are shown acting on the various members of the frame in Figure 4-14b.

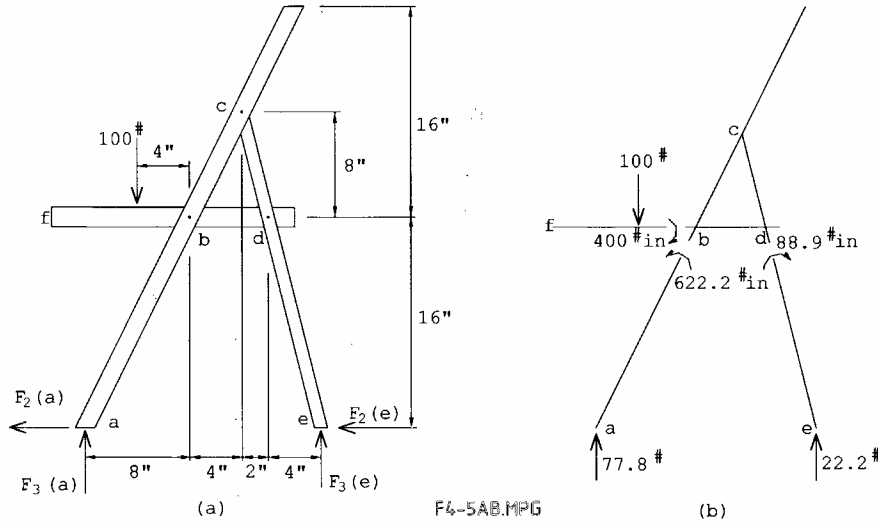


Figure 4-14. [f4-5.dxf]

Over Bed Table

The hospital over bed table shown in Figure (4-15a) appears to be a totally different type structure than those previously analyzed although it is actually nothing more than a simple table frame rotated 90 degrees counter-clockwise. Critical moments occur in this frame, Figure 4-15b, at joints (a) and (b). The vertical floor reaction force at joint (d) may be found by summing external moments at joint (a), i.e.,

$$F_3(d) \times 33 \text{ in} - 100 \text{ lb} \times 30 \text{ in} = 0$$

or,

$$F_3(d) = 100 \times 30 / 33 = 90.9 \text{ lbs.}$$

The internal resisting moment acting on foot rail (a-d) at joint (a), accordingly, is found by summing moments at joint (a), i.e.,

$$f_4(ad) + 90.9 \text{ lb} \times 33 \text{ in} = 0,$$

or,

$$f_4(ad) = -90.9 \text{ lb} \times 33 \text{ in} = -3000 \text{ in-lb}.$$

Likewise, the internal resisting moment acting on rail (b-c) at joint (b) is found by summing moments at joint (b), i.e.,

$$f_4(bc) - 100 \text{ lb} \times 30 \text{ in} = 0$$

or,

$$f_4(bc) = 100 \text{ lb} \times 30 \text{ in} = 3000 \text{ in-lb}$$

The internal moments acting on both these members along with those acting on the post are shown in Figure 4-15c.

Analyses Of Complex Frames By Approximate Methods

When the structural behavior of one or more members or joints in a frame is known, relatively good approximate analyses can often be made of furniture frames that are too complex to be solved by means of the laws of static equilibrium alone.

Flexible Member Assumption

One of the most commonly employed techniques in carrying out such analyses is to *assume that small slender members are too flexible to carry significant bending moments. Such members may then be treated as though they were pinned at each end rather than fixed, a technique that greatly simplifies analyses.* These assumptions are justified when the stiffness of the member in question is small relative to the stiffnesses of the other members in the frame since the amount of moment a member can carry is directly related to its stiffness. By way of illustration, the stiffness of a member of rectangular cross section may be calculated by means of the expression

$$K = 4td^3E/(12L)$$

where K = the stiffness of the member, lb.-in; t = the thickness, in; d = depth, in; E = modulus of elasticity of the material, psi; and L = length of the member, in.

Thus, a 3/4-inch square stretcher in a chair side frame would have a stiffness of $4 \times (0.75)^4 E / (12 \times L)$, whereas a rail with a cross section of 3/4-inch by 3-1/2 inch would have a stiffness of $4 \times (0.75 \times (3.5)^3) / (12 \times L)$. The ratio of the stiffness of the stretcher to the rail, accordingly, is $(0.75)^4 / (0.75 \times (3.5)^3)$, or 1/102. Although these calculations do not provide exact estimates of the relative moment carried by each of the two members, they do indicate that the force carried by the stretcher may be

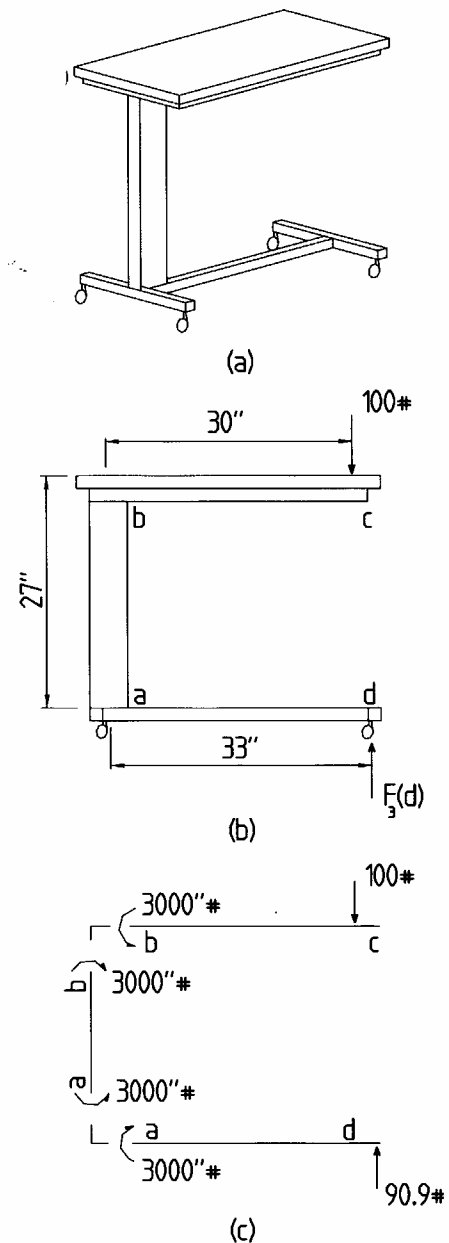


Figure4-15. [f4-6.dxf]

expected to be quite small compared to that carried by the rail. Under these conditions, to provide first estimates, the bending moment carried by the stretcher reasonably could be ignored and the stretcher treated as though it had pinned rather than fixed ends.

Semi-Rigid Joint Assumption

Semi-rigid joints that are relatively flexible may also be treated as though they were pinned. When a dowel joint is constructed and loaded as shown in Figure 4-16a, for example, it is quite stiff and no simplifying assumptions may be made. When it is constructed and loaded as illustrated in Figure 4-16b, however, it may be treated as though it were pinned—at least as a first approximation.

In upholstered furniture frame construction, these joints are often loosely fitted so that the members do not bear tightly against one another, and often, the members are slightly separated as shown in Figure 4-16c. In such cases, the connections clearly may be treated as flexible joints.

Analysis of Chair Side Frames

Side Frame without Stretcher

Although a chair side frame without stretcher can be readily analyzed by summing the forces acting on the joints, it is useful to develop an expression that can be used to estimate the bending moment acting on the critical side rail to back post joint.

Consider the side frame shown in Figure 4-17. When a user tilts backwards in the chair, forces similar to those shown in Figure 4-17a are imposed on the chair side frame. The magnitude of the moment imposed on the seat rail to back post joint by a vertical force applied to the top of the front leg when the chair is supported as shown is given by the expression

$$f_4(d) = F_v \times L_1 \text{ in} - lb$$

where $f_4(d)$ is the bending moment acting on the d-end of the side rail, member (c-d). Note that F_v is assumed positive (i.e., acts upward) in the above expression.

