

### Relevant equations:

Proper time to travel a given distance at constant acceleration:

$$\tau = \frac{c}{a} \cosh^{-1} \left( 1 + \frac{aD}{c^2} \right)$$

Rapidity:

$$\phi = \frac{a\tau}{c}$$

Velocity at a given rapidity:

$$v = c \tanh \phi$$

Distance traveled while accelerating up to a given rapidity:

$$D = \frac{c^2}{a} (\cosh \phi - 1)$$

Lorentz factor:

$$\gamma = \cosh \phi$$

Proper time experienced during a given Earth-time and Lorentz factor:

$$\tau = \frac{t}{\gamma}$$

### Scenarios:

1. Constant acceleration of 1.5 g's.
2. Constant acceleration of 0.9 g's.
3. Acceleration of 1.5 g's up to the max rapidity of scenario 2, coasting, and negative acceleration of 1.5 g's.

**Scenario 1:** Proper time to travel 5.95 light years (half of 11.9 ly, the distance to Tau Ceti) with constant acceleration of 1.5 g's.

$$\tau_{half} = \frac{c}{1.5g} \cosh^{-1} \left( 1 + \frac{(1.5g)(5.95ly)}{c^2} \right) \approx 1.95y$$

Total trip duration:

$$\tau_{total} = 2 * \tau_{half} \approx 3.9y$$

**Scenario 2:** Proper time to travel 5.95 light years (half of 11.9 ly, the distance to Tau Ceti) with constant acceleration of 0.9 g's.

$$\tau_{half} = \frac{c}{0.9g} \cosh^{-1} \left( 1 + \frac{(0.9g)(5.95ly)}{c^2} \right) \approx 2.76y$$

Total trip duration:

$$\tau_{total} = 2 * \tau_{half} \approx 5.5y$$

**Scenario 3:** Using the same fuel limitation from scenario 2, but accelerating at 1.5 g's until reaching maximum rapidity.

Maximum rapidity from Scenario 2:

$$\phi_{max} = \frac{(0.9g)(2.76y)}{c} = 2.5633$$

Rearrange to get the proper time to reach max rapidity with 1.5 g acceleration.

$$\tau_{burn} = \frac{c}{a} \phi = \frac{c}{1.5g} 2.56 = 1.66y$$

Distance traveled during each burn:

$$D_{burn} = \frac{c^2}{1.5g} (\cosh 2.56 - 1) \approx 3.6ly$$

Distance to coast:

$$D_{coast} = 11.9ly - 2(3.6ly) = 4.7ly$$

Maximum speed:

$$v = c \tanh 2.56 = 0.988c$$

Earth time to coast 4.7ly at 0.988c:

$$t = \frac{4.7ly}{0.988c} = 4.76y$$

Lorentz factor at a rapidity of 2.5633:

$$\gamma = \cosh 2.5633 = 6.53$$

Proper time experienced while coasting:

$$\tau_{coast} = \frac{t}{\gamma} \approx \frac{4.76y}{6.53} \approx 0.73y$$

Total proper time for Scenario 3:

$$\tau_{total} = \tau_{burn} + \tau_{coast} + \tau_{burn} \approx 1.66y + 0.73y + 1.66y = 4.05y$$