

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(x-x')-\alpha|k|} = \frac{1}{2\pi} \int_{-\infty}^0 dk e^{ik(x-x')+\alpha k} + \frac{1}{2\pi} \int_0^{\infty} dk e^{ik(x-x')-\alpha k}$$

a)

let  $u = ik(x + \alpha - x')$  then  $du = i(x + \alpha - x')dk$ ; so  $dk = \frac{1}{i(x+\alpha-x')} du$

$$\begin{aligned} \frac{1}{2\pi} \int_{-\infty}^0 dk e^{ik(x+\alpha-x')} &= \frac{1}{2\pi} \frac{1}{i(x + \alpha - x')} \int_{-\infty}^0 du e^u = \frac{1}{2\pi} \frac{e^{ik(x+\alpha-x')}}{i(x + \alpha - x')} \Bigg|_{-\infty}^0 \\ &= \frac{1}{2\pi} \frac{e^0}{i(x + \alpha - x')} - \frac{1}{2\pi} \frac{e^{-\infty}}{i(x + \alpha - x')} = \frac{1}{2\pi} \frac{1}{i(x + \alpha - x')} \end{aligned}$$

b)

let  $u = ik(x - \alpha - x')$  then  $du = i(x - \alpha - x')dk$ ; so  $dk = \frac{1}{i(x-\alpha-x')} du$

$$\begin{aligned} \frac{1}{2\pi} \int_0^{\infty} dk e^{ik(x-\alpha-x')} &= \frac{1}{2\pi} \frac{1}{i(x - \alpha - x')} \int_0^{\infty} du e^u = \frac{1}{2\pi} \frac{e^{-ik(-x+\alpha+x')}}{i(x - \alpha - x')} \Bigg|_0^{\infty} \\ &= \frac{1}{2\pi} \frac{e^{-\infty}}{i(x - \alpha - x')} - \frac{1}{2\pi} \frac{e^0}{i(x - \alpha - x')} = \frac{1}{2\pi} \frac{-1}{i(x - \alpha - x')} \end{aligned}$$

c) adding a) + b) we get

$$\frac{x-\alpha-x'-x-\alpha+x'}{2\pi i(x+\alpha-x')(x-\alpha-x')} = \frac{-\alpha}{i\pi(x+\alpha-x')(x-\alpha-x')} \text{ or } \frac{i\alpha}{\pi(x+\alpha-x')(x-\alpha-x')}$$