An example of this type of loading is given in Figure 4-17b. In this example, a vertical force of 100 pounds is shown acting downward (in the negative direction) on the front post of the side frame; length of the side rail is 16 inches. Substituting the appropriate values into the above expression and solving gives

$$f_4 = -100 \text{ lb} \times 16 \text{ in} = -1600 \text{ in} - lb.$$

This result indicates that for the loading given, a moment of -1600 in-lb acts on the d-end of the side rail and +1600 in-lb on the side rail to back post joint. Thus, both the side rail

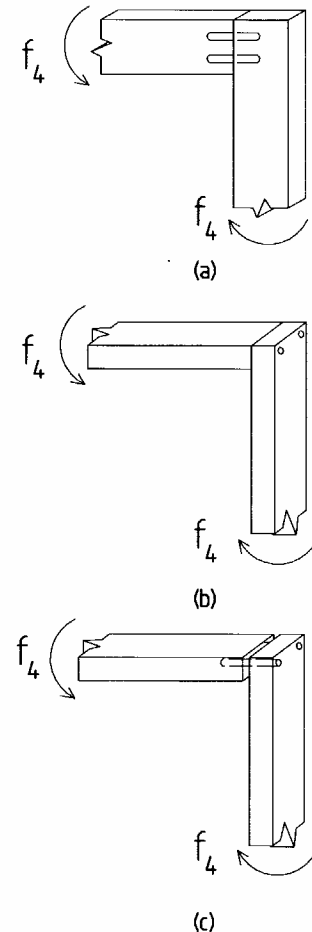


Figure 4-16. [f4-7.dxf]

and the side rail to back post joint must be designed to resist this bending moment.

A different loading of a chair side frame without stretcher is shown in Figure 4-17(c). In this case, a horizontal front to back force is shown acting on the front post in line with the horizontal axis of the side rail. It should be noted that this type of loading is frequently used in the performance testing of chairs.

For this type of loading, the vertical reaction force, F_V , acting on the top of the front post is determined by summing moments about point (b), i.e.,
 $-F_V \times (L_1 + L_2) - F_H \times h_1 = 0$,
 or,

$$F_V = \frac{-F_H \times h_1}{L_1 + L_2}$$

where the negative sign indicates that the floor reaction force acts downward (in order to keep the chair from overturning). The moment acting on the side rail to back post joint is found by summing bending moments about joint (d), i.e.,

$$f_4 = \frac{-F_H \times h_1}{L_1 + L_2} \times L_1$$

where the minus sign indicates that the moment is acting in the clockwise direction on the d-end of the side rail.

As an example, for the loading shown in Figure 4-17d, the moment acting on the c-end of the side rail amounts to

$$f_4 = \frac{-100 \text{ lb} \times 16 \text{ in} \times 16 \text{ in}}{16 \text{ in} + 3 \text{ in}} = -1347 \text{ in} \cdot \text{lb}$$

When a horizontal front to back force is applied to the top of the back post of a side frame, Figure 4-17(e), the vertical reaction force acting on the top of the front post is determined by summing moments about joint (b), i.e.,

$$-F_V \times (L_1 + L_2) - F_H \times (h_1 + h_2) = 0, \text{ or, } F_V = -\frac{F_H \times (h_1 + h_2)}{L_1 + L_2} \text{ lb.}$$

The corresponding moment acting on the d-end of the side rail may be found from the expression

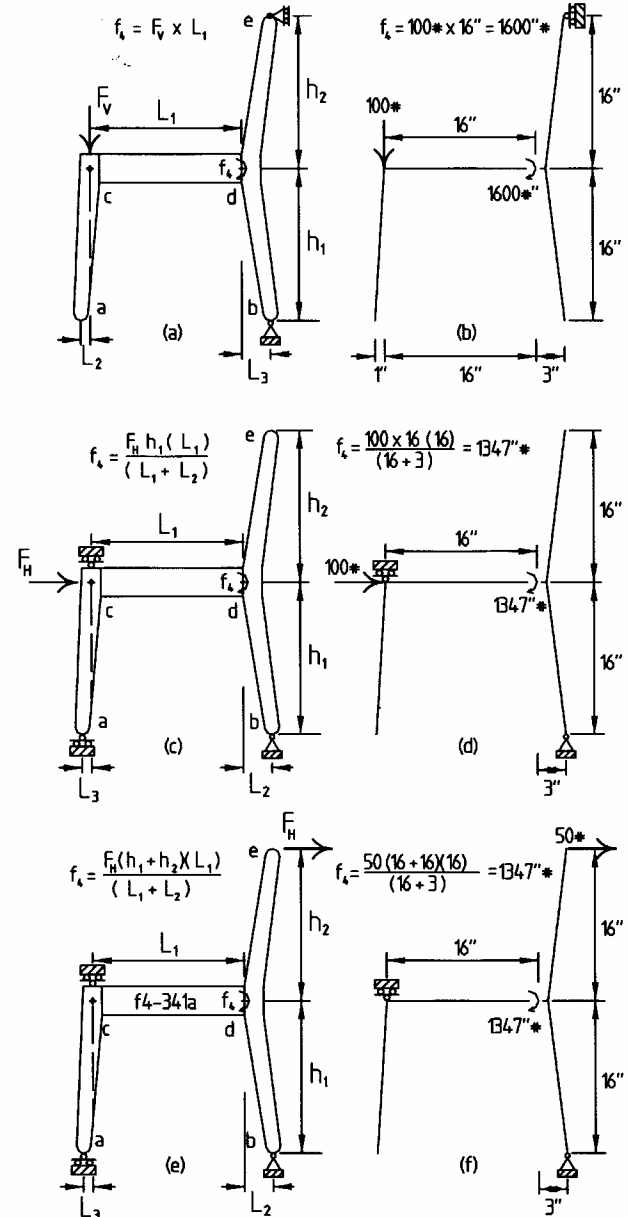


Figure 4-17. [f4-9..dxf-100]

$$f_4(dc) = \frac{-F_H \times (h_1 + h_2)}{L_1 + L_2} \times L_1$$

To illustrate, for the loading shown in Figure 4-17f, the moment acting on the d-end of the side rail as a result of a positive 50 lb horizontal front to back force applied to the top of the back post amounts to

$$f_4(dc) = \frac{-50 \text{ lb} \times (16 \text{ in} + 16 \text{ in}) \times 16 \text{ in}}{16 \text{ in} + 3 \text{ in}} = -1347 \text{ in} - \text{lb}$$

As a check, for the loading shown in Figure 4-17f, the vertical reaction force amounts to

$$F_v = \frac{-50 \text{ lb} \times (16 \text{ in} \times 16 \text{ in})}{16 \text{ in} + 3 \text{ in}} = -84.2 \text{ lb}.$$

The moment acting on the side rail to back post joint is then found by summing moments about joint (d), i.e.,

$$f_4(dc) + 84.2 \text{ lb} \times 16 \text{ in} = 0, \text{ or, } f_4(dc) = -84.2 \text{ lb} \times 16 \text{ in} = -1347 \text{ in} - \text{lb}$$

as was previously determined.

Side Frame with Stretcher

A chair side frame with slender stretcher may also be treated by approximate methods. In evaluating such frames, the analyst is usually most interested in determining the moment acting on the ends of the side rail and the side rail to back post joint when the frame is loaded in a manner similar to that which occurs when a chair is tilted backward or when it is subjected to the front to back loads used in various performance tests, Figure 4-18.

If the stretcher is small enough in cross section that it can be treated as though it were pinned at each end, the moment acting on each end of the side rail is related to the vertical force acting on the top of the front post by the expression

$$f_4(ef) + f_4(fe) - F_v \times L_1 = 0$$

where $f_4(ef)$ and $f_4(fe)$ refer to the internal forces acting on the e and f ends, respectively, of member (e-f), F_v refers to the vertical external force acting in the positive direction (upwards) on the top of the front post, and L refers to the horizontal distance measured from the line of action of F_v to the side rail to back post joint.-

As an approximation let us assume that $f_4(ef) = f_4(fe)$. Under this condition we may then write

$$f_4(ef) = f_4(fe) = F_v \times L_1 / 2$$

As an example, substituting the appropriate values for the frame shown in Figure 4-18(a) into the above expression and solving gives

$f_4(e f) = f_4(f e) = -100 \times 16 / 2 = -800 \text{ in-lb}$ for the moments acting on the ends of the seat side rail.

