I understand how one can compute the elements of $\mathbf{su(2)}$ (i.e. the Lie Algebra of $\mathbf{SU(2)}$) using what the book says: the Lie Algebra $\mathbf{g}$ of the matrix Lie Group $\mathbf{G}$ is the set of matrices X such that $exp(tX)$ is in $\mathbf{G}$ for all real t. However, I do not see how this is related to what physicists usually do when they try to find the elements of the Lie Algebra of a certain Lie Group: they take an "infinitesimal transformation" U of $\mathbf{G}$

$$U = I + i\epsilon K + O(\epsilon^2) \approx I + i\epsilon K$$

And whatever properties they find for K, they generalize them for all matrices of $\mathbf{su(2)}$. Why is that? Is K a generic $\mathbf{su(2)}$ element? Why?

I feel like this has to do with the following facts:

1. Any element A of a connected matrix Lie Group $\mathbf{G}$ can be written as $A = e^{X_1}...e^{X_m}$, for some $\{X_i\}$ in $\mathbf{g}$.
2. Let $X \in \mathbf{M}_n(\mathbb{C})$. Then, $\frac{d}{dt}e^{tX} \mid_{t=0}= X$.

But I don't know exactly how...