EXERCISES

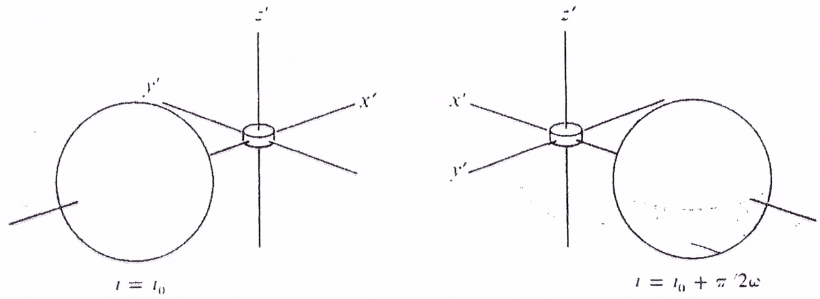

Figure 12.1.
The coordinate system carried by an orbital laboratory as it moves in a circular orbit about the Earth.

Exercise 12.4. CONNECTION COEFFICIENTS FOR ROTATING, ACCELERATING COORDINATES

Beginning with equation (12.4) for the connection coefficients of a Galilean coordinate system $\{x^\alpha(\mathscr{P})\}$, derive the connection coefficients (12.14) of the coordinate system $\{x^{\alpha'}(\mathscr{P})\}$ of equations (12.13). From this, verify that (12.15) are necessary and sufficient for $\{x^{\alpha'}(\mathscr{P})\}$ to be Galilean.

Exercise 12.5. EINSTEIN'S ELEVATOR

Use the formalism of this chapter to discuss "Einstein's elevator"—i.e., the equivalence of "gravity" to an acceleration of one's reference frame. Which aspects of "gravity" are equivalent to an acceleration, and which are not?

Exercise 12.6. GEODESIC DEVIATION ABOVE THE EARTH

A manned orbital laboratory is put into a circular orbit about the Earth [radius of orbit $= r_0$, angular velocity $= \omega = (M/r_0^3)^{1/2}$—why?]. An astronaut jetisons a bag of garbage and watches it move along its geodesic path. He observes its motion relative to (non-Galilean) space coordinates $\{x^{j'}(\mathscr{P})\}$ which—see Figure 12.1—(1) are Euclidean at each moment of universal time $[(\partial/\partial x^{j'}) \cdot (\partial/\partial x^{k'}) = \delta_{jk}]$, (2) have origin at the laboratory's center, (3) have $\partial/\partial x'$ pointing away from the Earth, (4) have $\partial/\partial x'$ and $\partial/\partial y'$ in the plane of orbit. Use the equation of geodesic deviation to calculate the motion of the garbage bag in this coordinate system. Verify the answer by examining the Keplerian orbits of laboratory and garbage. *Hints:* (1) Calculate $R^{\alpha'}{}_{\beta'\gamma'\delta'}$ in this coordinate system by a trivial transformation of tensorial components. (2) Use equation (12.14) to calculate $\Gamma^{\alpha'}{}_{\beta'\gamma'}$ at the center of the laboratory (i.e., on the fiducial geodesic).