When a horizontal front to back or back to front force is applied to the seat of the chair (or top of the front post) as shown in Figure 4-18c, the vertical reaction force acting on the top of the front post may be found from the expression

$$-F_V \times (L_1 + L_2) - F_H \times h_1 = 0,$$

$$\text{or, } F_V = \frac{-F_H \times h_1}{L_1 + L_2}.$$

The moments acting on the ends of the side rail then may be estimated from the expression

$$f_4(f e) = \frac{F_V \times L_1}{2} = \frac{-F_H h_1 L_1}{2(L_1 + L_2)}$$

Applying this relationship to the example shown in Figure 4-18(e), the estimated moment acting on each end of the rail amounts to

$$f_4 = \frac{-100 \text{ lb} \times 16 \text{ in} \times 16 \text{ in}}{2 \times (16 + 3) \text{ in}} = -674 \text{ in-lb}.$$

For the case in which a horizontal front to back force is applied to the top of the back post, the vertical force, F_V , acting on the top of the front post may be found from the expression

$$-F_V \times (L_1 + L_2) - F_H \times (h_1 + h_2) = 0,$$

or,

$$F_V = \frac{-F_H \times (h_1 + h_2)}{L_1 + L_2}$$

The moment acting on the ends of the side rail then may be estimated by means of the expression

$$f_4 = \frac{-F_H \times (h_1 + h_2) \times L_1}{2(L_1 + L_2)}$$

As an example, substituting the appropriate values from Figure 4-18(f) into the above expression, gives the following

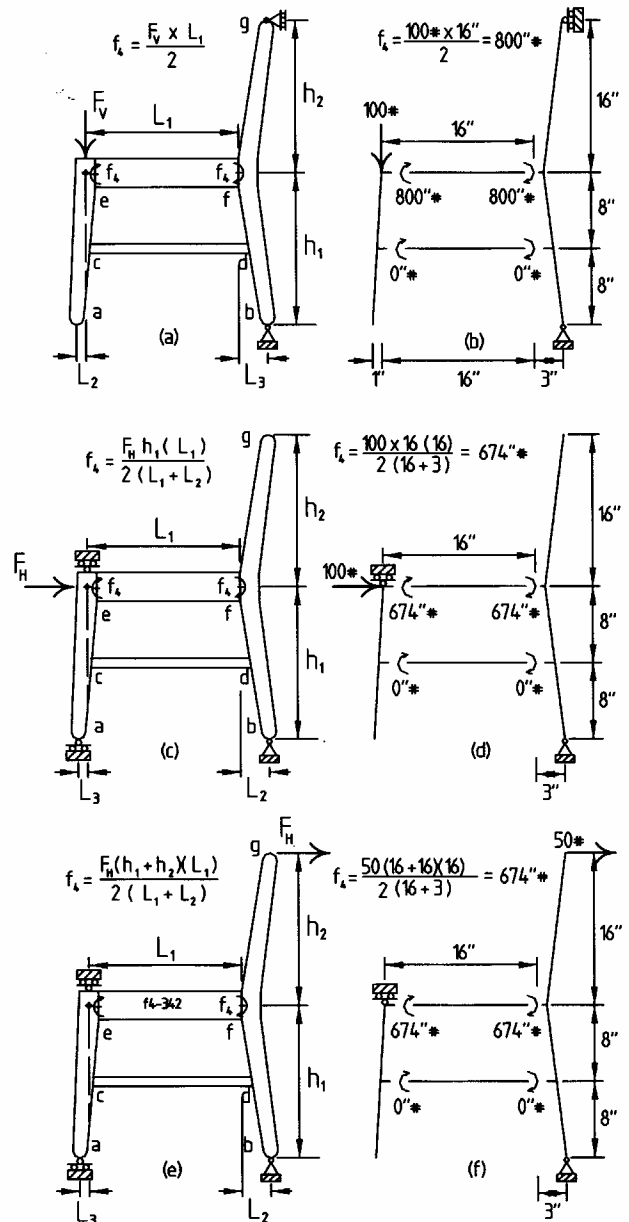


Figure 4-18. [f4-10.dxf]

$$f_4(fe) = \frac{-100 \text{ lb} \times (16 \text{ in} + 16 \text{ in}) \times 16 \text{ in}}{2 \times (16 + 3) \text{ in}} = -674 \text{ in} \cdot \text{lb}$$

The sample chair shown in Figure 4-19 further illustrates the use of the last expression developed above. Substituting the appropriate values into the moment expression gives

$$f_4(cb) = \frac{-62.5 \text{ lb} \times (16 \text{ in} + 16 \text{ in}) \times 16 \text{ in}}{2 \times (16 + 4) \text{ in}} = -800 \text{ in} \cdot \text{lb}$$

$$f_4(cb) = \frac{-100 \text{ lb} \times 16 \text{ in}}{2} = -800 \text{ in} \cdot \text{lb}$$

An exact analysis of this frame gives the values listed in Figure (4-19c). The formula over-predicted the moment at joint (b) by 37 percent and under-predicted the moment at joint (c) by -3.3 percent.

Side Frame with Equal Size Rail and Stretcher

When the side rail and the stretcher in a side frame are of the same size, as is often the case with sled base chairs, Figure 4-20, as a first approximation, it may be assumed that the moments acting on the ends of the

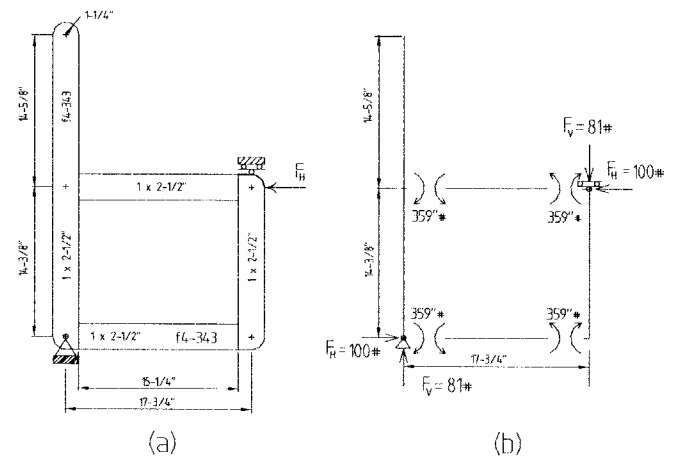


Figure 4-20. [f4-12.dxf]

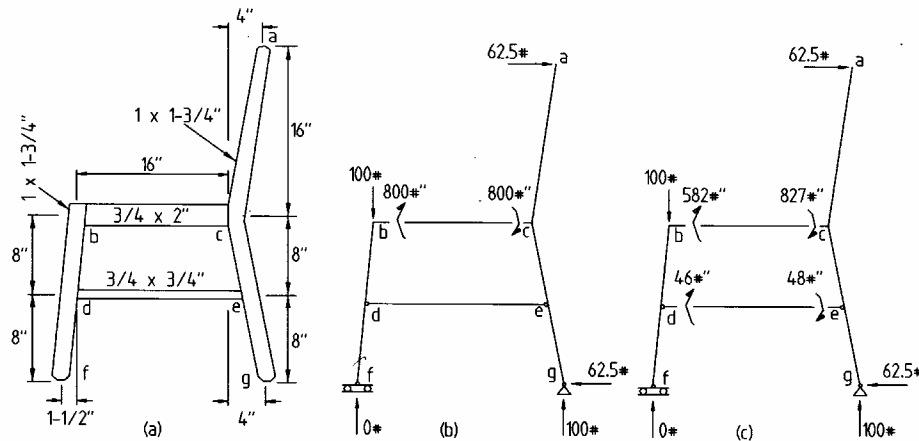


Figure 4-19. [f4-11.dxf]

horizontal side frame members are equal. Under this assumption, the moments acting on the ends of the side rail and sled rail are given by the expression

$$f_4 = \frac{-F_H \times h_1}{4}$$

Thus, for the loading shown, the bending moment acting on the ends of the horizontal members and the corresponding joints amounts to

$$f_4(2) = f_4(5) = f_4(8) = \frac{-F_H \times L_V}{6}$$

This simple expression also can be used to provide first estimates of internal bending moments in chairs with slanted back legs – but with increased error. In the case of the chair shown in Figure 4-22, for example, this expression estimates internal bending moments of

$$f_4(8) = \frac{-100 \times 16}{6} \times \frac{16}{18} = -237 \text{ in} \cdot \text{lb}.$$

Referring to Table 4-22, it can be seen that this value differs from the "exact" values by a maximum of $100\% \times (237 - 250.5) / 250.5$, or, -5.5%.

To further illustrate the use of the above expression, consider the chair shown in Figure 4-23. In this chair, the side frame members are not slender, but all members have the same cross section. For the loading shown, the above expression estimates a bending moment of

$$f_4 = \frac{-100 \text{ lb} \times 29 \text{ in}}{6} = -483.3 \text{ in} \cdot \text{lb}$$

acting on the ends of the arm, seat rail, and stretcher. This value may be compared with the exact values computed for this frame that also are shown in Figure 4-23. As can be seen, the simplified estimated value is 21 percent lower than the highest exact value and 25 percent higher than the lowest exact value.

