$$\mathcal{L} = -\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + j^\mu A_\mu$$

$$= -\frac{1}{4} F_{\alpha\beta}\, \eta^{\alpha\sigma} \eta^{\beta\rho} F_{\sigma\rho}$$

$$= -\frac{1}{4}\left(\partial_\alpha A_\beta - \partial_\beta A_\alpha\right) \eta^{\alpha\sigma} \eta^{\beta\rho} \left(\partial_\sigma A_\rho - \partial_\rho A_\sigma\right)$$

$$\therefore \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = -\frac{1}{4}\Big\{ \eta^{\mu\sigma} \eta^{\nu\rho} \left(\partial_\sigma A_\rho - \partial_\rho A_\sigma\right) - \eta^{\nu\sigma}\eta^{\mu\rho}\left(\partial_\sigma A_\rho - \partial_\rho A_\sigma\right)$$

$$+ \left(\partial_\alpha A_\beta - \partial_\beta A_\alpha\right)\eta^{\alpha\mu}\eta^{\beta\nu} - \eta^{\alpha\nu}\eta^{\beta\mu}\left(\partial_\alpha A_\beta - \partial_\beta A_\alpha\right)\Big\}$$

$$= -\frac{1}{4}\Big\{\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) - \left(\partial^\nu A^\mu - \partial^\mu A^\nu\right) + \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) - \left(\partial^\nu A^\mu - \partial^\mu A^\nu\right)\Big\}$$

$$= -\frac{1}{4}\left(F^{\mu\nu} - F^{\nu\mu} + F^{\mu\nu} - F^{\nu\mu}\right) = -\left(\frac{1}{4}\right) 2 \left(F^{\mu\nu} - F^{\nu\mu}\right)$$

$$= -\frac{1}{2}\left[F^{\mu\nu} - \left(-F^{\mu\nu}\right)\right] \quad \because F^{\mu\nu} \text{ is antisym.}$$

$$= -\left(\frac{1}{2}\right) 2 F^{\mu\nu} = -F^{\mu\nu}.$$

check

But E.L. eqn. for fields A_μ is $\dfrac{\partial \mathcal{L}}{\partial A_\mu} - \partial_\nu\left(\dfrac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)}\right) = 0$

$$\frac{\partial \mathcal{L}}{\partial A_\mu} = j^\mu$$

$$\therefore \text{E.L. eqn} \Rightarrow j^\mu - \partial_\nu\left(-F^{\mu\nu}\right) = 0 \Rightarrow \boxed{\partial_\nu F^{\mu\nu} = j^\mu}$$