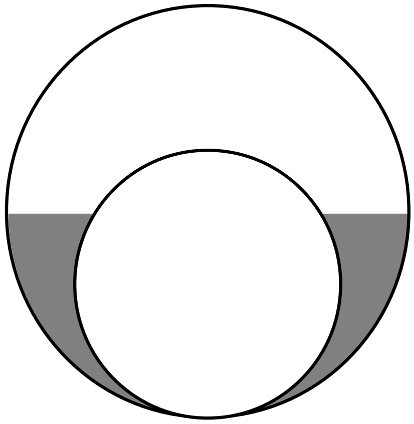

Figure 5: Position of tubing within casing showing area (shaded region) occupied by fluid.

For channel flow the Chézy formula[9] relates velocity, friction factor, wetted perimeter, and area:

$$v^2 \Big/ \left(\frac{8g}{f} \right) = A_1 / P \quad .$$

The Reynolds number and friction factor can be written in terms of the area as

$$\mathrm{Re} = \frac{4\rho Q_1}{\mu\beta\sqrt{A_1}} \quad \text{and} \quad f = 0.184\,\mathrm{Re}^{-.02} = 0.184\left(\frac{4\rho Q_1}{\mu\beta\sqrt{A_1}} \right)^{-0.2}$$

These equations can be combined and solved for the area to get

$$A_1 = \sqrt[2.4]{Q_1^{\,2}\left(\frac{0.184\beta}{8g} \right)\left(\frac{4\rho Q_1}{\mu\beta} \right)^{-0.2}}$$

Annular Heat Transfer Coefficient

The annular heat transfer coefficient can be calculated using the following formulas:[10]

$$D_h = 4A_1/P$$
$$\mathrm{Re} = D_h \langle v \rangle \rho / \mu$$
$$\langle v \rangle = Q_1 / A_1$$
$$P_r = C\mu/k$$
$$Nu_d = 0.0296\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$
$$h = Nu_d\, k / D_h$$

Heat Loss to the Earth

Ramey[2] showed how the heat lost to the earth can be approximated by a parameter

$$A[t] = \frac{Q_1 C\left[k + R_{co} U_{co} f(t)\right]}{2\pi R_{co} k}$$

where C is the heat capacity of the fluid in the annulus, k is the conductivity of the surrounding earth, and U is the overall heat transfer coefficient as discussed by Willhite.[11] For hot oil/watering jobs, U is typically large enough that *f(t)* is closer to a constant temperature than constant flux boundary condition. The analytic solution for this boundary condition is[12]

$$f(y) = -\left(\frac{4}{\pi^2}\right)\int_0^\infty e^{-yz^2} \frac{dz}{z\left[J_0^2(z) + Y_0^2(z)\right]},$$

which can be approximated as (y<5):

$$f(y) \cong \frac{1}{(\pi y)^{-1/2} + 0.35 - 0.067(y/\pi)^{1/2}}$$

For hot oil/watering jobs the time of heat transfer is short enough that the difference in thermal conductivity of the "cement" surrounding casing and the earth can be important. This can be corrected for using the following empirical equation:

$$f(y) \rightarrow f(y)\frac{m + (k_c / k_e) f^2(y) \cdot (1 - k_c / k_e)}{m + f^2(y) \cdot (1 - k_c / k_e)}$$

where k_c is the conductivity of the cement, k_e is the conductivity of the earth, and

$$m = (1.976 - 1.69 R_{co} / R_h) Ln[R_h / R_{co}] \quad .$$

Joule-Thomson Cooling

For oil and water free gas expanding from a high formation pressure to a low flow-line pressure the Joule-Thomson cooling can be approximated as

$$\Delta T_{J-T} = (0.048°F / psi)\Delta P \quad .$$

QUASI-STATIC APPROXIMATION

Including the effects of the heat capacity of the fluid in the annulus results in the following time dependent generalization of Ramey's[1] differential equation:

$$\frac{\partial T}{\partial z} + \varepsilon \frac{\partial T}{\partial t} + \frac{sign}{J \cdot C} + \frac{T - T_e}{A[t]} = 0$$

where Q is positive for flow down the well and negative for flow up the well, sign = 0 for liquid flow in the well, sign = -1 for gas flow down the well, sign = +1 for gas flow up the well, and

$$\begin{aligned} \varepsilon &= A_1 / Q_1 \\ T_e &= a \cdot z + T_s \end{aligned} \quad .$$

Let

$$T = a \cdot z + T_s + F_0[t] + B[z, t]$$

where $B[z,t]$ is a function that goes to zero far from where fluid enters the well and $F_o[t]$ is a function governing the shift in temperature of a flowing well with no fluid injection, thus

$$\begin{aligned} F_0[0] &= 0 \\ T\big|_{z=0} &= T_{ho} \end{aligned} \quad .$$

The equation can be solved by Laplace transform techniques to give

$$F_0[t] = -\left(\frac{a}{\varepsilon} + \frac{sign}{C\varepsilon J}\right) \frac{\int_0^t e^{\int 1/(\varepsilon A[t'])dt'} dt'}{e^{\int 1/(\varepsilon A[t])dt}} \quad \text{and}$$

$$B[z, t] = \frac{T_{ho} - T_s - F_0[t - \varepsilon \cdot z]}{e^{\int_{t-\varepsilon \cdot z}^{t} 1/(\varepsilon A[t])dt}} \quad .$$

Unfortunately these integrals can not be solved in closed form. In the quasi-static approximation, these integrals are solved assuming that $A[t]$ is a constant independent of time and then the appropriate time dependent value of $A[t]$ is plugged into the approximate solutions. For constant $A[t]$ or the quasi-static approximation, the above expressions become

$$F_{0qs}[t] \to -\left(\frac{a}{\varepsilon} + \frac{sign}{C\varepsilon J}\right) A\varepsilon\left[1 - e^{-t/(A\varepsilon)}\right] \quad \text{and}$$

$$B_{qs}[z, t] \to \frac{T_{ho} - T_s - F_{0qs}[t - \varepsilon \cdot z]}{e^{-(z/A)}}$$

for z < t/ε and **zero** for z > t/ε. The depth t/ε is the depth the transient solution has propagated