Analyses of Chair Frames with Semi-Rigid Joints

Chair frames of the type shown in Figure 4-24a may be analyzed if certain joints are assumed pinned. Joints (b), (c), and the back post to side rail joint (e) of this side frame are all loaded flatwise in bending so that their structural behavior may be approximated reasonably by pinned joints. The front post to side rail connection and the back leg to side rail connections, on the other hand, are rigid. For the one-hundred-pound

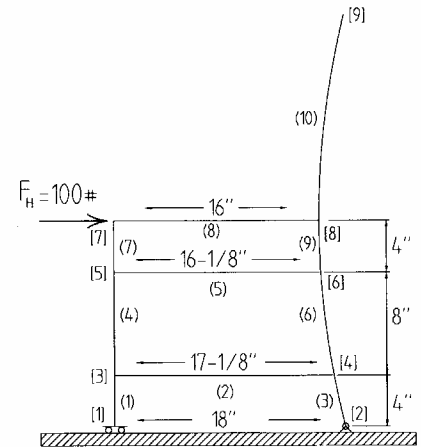


Figure 4-22. [f4-14.dxf] Side frame of chair constructed with round side stretchers. Post diameter is 2 inches; stretcher diameter is 1 inch.

Table 4-22.

Member	Moment	Simplified Analysis	Exact Analysis	Diff.
No.	Location	in-lb	in-lb	%
2	f(3-4)	-232.5	-235.6	-1.32
	f(4-3)	-232.5	-243.0	-4.32
5	f(5-6)	-246.9	-250.5	-1.44
	f(6-5)	-246.9	-248.4	-0.6
8	f(7-9)	-248.8	-241.0	3.24
	f(9-7)	-248.8	-239.0	4.1

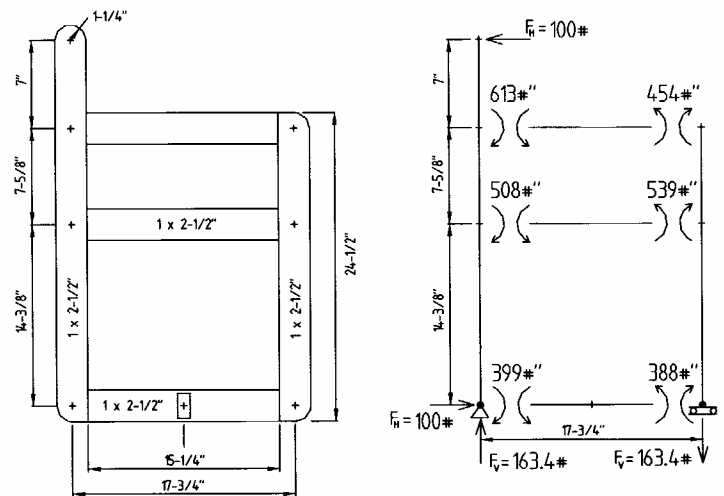


Figure 4-23. [f4-15.dxf]

front to back force shown, a summation of forces about joint (e), Figure 4-24b, gives an axial force acting along the axis of the arm of

$$f_2(bc) = 100 \text{ lb} \times 20 \text{ in} / 10 \text{ in} = 200 \text{ lb}.$$

The bending moment acting on the b-end of the stump, member (d-b), therefore, is

$$f_4(db) = 200 \text{ lb} \times 10 \text{ in} = 2000 \text{ lb} \cdot \text{in}.$$

Equilibrium requires that the bending moment acting on the d-end of the seat side rail be equal but opposite in sign, i.e., $f_4(de) = -2000 \text{ in} \cdot \text{lb}$.

The bending moment acting on the top of the back leg is

$$f_4(eg) = 100 \text{ lb} \times 14 \text{ in} - 170 \text{ lb} \times 4 \text{ in} = 720 \text{ lb} \cdot \text{in}.$$

This result can also be obtained by summing moments about the e-end of the side rail, i.e.,

$$f_4(ed) + 170 \text{ lb} \times (20 - 4) \text{ in} - 200 \text{ lb} \times 10 \text{ in} = 0$$

or,

$$f_4(ed) = -720 \text{ in} \cdot \text{lb}.$$

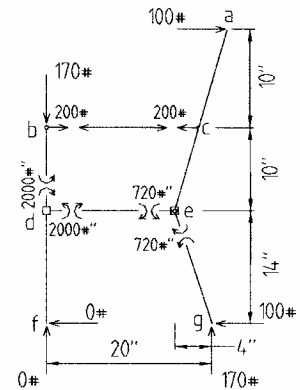
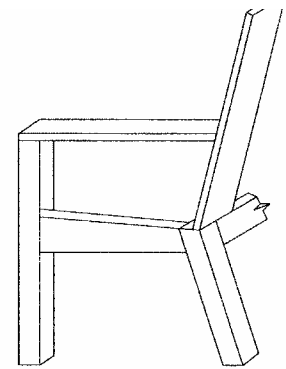


Figure 4-24. [f4-18.dxf]

Force Equilibrium in Side Frames

When an external force is applied to a chair side frame such as the horizontal force applied to the top rail or to the seat as shown in Figure 4-25, it is resisted by internal forces developed in the frame. These internal forces are always in equilibrium with, i.e., they just balance, the external forces.

Consider, for example, the chair shown in Figure 4-25b in which a 100 lb horizontal load is applied to the top rail. This force tends to cause the chair to overturn, but it is resisted by the 200 vertical seat load force. This loading simulates the actions created by a user sitting down in a chair and just starting to tilt backward. Under the action of these external loads, internal resisting bending moments are developed in the stretchers. Specifically, internal bending moments of 544 and 571 lb-in, 526 and 525 lb-in, and 513 and 521 lb-in are developed at the ends of the top, middle, and bottom stretchers, respectively. The magnitudes of the vertical internal shear forces acting on the ends of the stretchers may be found by summing the internal bending moments acting on the ends of a stretcher and dividing the sum by the length of the stretcher. Thus, the vertical internal shear force acting on the left end of the top stretcher is given by the expression

$$f_3 = \frac{544 + 557}{16} = 69.7 \text{ lb}.$$

Likewise, the internal vertical shear force acting on the left end of the middle stretcher is

$$f_3 = \frac{526 + 525}{16} = 65.7 \text{ lb},$$

and finally, the internal shear force acting on the left end of the bottom stretcher is

$$f_3 = \frac{513 + 521}{16} = 64.6 \text{ lb}.$$

If we add these internal shear forces together, we obtain $\sum f_2 = 69.7 + 65.7 + 64.6$ or, 200 lb. Thus, the sum of the vertical internal resisting shear forces just equals the vertical seat load force.

Similar calculations apply to the side when loaded as shown in Figure 4-25b. In both cases, note how the 200 lb vertical force applied to the front leg is transferred to the floor through the back leg.

Analyses of Sofa Frames with Semi-Rigid Joints

Sofa frames also contain joints that are sufficiently flexible that they may be treated as though they were pinned. The side frame shown in Figure (4-26a), for example, is constructed in such a way that all of the members are subjected to bending moments about their faces rather than their edges. These joints have relatively little stiffness when loaded in this manner, and they may, therefore, be treated as pinned joints. This is particularly true in the case of the side slats which are usually jointed to the frame by means of a single dowel at each end. In addition, a gap often exists between the ends of the slat and the posts so that these joints are only as stiff as the dowels themselves. Since a single dowel will ordinarily have little stiffness relative to the members it joins together, these joints can properly be treated as pinned joints. The arm to back, arm

