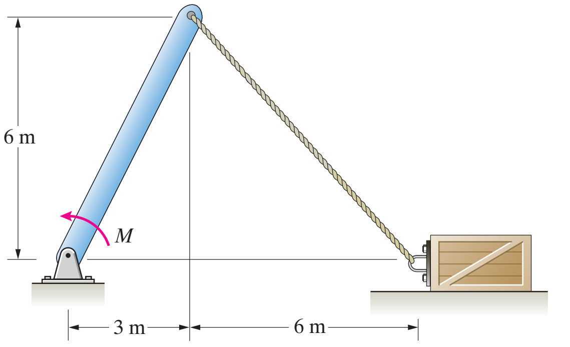
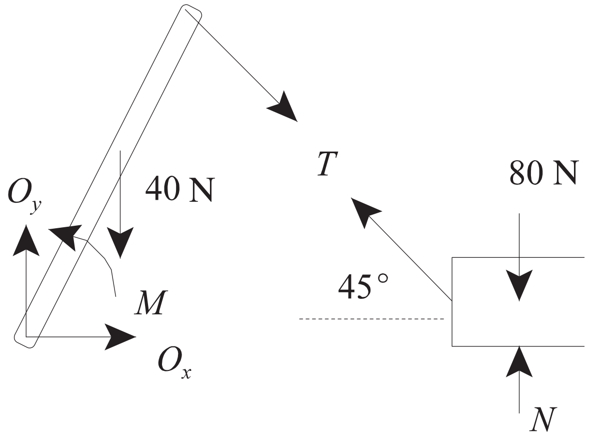
Problem 18.56 The slender bar weighs 40 N and the crate weighs 80 N. At the instant shown, the velocity of the crate is zero and it has an acceleration of 14 m/s^2 toward the left. The horizontal surface is smooth. Determine the couple M and the tension in the rope.



Solution: There are six unknowns $(M, T, N, O_x, O_y, \alpha)$, five dynamic equations, and one constraint equation. We use the following subset of the dynamic equations.



$$\Sigma M_O : M - (40\text{ N})(1.5\text{ m})$$

$$- T\cos 45^\circ(6\text{ m})$$

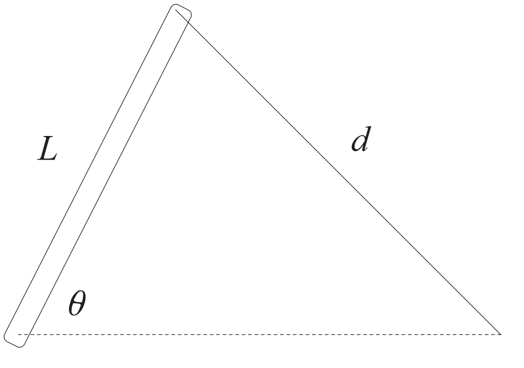
$$- T\sin 45^\circ(3\text{ m})$$

$$= \frac{1}{3}\left(\frac{40\text{ N}}{9.81\text{ m/s}^2}\right)(45\text{ m}^2)\alpha,$$

$$\Sigma F_x : -T\cos 45^\circ = -\left(\frac{80\text{ N}}{9.81\text{ m/s}^2}\right)(14\text{ m/s}^2)$$

The constraint equation is derived from the triangle shown. We have

$$L = \sqrt{45}\text{ m},\ d = 6\sqrt{2}\text{ m},\ \theta = 63.4^\circ.$$



$$x = L\cos\theta + \sqrt{d^2 - L^2\sin^2\theta}$$

$$\dot{x} = \left(-L\sin\theta - \frac{L^2\cos\theta\sin\theta}{\sqrt{d^2 - L^2\sin^2\theta}}\right)\dot{\theta}$$

Since the velocity $\dot{x} = 0$, then we know that the angular velocity $\omega = \dot{\theta} = 0$. Taking one more derivative and setting $\omega = 0$, we find

