

$$\begin{aligned} \therefore \int_0^{\pi} \frac{2}{3+\cos(2t)} dt &= \int_0^{2\pi} \frac{du}{3+\cos(u)} \\ &= \int_0^{\pi} \frac{du}{3+\cos(u)} + \int_{\pi}^{2\pi} \frac{du}{3+\cos(u)} \end{aligned}$$

If $u = \tan^{-1}\left(\frac{v}{z}\right)$, then $\cos(u) = \frac{1-z^2}{1+z^2}$

and $du = \frac{2}{1+z^2} dz$. Thus;

$$\begin{aligned} \int_0^{\pi} \frac{du}{3+\cos(u)} &= \int_0^{+\infty} \frac{1}{3+\frac{1-z^2}{1+z^2}} \cdot \frac{2}{1+z^2} dz \\ &= \int_0^{+\infty} \frac{2dz}{4+z^2} \\ &= \int_0^{+\infty} \frac{dz}{2+z^2} = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{z}{\sqrt{2}}\right) \Big|_0^{+\infty} \end{aligned}$$

Analogous;

$$\int_{\pi}^{2\pi} \frac{du}{3+\cos(u)} = -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{z}{\sqrt{2}}\right) \Big|_0^{+\infty}$$

$$\therefore \int_0^{2\pi} \frac{du}{3+\cos(u)} = 0$$