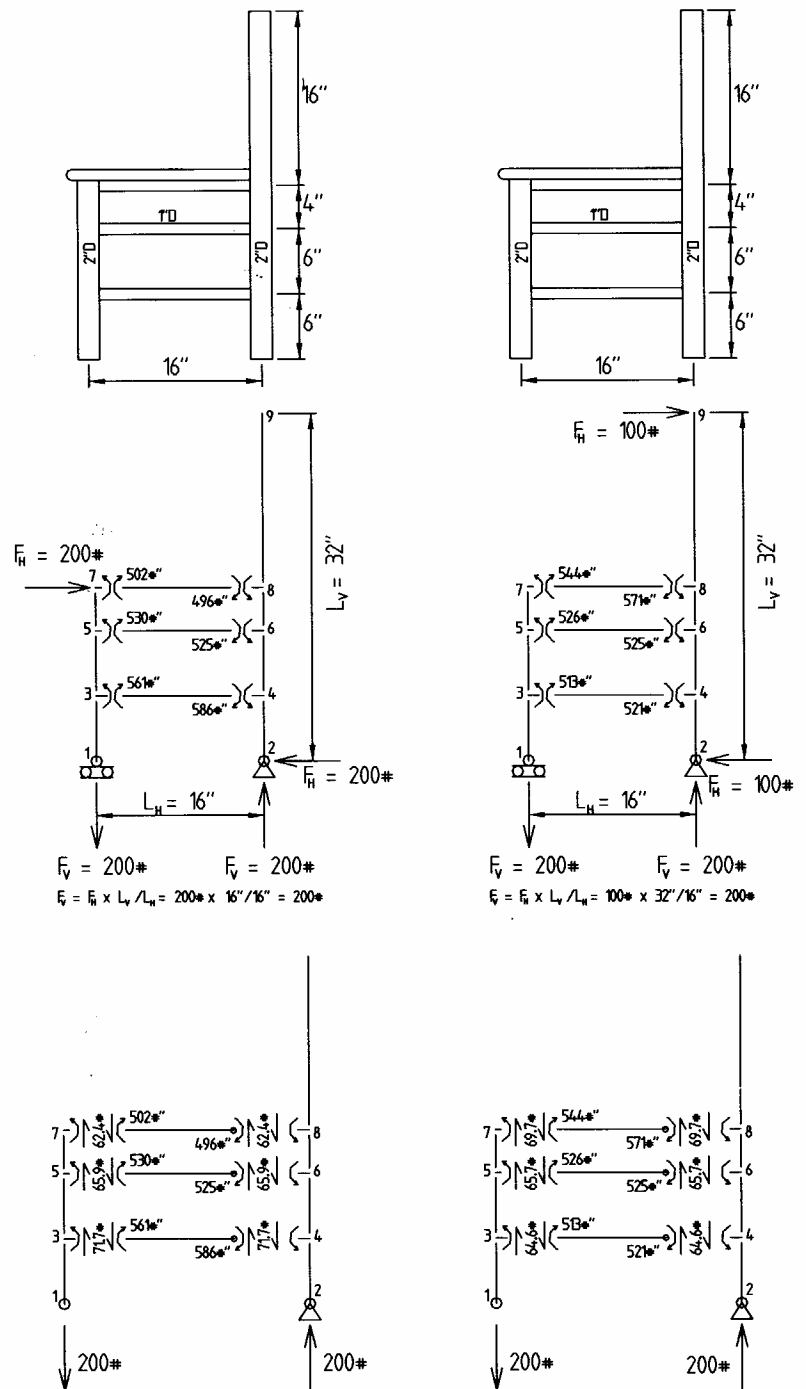


Figure 4-25. [f4-16.dxf]

to stump, and side rail to back post joints will ordinarily be somewhat stiffer than the slat joints, but often, they may still be treated as pinned joints. An idealized line drawing of the frame with a 100 lb horizontal front to back force applied to joint (a) is given in Figure (4-26b). Since it is assumed that the arm connections will carry essentially all of the axial forces, the side slat has been omitted in Figure (4-26c) and Figure (4-26d).

The magnitude of the vertical floor reaction force required to keep the frame from overturning may be found by summing forces about the bottom of the rear leg, i.e.,

$$-F_v \times 31 \text{ in} - 100 \text{ lb} \times 33 \text{ in} = 0,$$

or,

$$F_v = -(100 \text{ lb} \times 33 \text{ in}) / 31 \text{ in} = -106.5 \text{ lb}.$$

In order to keep the frame from collapsing, one of the joints must be rigid. Intuitively, it might be thought that the side rail to back post joint should be reinforced. The stump to side rail joint, joint (f), may just as well be reinforced, however, as was done in this problem.

The absolute value of the force exerted on joint (c) by the arm may be found by summing forces about point (g), i.e.,

$$-f_2(c) \times 18 \text{ in} - 100 \text{ lb} \times 28 \text{ in} = 0,$$

or,

$$f_2(c) = -(100 \text{ lb} \times 28 \text{ in}) / 18 \text{ in} = -155.6 \text{ lb}$$

The force acting on joint (b) is equal in value but opposite in sign to the force acting on joint (c).

The bending moment acting on joint (f) is found by summing forces about that point, i.e.,

$$f_4(f) - 155.6 \text{ lb} \times 18 \text{ in} = 0,$$

or,

$$f_4(f) = 2800 \text{ in} \cdot \text{lb}$$

The distribution of axial and bending moments acting on the side frame under the action of the 100 pound front to back force applied to the top of the back post is given in Figure (4-26c). It can be seen that the arm carries a substantial axial force; the arm to stump and arm to back post joints must, therefore, be designed carefully to resist these forces. The bending moment acting on the bottom of the stump, $f_4(fb)$, is 2800 pound inches. This is a substantial bending moment and some difficulty may be encountered in designing the joint to resist a force of this magnitude. A good solution to this problem is to reinforce both the stump to side rail and the back post to side rail joints,

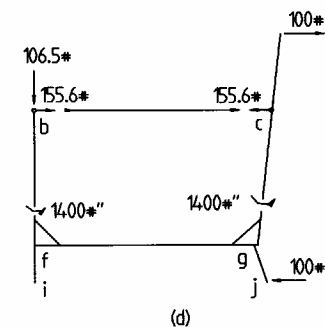
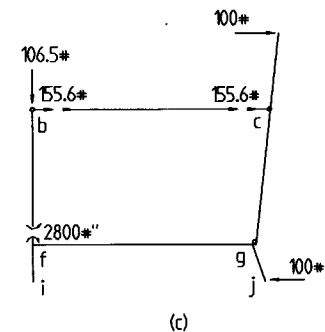
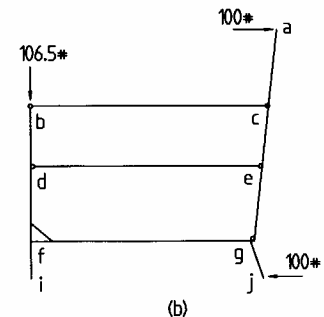
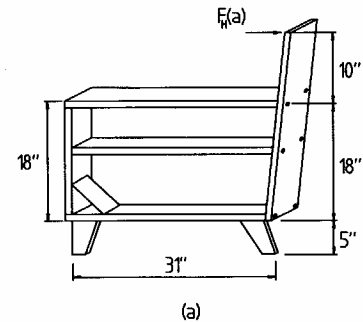


Figure 4-26. [f4-17.dxf]

Figure (4-26d). Under these conditions, the bending moments are approximately equally distributed between the two joints, so that each carries a bending moment of only 1400 pound inches.

Analysis of Table Side Frame with Slender Stretcher

A table side frame with a long slender stretcher provides an excellent example of a structure which can be treated by approximate methods. The stretcher in the side frame shown in Figure 4-27a is only 1 inch square as opposed to the top rail or apron that is 1 x 4 inches in cross section. Under the action of the forces shown, the left leg is just ready to lift off the floor so that $F_2(a)$ and $F_3(a)$ are both zero.

When the stretcher is small in size relative to the rail, good estimates of the moments acting on the ends of the rail, member (c-d), can be obtained from the expression

$$f_4(cd) + f_4(fdc) - F_H \times h = 0,$$

where h refers to the distance from the floor to the centerline of the rail. Solving, gives

$$f_4(cd) = f_4(fdc) = -F_H \times h/2$$

Substituting the appropriate values into the above expression and solving gives

$$f_4(cd) = f_4(fdc) = -100 \text{ lb} \times 30 \text{ in} / 2 = -1500 \text{ in} - \text{lb}.$$

In this example, the loads were chosen so that the entire side thrust force was carried by one leg. It should be noted, however, that the same result is obtained regardless of the manner in which the floor reaction forces are distributed between the two legs. This approximate analysis provides us with estimates of the internal moments acting on the top of each leg, the internal moments acting on each end of the rail, and perhaps most importantly of all, the internal moments acting on the leg to rail joints. All of these forces must, of course, be equal in absolute value in order to satisfy the laws of equilibrium.

An exact analysis of this frame is given in Figure 4-27c. The moment at joint (c) differs from the predicted value by only 0.6 percent. At the right end, the bending moment amounts to 84 percent of the predicted value. This technique, therefore, provides reasonable estimates of the bending moments acting on these two critical joints when the cross section of the stretcher in the frame is relatively small.