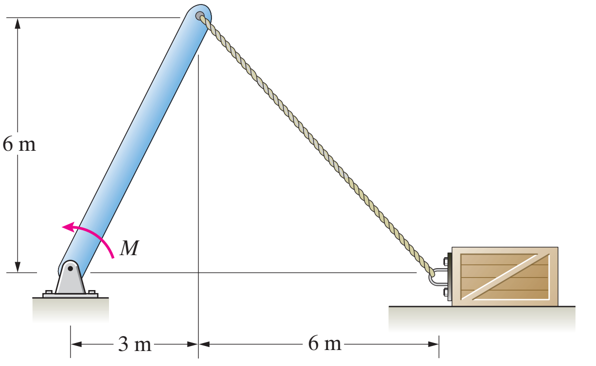
$$\ddot{x} = \left(-L\sin\theta - \frac{L^2\cos\theta\sin\theta}{\sqrt{d^2 - L^2\sin^2\theta}}\right)\ddot{\theta} \Rightarrow -(14\text{ m/s}^2)$$

$$= \left(-L\sin\theta - \frac{L^2\cos\theta\sin\theta}{\sqrt{d^2 - L^2\sin^2\theta}}\right)\alpha$$

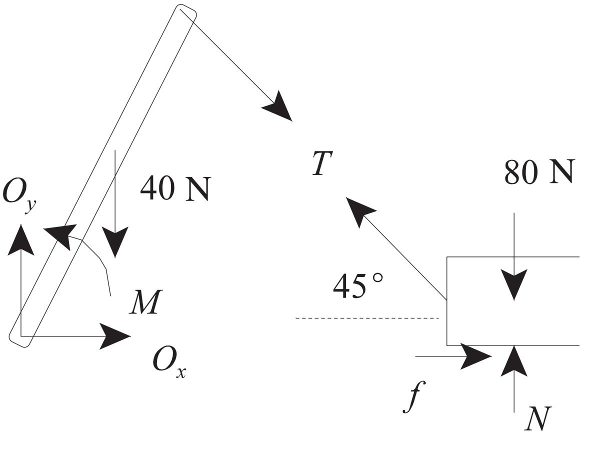
Solving these equations, we find that

$$\alpha = 1.56\text{ rad/s}^2,\quad \boxed{M = 1149\text{ N-m},\ T = 161.5\text{ N.}}$$

Problem 18.57 The slender bar weighs 40 N and the crate weighs 80 N. At the instant shown, the velocity of the crate is zero and it has an acceleration of 14 m/s^2 toward the left. The coefficient of kinetic friction between the horizontal surface and the crate is $\mu_k = 0.2$. Determine the couple M and the tension in the rope.



Solution: There are seven unknowns $(M, T, N, O_x, O_y, \alpha, f)$, five dynamic equations, one constraint equation, and one friction equation. We use the following subset of the dynamic equations.



$$\Sigma M_O : M - (40\text{ N})(1.5\text{ m})$$

$$- T\cos 45^\circ(6\text{ m})$$

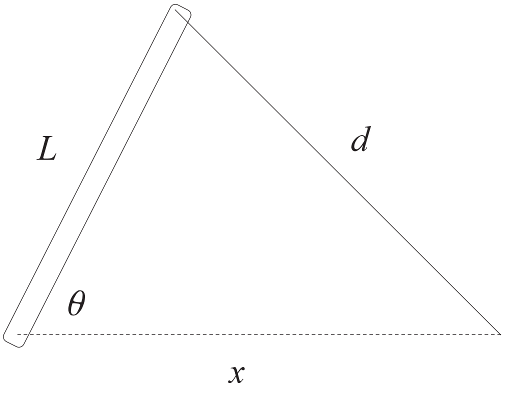
$$- T\sin 45^\circ(3\text{ m})$$

$$= \frac{1}{3}\left(\frac{40\text{ N}}{9.81\text{ m/s}^2}\right)(45\text{ m}^2)\alpha,$$

$$\Sigma F_x : -T\cos 45^\circ + (0.2)N = -\left(\frac{80\text{ N}}{9.81\text{ m/s}^2}\right)(14\text{ m/s}^2)$$

$$\Sigma F_y : T\sin 45^\circ + N - (80\text{ N}) = 0.$$

The constraint equation is derived from the triangle shown. We have $L = \sqrt{45}$ m, $d = 6\sqrt{2}$ m, $\theta = 63.4^\circ$.



$$x = L\cos\theta + \sqrt{d^2 - L^2\sin^2\theta}$$

$$\dot{x} = \left(-L\sin\theta - \frac{L^2\cos\theta\sin\theta}{\sqrt{d^2 - L^2\sin^2\theta}}\right)\dot{\theta}$$

Since the velocity $\dot{x} = 0$, then we know that the angular velocity $\omega = \dot{\theta} = 0$. Taking one more derivative and setting $\omega = 0$, we find

