$$\therefore \int_0^{\pi} \frac{2}{3+\cos(2t)}\, dt = \int_0^{2\pi} \frac{du}{3+\cos(u)}$$

$$= \int_0^{\pi} \frac{du}{3+\cos(u)} + \int_{\pi}^{2\pi} \frac{du}{3+\cos(u)}$$

If $v = \tan\left(\frac{u}{2}\right)$, then $\cos(u) = \frac{1-v^2}{1+v^2}$

and $du = \frac{2}{1+v^2} dv$. Thus;

$$\int_0^{\pi} \frac{du}{3+\cos(u)} = \int_0^{+\infty} \frac{1}{3+\frac{1-v^2}{1+v^2}} \cdot \frac{2}{1+v^2}\, dv$$

$$= \int_0^{+\infty} \frac{2\,dv}{4+2v^2}$$

$$= \int_0^{+\infty} \frac{dv}{2+v^2} = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{v}{\sqrt{2}}\right)\Big|_0^{+\infty}$$

Analougus;

$$\int_{\pi}^{2\pi} \frac{du}{3+\cos(u)} = -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{v}{\sqrt{2}}\right)\Big|_0^{+\infty}$$

$$\therefore \int_0^{2\pi} \frac{du}{3+\cos(u)} = 0$$