For frames similar to the one shown here, the bending moment acting on the two critical leg joints may, therefore, be estimated by means of the expression

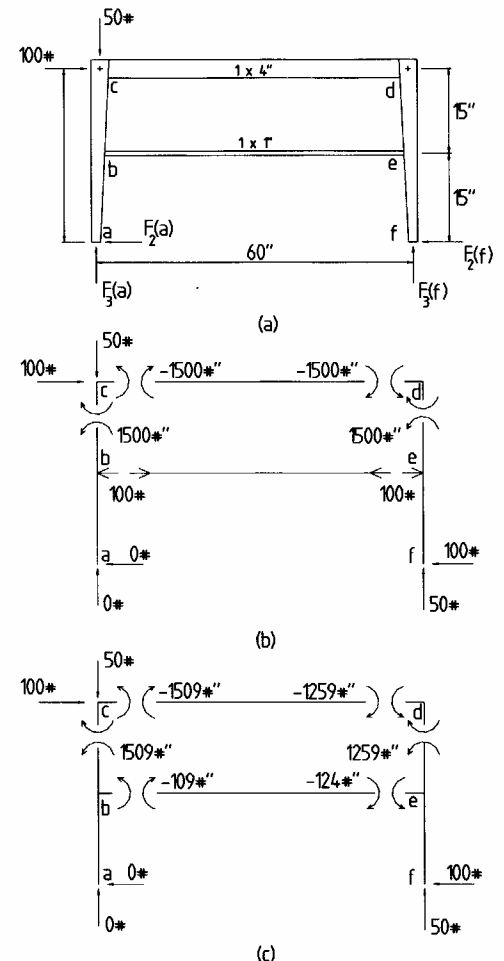


Figure 4-27. [f4-8.dxf]

$$f_4 = F_H \times L / 2$$

where f_4 = the bending moment acting on the leg to rail joints; F_H = the side thrust force; L = the length of the leg.

If the legs of the table are slanted, the bending moments acting on the joints may change somewhat, but the expression will still give close enough results for most practical design problems.

Deflection Analysis of Case Construction

The structural behavior of case and carcass furniture was discussed in some detail in Chapter II. As was noted there, this type of furniture ordinarily contains only two basic components—a frame and panels. In some instances the frame may be entirely lacking and the case made up entirely of panels, whereas in other instances the frame may be the most important structural component and the panels may be structurally inoperative. When a frame and panels are both present, they interact in varying degrees as a composite structure and the resulting case is thus structurally indeterminate. An exact analysis of composite structures is quite complex and will not be discussed here—first, because of the complexity of the analysis, and second, because so little is known about the elastic behavior of this type of furniture that the number of assumptions that must be made to carry out such an analysis would likely render the results obtained meaningless when applied to the real structure. Certain types of essentially pure panel construction that have a minimal framework and which depend on the stiffness of the individual panels for their overall rigidity can be analyzed relatively simply, however.

Consider the simple five-sided case supported at three corners as shown in Figure 4-28. Let us assume that the case is constructed of flat panels which are bonded together at the edges and that a framework is either lacking, or if present, does not make a significant contribution to the strength of the case. Free-standing wardrobes constructed of particleboard which are frequently used in college dormitories are good examples of this type of construction.

Under the action of the vertical load F_v , the deflection of the free corner may be calculated by means of the case stiffness expression (Eckelman, 1967),

$$y_d = \frac{F_v}{f_v(B)/y_v(B) + f_v(T)/y_v(T) + (2b^2/a^2)(f_v(S)/y_v(S) + (b^2/c^2)(f_v(R)/y_v(R))}$$

where a , b , c = the width, height, and depth of the case, respectively; y_d = the deflection of the unsupported corner of the case under the action of the vertical load F_v ; and $f_v(B)/y_v(B)$, $f_v(T)/y_v(T)$, $f_v(S)/y_v(S)$, and $f_v(R)/y_v(R)$ refer to the torsional rigidity of the bottom, top, side, and rear panels, respectively when each is loaded as illustrated in Figure 4-29.

A somewhat different notation has been employed here than has previously been used in that the letters contained in parentheses, B, T, S, and R, refer to the back, top, side, and rear panels, respectively rather than to a point or joint. Also, note that the symbol f_v is used to designate the vertical load applied to each panel rather than the symbol F_v that represents the vertical load applied to the assembled case.

The quantities of $F_v(B)/y_v(B)$, $f_v(T)$, $f_v(S)/y_v(S)$, and $f_v(R)/y_v(R)$ may be determined experimentally as is illustrated in Figure 4-29. A panel is supported at three points and a vertical load, f_v , is applied to the fourth. The deflection, y , of the fourth corner is measured, and the quantity f/y is then calculated. Alternatively, provided that the material properties of each panel are known, the rigidity of each panel may be calculated theoretically by means of the following panel stiffness expression

$$\frac{f_v}{y_v} = \frac{t^3 G}{3wL}$$

where f_v = the vertical load applied to one corner of the panel, y_v = deflection of the panel at that corner, t = the thickness of the panel, L = length of the panel, w = width of the panel, and G = modulus of rigidity of the material of which the panel is constructed.

If we assume that the case we are considering is constructed of 3/4-inch thick particleboard that has a modulus of rigidity of 190,000 psi (McNatt, 1973). Let us further assume that the height of the case, b , is 96 in; depth of the case, c , is 24 inches; and the width of the case, a , is 48 inches. Substituting these values into the panel stiffness expression gives the following stiffnesses for the panels:

$$\frac{f_v(T)}{y_v(T)} (top) = \frac{f_v(B)}{y_v(B)} (bottom) = \frac{(3/4)^3 \times 190,000}{3 \times 24 \times 48} = 23.19 \text{ lb/in.}$$

$$\frac{f_v(S)}{y_v(S)} (sides) = \frac{(3/4)^3 \times 190,000}{3 \times 24 \times 96} = 11.60 \text{ lb/in.}$$

$$\frac{f_v(R)}{y_v(R)} (back) = \frac{(3/4)^3 \times 190,000}{3 \times 48 \times 96} = 5.80 \text{ lb/in.}$$

Substituting these values into the case deflection expression gives

$$y(case) = \frac{F_v}{23.19 + 23.19 + 2 \frac{(96)^2}{(48)^2} \times 11.60 + \frac{(96)^2}{(24)^2} \times 5.80} = \frac{F_v}{232} = 0.0043 \times F_v \text{ in/lb}$$

In other words, it would require a 232 pound force to deflect the free

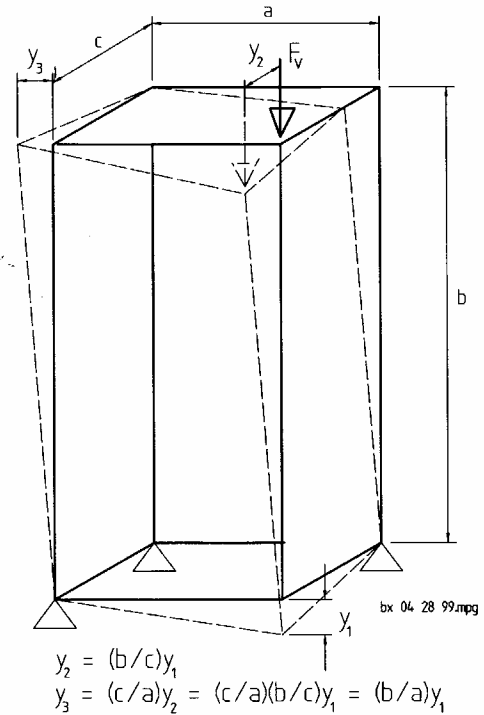


Figure 4-28. [f4-19.dxf]

corner of the case one inch when loaded as shown in Figure 4-28. Had this case been constructed of 3/4-inch thick plywood which has a modulus of rigidity of 90,000 psi, (Anon., 1974), presumably the case would have been only 90,000/190,000, or 47 percent as stiff so that

$$y(case) = \frac{F_v}{0.47} \times 232 = \frac{F_v}{110} = 0.0091 \times F_v \text{ in/lb}$$