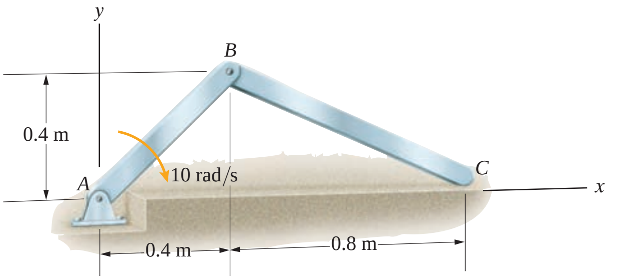
$$\ddot{x} = \left(-L\sin\theta - \frac{L^2\cos\theta\sin\theta}{\sqrt{d^2 - L^2\sin^2\theta}}\right)\ddot{\theta} \Rightarrow -(14\text{ m/s}^2)$$

$$= \left(-L\sin\theta - \frac{L^2\cos\theta\sin\theta}{\sqrt{d^2 - L^2\sin^2\theta}}\right)\alpha$$

Solving these equations, we find that

$$\alpha = 1.56\text{ rad/s}^2,\ N = -28.5\text{ N},\ \boxed{M = 1094\text{ N-m},\ T = 152.8\text{ N.}}$$

Problem 18.58 Bar AB is rotating with a constant clockwise angular velocity of 10 rad/s. The 8-kg slender bar BC slides on the horizontal surface. At the instant shown, determine the total force (including its weight) acting on bar BC and the total moment about its center of mass.



Solution: We first perform a kinematic analysis to find the angular acceleration of bar BC and the acceleration of the center of mass of bar BC. First the velocity analysis:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = 0 + (-10\mathbf{k}) \times (0.4\mathbf{i} + 0.4\mathbf{j}) = (-4\mathbf{i} + 4\mathbf{j})$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B} = (-4\mathbf{i} + 4\mathbf{j}) + \omega_{BC}\mathbf{k} \times (0.8\mathbf{i} - 0.4\mathbf{j})$$

$$= (-4 + 0.4\,\omega_{BC})\mathbf{i} + (4 + 0.8\,\omega_{BC})\mathbf{j}$$

Since C stays in contact with the floor, we set the $\mathbf{j}$ component to zero $\Rightarrow \omega_{BC} = -5$ rad/s. Now the acceleration analysis.

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - {\omega_{AB}}^2\mathbf{r}_{B/A}$$

$$= 0 + 0 - (10)^2(0.4\mathbf{i} + 0.4\mathbf{j}) = (-40\mathbf{i} - 40\mathbf{j})$$

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - {\omega_{BC}}^2\mathbf{r}_{C/B}$$

$$= (-40\mathbf{i} - 40\mathbf{j}) + \alpha_{BC}\mathbf{k} \times (0.8\mathbf{i} - 0.4\mathbf{j}) - (-5)^2(0.8\mathbf{i} - 0.4\mathbf{j})$$

$$= (-60 + 0.4\alpha_{BC})\mathbf{i} + (-30 + 0.8\alpha_{BC})\mathbf{j}$$

Since C stays in contact with the floor, we set the $\mathbf{j}$ component to zero $\Rightarrow \alpha_{BC} = 37.5$ rad/s^2. Now we find the acceleration of the center of mass G of bar BC.

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{G/B} - {\omega_{BC}}^2\mathbf{r}_{G/B}$$

$$= (-40\mathbf{i} - 40\mathbf{j}) + (37.5)\mathbf{k} \times (0.4\mathbf{i} - 0.2\mathbf{j}) - (-5)^2(0.4\mathbf{i} - 0.2\mathbf{j})$$

$$= (-42.5\mathbf{i} - 20\mathbf{j})\ \text{m/s}^2.$$

The total force and moment cause the accelerations that we just calculated. Therefore

$$\mathbf{F} = m\mathbf{a}_G = (8\ \text{kg})(-42.5\mathbf{i} - 20\mathbf{j})\ \text{m/s}^2 = (-340\mathbf{i} - 160\mathbf{j})\ \text{N},$$

$$M = I\alpha = \frac{1}{12}(8\ \text{kg})([0.8\ \text{m}]^2 + [0.4\ \text{m}]^2)(37.5\ \text{rad/s}^2) = 20\ \text{N-m}.$$

$$\boxed{\mathbf{F} = (-340\mathbf{i} - 160\mathbf{j})\ \text{N},\ \ M = 20\ \text{N-m counterclockwise.}}$$