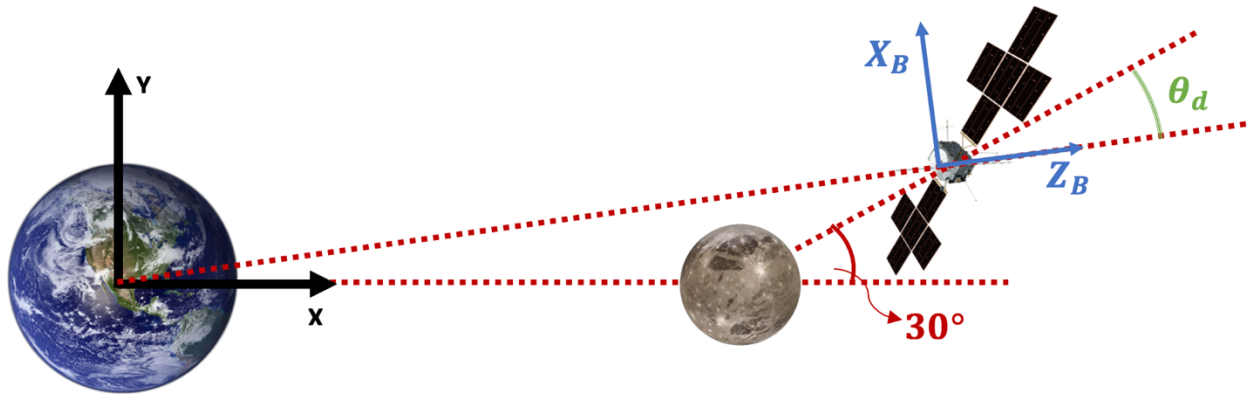


## Space Missions and Systems 2025: Homework #2

A lunar orbiter is flying in a **circular orbit around the Moon** at an altitude of 8000 km. The Moon itself is assumed to follow a **circular orbit around Earth** with radius 361847 km. The inertial reference frame is centered at the Earth's center of mass. The orbital planes of the spacecraft and the Moon coincide. **(This is a 2D problem – for the geometry, refer to the figure. The angle reported is the angle that, at  $t_0 = 0$ , the spacecraft forms with the Earth-Moon line).**



The orbiter carries a high-gain antenna mounted such that its **pointing direction corresponds to the negative direction of the spacecraft body-fixed Z-axis**. To ensure continuous telemetry and command relay to ground stations on Earth, the spacecraft must maintain an Earth-pointing attitude during specific orbital phases.

Power is provided by two identical solar arrays with specular reflectivity coefficient 0.8. Their total area is  $34 \text{ m}^2$  ( $17 \text{ m}^2$  each). The center of pressure is offset by 60 cm in the negative direction along the Z-axis relative to the center of mass. Assume that the solar panels maintain a constant perpendicular orientation to the direction of the Sun, ensuring a consistent disturbance torque.

To maintain Earth-pointing, the attitude control system uses momentum wheels and thrusters to provide stabilization. A momentum wheel has its axis along the Y-axis of the spacecraft. Given the geometry of the problem, this is the wheel that effectively controls the attitude. Two pairs of 3 N thrusters, located 1.25 m from the center of mass of the spacecraft, can be used to generate a balanced torque along the body-fixed Y-axis.

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At time  $t_0 = 0$ , the initial conditions are  $[\psi, \theta, \varphi] = [0^\circ, -29.2537^\circ, 0^\circ]$  and  $[\dot{\psi}, \dot{\theta}, \dot{\varphi}] = [0^\circ/s, -0.0039^\circ/s, 0^\circ/s]$ .  $[\psi, \theta, \varphi]$  are the yaw, pitch and roll axes. Note that these initial conditions are close to the desired Earth-pointing configuration at  $t_0$ . The geometry at  $t_0$  is shown in the figure above. The spacecraft must correct this misalignment and **point toward the Earth** within **5 minutes**, with a maximum error of 20 arcsec using **only the momentum wheel torque**. Once Earth-pointing is acquired, the spacecraft must maintain it continuously for 1 hour.

1. Evaluate all the disturbance torques acting on the spacecraft along the pitch axis during this time. Plot the evolution of the pitch angle in the absence of control torques.
2. Design an attitude control system compliant with the flight rules. Provide in the report the constants describing the control algorithm. Comment on your choice and plot all relevant quantities (offset angle and rate, wheel angular speed, torque, current and power).

After 1 hour the spacecraft experiences a minor malfunction of the attitude control thrusters that causes a constant disturbance torque of 0.008 Nm along the spacecraft Y-axis.

3. Calculate how long the pointing window can be extended using the same control law designed before, considering this new disturbance acting on the spacecraft. Report at which time from  $t_0$  a desaturation maneuver becomes necessary.
4. Design the maneuver to maximize the interval between two consecutive desaturation maneuvers. Consider a maximum pointing error of  $1^\circ$  only during the desaturation maneuver. Report the time needed for the desaturation, plot and comment the results.

Read carefully the following information:

- Solar flux at 1 AU ( $\Phi$ ): 1371 W/m<sup>2</sup>
- Speed of light ( $c$ ): 299792.458 km/s
- Spacecraft moment of inertia: [21000,20000,22500] kg·m<sup>2</sup>
- Wheel moment of inertia: 0.11 kg·m<sup>2</sup>
- Wheel initial angular velocity: +271 rpm (revolutions per minute)
- Wheel maximum angular speed:  $\pm 2000$  rpm
- Wheel maximum torque:  $\pm 0.236$  N·m
- Moon radius: 1737 km
- Moon GM: 4903 km<sup>3</sup>/s<sup>2</sup>
- Earth GM: 398600 km<sup>3</sup>/s<sup>2</sup>

The electric equations of the wheel are:

$$i_W = \frac{T_C}{K_M}$$

$$P_W = V_M \cdot i_W = (K_W \cdot \omega_W + R \cdot i_W) i_W$$

$$K_M = 0.118 \frac{Nm}{A}; \quad K_W = 0.003 \frac{V}{rpm}; \quad R = 1.5 \text{ Ohm}; \quad P_{max} = 18 \text{ W}$$

Assume that the force due to solar pressure is described by:

$$F_{SRP}(t) = - \frac{\Phi}{c} \cdot A \cdot (1 + C_s)$$

where  $\Phi$  is the solar flux at 1AU,  $c$  is the speed of light,  $A$  is the effective cross area, and  $C_s$  is the specular coefficient of the solar panels. Assume that the solar radiation pressure acts along the body fixed X-axis.

The gravity gradient torque along the controlled axis can be modeled as follows:

$$G_y = \frac{3}{2} \frac{GM_{Moon}}{R^3} (I_z - I_x) \sin(2\theta) \cos(\varphi)$$

Where R is the distance from the Moon.

**Submission Guidelines:**

- Upload on Google Classroom a zip folder containing a working computer code printing and plotting the relevant results (MATLAB is recommended) and a comprehensive and short note in English (a pdf file outside the zip folder) containing ALL the relevant mathematical procedure, results, comments, tables and figures. ALWAYS put labels and units on the plot axes.
- Deadline: Sunday 11 May 23:59.
- In the first page of your pdf indicate your first name, last name, and student id (a.k.a. “numero di matricola”).
- Only questions pertaining to the text will be answered. Please ask questions publicly on Classroom (in English).