The relative efficiencies of various panel thicknesses can readily be evaluated by means of the previous equations. Suppose, for instance that in an effort to stiffen the case, all of the panels had been constructed of 1-inch instead of 3/4-inch thick particleboard. The panels would then have stiffnesses of;

$$\frac{f_v(T)}{y_v(T)}(top) = \frac{f_v(B)}{y_v(B)}(bottom) = \frac{(1)^3 \times 190,000}{3 \times 24 \times 48} = 54.98 \text{ lb/in}$$

$$\frac{f_v(S)}{y_v(S)}(sides) = \frac{(1)^3 \times 190,000}{3 \times 24 \times 96} = 27.49 \text{ lb/in}$$

$$\frac{f_v(R)}{y_v(R)}(back) = \frac{(1)^3 \times 190,000}{3 \times 48 \times 96} = 13.74 \text{ lb/in}$$

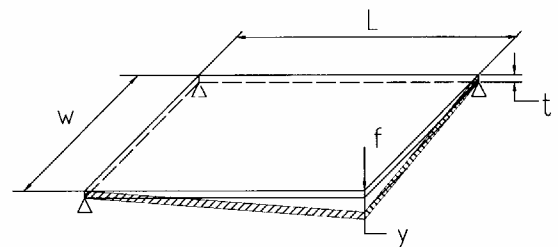
Carrying out the necessary calculations gives

$$y(case) = \frac{F_v}{54.98 + 54.98 + 2 \frac{(96)^2}{(48)^2} \times 27.49 + \frac{(96)^2}{(24)^2} \times 13.74} = \frac{F_v}{550} = 0.0018 \cdot F_v \cdot \text{in/lb}$$

Increasing the thickness of all of the panels by 33-1/3 percent increased the stiffness of the case, therefore, by a factor of 550/232, or, 237 percent.

Had all this added material been put into just one panel, however, say the bottom, the case could have been made even stiffer. The total external area of the case is 11520 sq. in. so that if we increase its thickness by 1/4 inch, we add 2880 cu. in. to its volume. Since the area of the bottom is 24 x 48, or 1152 sq. in., this volume would add 2880/1152, or, 2-1/2 inches to its thickness. Total thickness of the bottom would thus be 3/4 + 2-1/2, or 3-1/4 inches. If we now determine the stiffness of the case with a 3-1/4 inch thick bottom, we obtain

$$y(case) = \frac{F_v}{1887 + 23 + 2 \frac{(96)^2}{(48)^2} \times 11.6 + \frac{(96)^2}{(24)^2} \times 5.8} = \frac{F_v}{2096}$$



$$f/y = t^3 G / (3wL)$$

By putting all the material into one panel rather than uniformly increasing the thickness of all the panels, the stiffness of the case was increased by a factor of 2096/232, or, 9.03. These simple calculations clearly demonstrate the value of substantially increasing the thickness of one panel rather than increasing the thickness of all the panels only

Figure 4-29. [f4-20.dxf]

slightly. The reason for this, of course, is that the stiffness of a panel is proportional to the cube of its thickness. Since the panels are interconnected, if one panel is greatly stiffened, then the entire case will be stiffened. Additional discussions of the design of case furniture and the methods of stiffening it are given in the literature (Eckelman, 1967; Kotas, 1957 and 1958). The interested reader is referred to these resources for further information.

Deflection Analysis Of Shelves

When designing shelves, the most important design consideration is the deflection of the shelves at mid span. The general requirement here is that the shelves must not sag enough under load so as to be unsightly. Exactly how much sag can be tolerated is a moot point. Existing performance standards require that mid span deflections be held to less than 1/200 to 1/500 of the total length of the shelf after a few days loading (Sparkes, 1974; Anon., 1960 and 1969). The author has found that deflections equal to 1/180 of the span are not unsightly but deflections of 1/165 definitely are.

In considering the deflection of shelving there are three cases of particular interest. These include: a) deflection of the shelf at mid span under the action of a uniform load, b) deflection at mid span under the action of a concentrated load applied at mid span, and c) deflection at mid span under the action of a concentrated load applied at any point along the length of the shelf.

Deflection under Uniform Load

The deflection of a shelf at mid span under the action of a uniform load, Figure 4-30a, is given by the expression

$$y_c = \frac{5qL^4}{384EI}$$

where

$$I = \frac{wt^3}{12}$$

and the remaining symbols have the following meanings: y_c = the deflection of the shelf at mid span; q = the load applied to the shelf, pounds per inch of shelf length; L = length of the shelf in inches; E = modulus of elasticity of the material of which the shelf is constructed, psi;

I = moment of inertia of the shelf = $\frac{wt^3}{12} \text{ in}^4$; w = width of the shelf,

inches; and t = thickness of the shelf, inches.

To illustrate the use of this expression, let us consider the following example: A shelf in a bookcase is constructed of red oak which has a modulus of elasticity of 2,000,000 psi at 7 percent moisture content and measures 0.7 inches thick by 10-3/4 inches wide by 34-1/4 inches long. It

is loaded with furniture trade magazines which weigh 36 pounds per foot, or, 3 pounds per inch. What is the deflection of the shelf at mid span?
 Ans. Substituting the appropriate values into the above expression and solving gives

$$y_c = \frac{5 \times 3 \times (34.25)^4}{384 \times 2,000,000 \times \frac{10.75 \times (0.7)^3}{12}} = 0.0875 \cdot \text{in}$$

It should be noted here that this is the "short term" deflection of the shelf. Under continuous loading (which must be expected for bookshelves), the wood may "creep" until the final deflection is nearly double the original deflection of 0.175 inches (Hoyle, 1973). The initial deflection was equal to $0.087/34.25$, or $1/392$ of the span. The final expected deflection, however, would be $0.175/34.25$, or $1/196$ of the span.

Deflection under Concentrated Load at Mid Span

When a concentrated load is applied at mid span, Figure 4-30b, the deflection of the shelf beneath this load may be computed by means of the formula

$$y_c = \frac{F_v L^3}{48EI}$$

where F_v refers to the vertical load acting on the shelf. In the previous example, if a 50 pound typewriter had been stored on the shelf at mid span, the deflection would have been

$$y_c = \frac{50 \times (34.25)^3}{48 \times 2,000,000 \times \frac{10.75 \times (0.7)^3}{12}} = 0.068 \cdot \text{in.}$$

Deflection under Off-Center Concentrated Load

When a concentrated load is not applied at mid span, Figure 4-30c, the deflection at mid span is given by

$$y_c = \frac{F_v b \cdot (3L^2 - 4b^2)}{48EI}$$

where b is the distance from the point of load application to the right hand end of the beam. Had the typewriter in the previous problem been set at a point centered 14 inches from the right hand side of the shelf, for example, the deflection of the shelf at mid span would have been

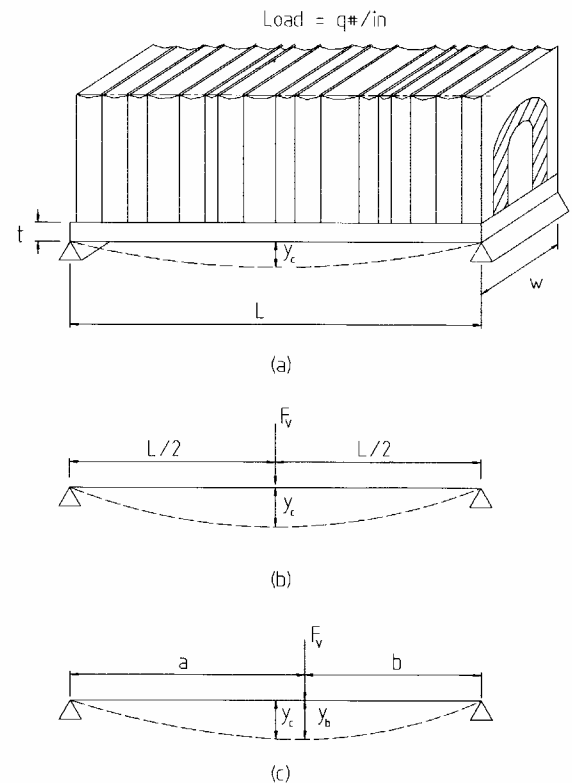
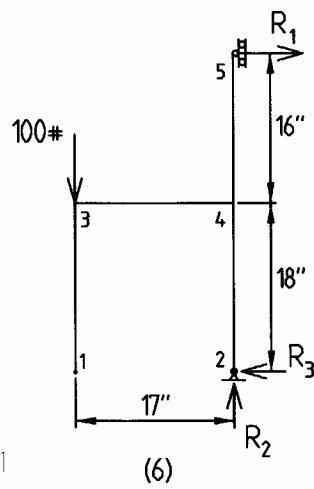
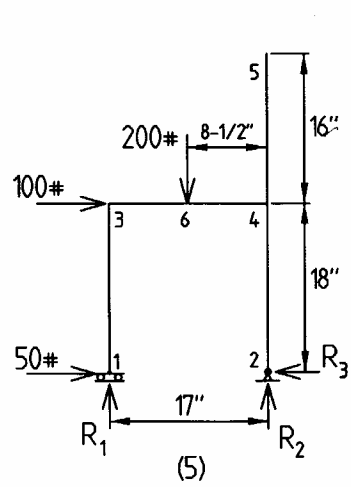
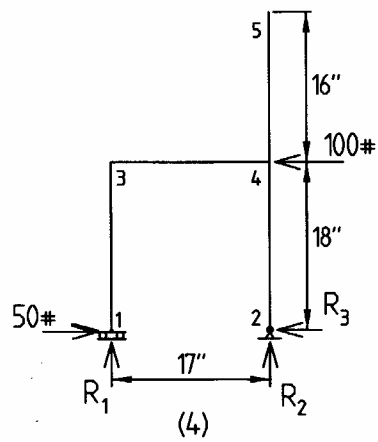
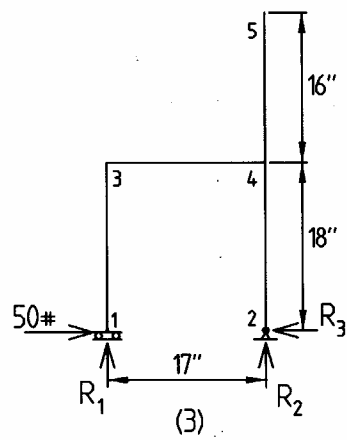
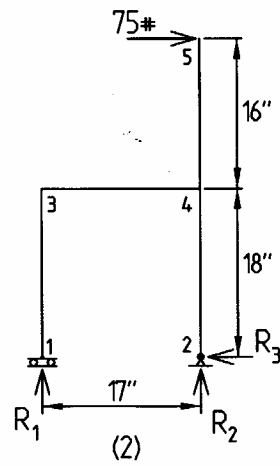
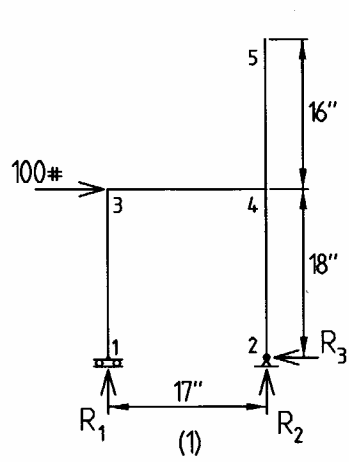


Figure 4-30. [f4-21.dxf]

$$y_c = \frac{50 \times 14 \times [3 \times (34.25)^2 - 4 \times (14)^2]}{48 \times 2,000,000 \times \frac{10.75 \times (0.7)^3}{12}} = 0.065 \cdot in$$

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pdp4-1

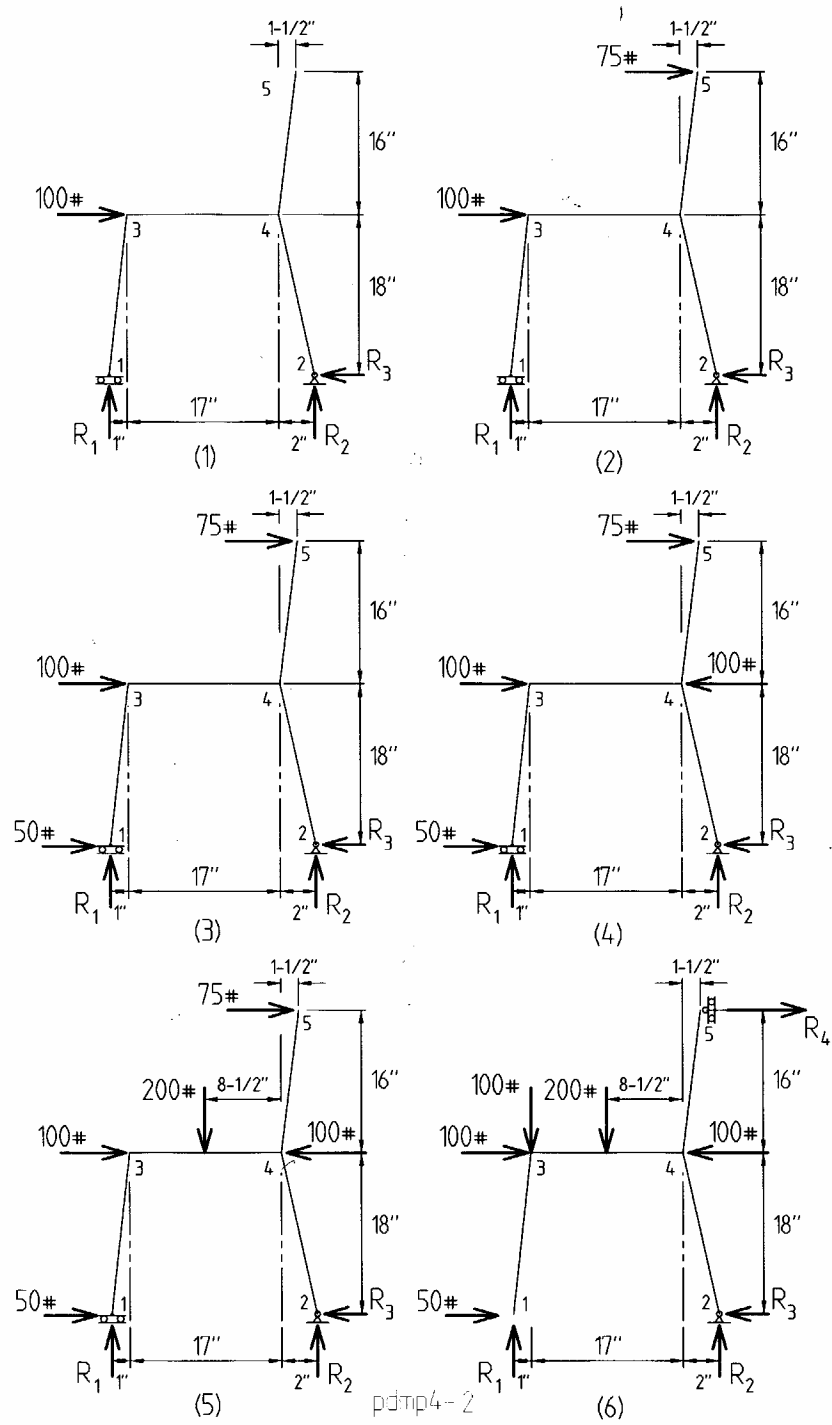


Figure 4-32. [fpdm4p-2]. Calculate the reaction forces and the internal bending moment acting on the end of each member. Use joint numbers to identify the member and the moment.

Answers: pdmp4-1

Bending	Chair No.					
Moment	(1)	(2)	(3)	(4)	(5)	(6)
$f_4(1-3)$	0	0	0	0	0	0
$f_4(3-1)$	0	0	-900	-900	-900	0
$f_4(3-4)$	0	0	900	900	900	0
$f_4(4-3)$	1800	-2550	-900	900	-2700	-1700
$f_4(2-4)$	0	0	0	0	0	0
$f_4(4-2)$	1800	1350	900	-900	2700	900
$f_4(4-5)$	0	1200	0	0	0	800
$f_4(5-4)$	0	0	0	0	0	0
R1	-105.88	-150	0	105.88	-5.88	50
R2	105.88	150	0	-105.88	205.88	100
R3	-100	-75	-50	-50	-150	-50

Answers: pdmp4-2

Bending	Chair No.					
Moment	(1)	(2)	(3)	(4)	(5)	(6)
$f_4(1-3)$	0	0	0	0	0	0
$f_4(3-1)$	-90	-217.5	-1117.5	-1027.5	-922.5	-900
$f_4(3-4)$		217.5	1117.5	1027.5	922.5	900
$f_4(4-3)$	90	-3915	-4815	-3195	-3005	-4300
$f_4(2-4)$	0	0	0	0	0	0
$f_4(4-2)$	1620	2715	3615	1995	1805	2418
$f_4(4-5)$	0	1200	1200	1200	1200	1882
$f_4(5-4)$	0	0	0	0	0	0
R1	-90	-217.5	-217.5	-127.5	-22.5	117.6
R2	90	217.5	217.5	127.5	222.5	300
R3	-100	-175	-225	-125	-125	167.6