

# PHYSICS

Registration No:

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February 19, 2012, Time: 10.00 A.M. to 1.00 P.M.

Booklet Series:

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Instructions to the candidates: PLEASE READ CAREFULLY

1. Please do not open the seal of the question paper before 10.00 A.M.
2. The set which is given to you contains question paper with some blank sheets and a machine readable Optical Mark Reader (OMR) sheet.
3. Enter your registration number in the boxes provided on the top of this question paper with blue/black ball point pen.
4. In the OMR sheet, enter your name, registration number and signature at the appropriate places. Strictly follow the instructions written on the OMR sheet.
5. **There are 50 multiple choice questions in the paper and only one of the four answers in each question is correct.**
6. **First 25 questions carry 3 (three) marks each and the last 25 questions carry 1 (one) mark each.**
7. **A correct answer gives full credit (3 for the first 25 questions and 1 for the last 25 questions) and a wrong answer *reduces* your marks by 1 for the first 25 questions and by one third (1/3) for the last 25 questions.**
8. In the OMR sheet, completely darken the bubble corresponding to your answer. Mark in the OMR sheet only after you are sure of the answer. Strictly follow the instructions written on the OMR sheet.
9. Use of pocket calculators is permitted but exchange of the calculators among the candidates is not permitted. Use of other gadgets (mobile phone, palmtops, laptops, pagers etc.,) are not permitted.
10. For rough work, use only the blank pages attached at the end of the question paper.
11. At the end of the examination, return the OMR sheet. You may keep the question paper and the copy of the OMR sheet for your records.

Speed of light, $c$	$3 \times 10^{10} \text{ cm s}^{-1}$
Planck's constant, $h$	$6.62 \times 10^{-27} \text{ erg sec}$
Boltzmann's constant, $k_B$	$1.381 \times 10^{-16} \text{ erg K}^{-1}$
Gas constant, $R$	$8.314 \times 10^7 \text{ erg K}^{-1} \text{ mol}^{-1}$
Stefan-Boltzmann constant, $\sigma$	$5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$
Electron charge, $e$	$1.6 \times 10^{-19} \text{ C}$
Electron mass, $m_e$	$9.11 \times 10^{-28} \text{ gm}$
Proton mass, $m_p$	$1.67 \times 10^{-24} \text{ gm}$
Acceleration due to gravity, $g$	$981 \text{ cm s}^{-2}$
Wien's displacement constant	$0.2898 \text{ cm-K}$
Avogadro's number, $N$	$6.02 \times 10^{23} \text{ mol}^{-1}$
Angstrom unit, $1 \text{ \AA}$	$10^{-8} \text{ cm}$
1 eV	$1.6 \times 10^{-12} \text{ erg}$
1 calorie	4.18 Joules

## PART A: THREE MARK QUESTIONS

1. A monoatomic ideal gas at  $17^\circ\text{C}$  is adiabatically compressed to  $1/8$  of its original volume. The temperature after compression is
  - (a)  $2.1^\circ\text{C}$
  - (b)  $17^\circ\text{C}$
  - (c)  $-200.5^\circ\text{C}$
  - (d)  $887^\circ\text{C}$
  
2. An observer in an inertial frame finds that at a point P the electric field vanishes but the magnetic field does not. This implies that in any other inertial frame the electric field  $\vec{E}$  and the magnetic field  $\vec{B}$  satisfy
  - (a)  $|\vec{E}|^2 = |\vec{B}|^2$
  - (b)  $\vec{E} \cdot \vec{B} = 0$
  - (c)  $\vec{E} \times \vec{B} = 0$
  - (d)  $\vec{E} = 0$
  
3. Consider a system of particles in three dimensions with momentum  $\vec{p}$  and energy  $E = c|\vec{p}|$ ,  $c$  being a constant. The system is maintained at inverse temperature  $\beta$ , volume  $V$  and chemical potential  $\mu$ . What is the grand partition function of the system?
  - (a)  $\exp[e^{\beta\mu}8\pi V/(\beta ch)^3]$
  - (b)  $e^{\beta\mu}6\pi V/(\beta ch)^2$
  - (c)  $\exp[e^{\beta\mu}6\pi V/(\beta ch)^3]$
  - (d)  $e^{\beta\mu}8\pi V/(\beta ch)^2$
  
4. The ground state (apart from normalization) of a particle of unit mass moving in a one-dimensional potential  $V(x)$  is  $\exp(-x^2/2) \cosh(\sqrt{2}x)$ . The potential  $V(x)$ , in suitable units so that  $\hbar = 1$ , is (up to an additive constant)
  - (a)  $x^2/2$
  - (b)  $x^2/2 - \sqrt{2}x \tanh(\sqrt{2}x)$
  - (c)  $x^2/2 - \sqrt{2}x \tan(\sqrt{2}x)$
  - (d)  $x^2/2 - \sqrt{2}x \coth(\sqrt{2}x)$
  
5. A magnetic field  $\vec{B} = B_0(\hat{i} + 2\hat{j} - 4\hat{k})$  exists at a point. If a test charge moving with a velocity,  $\vec{v} = v_0(3\hat{i} - \hat{j} + 2\hat{k})$  experiences no force at a certain point, the electric field at that point in SI units is
  - (a)  $\vec{E} = -v_0 B_0(3\hat{i} - 2\hat{j} - 4\hat{k})$
  - (b)  $\vec{E} = -v_0 B_0(\hat{i} + \hat{j} + 7\hat{k})$
  - (c)  $\vec{E} = v_0 B_0(14\hat{j} + 7\hat{k})$
  - (d)  $\vec{E} = -v_0 B_0(14\hat{j} + 7\hat{k})$
  
6. For small angular displacements (i.e.,  $\sin \theta \approx \theta$ ), a simple pendulum oscillates harmonically. For larger displacements, the motion
  - (a) becomes aperiodic
  - (b) remains periodic with the same period
  - (c) remains periodic with a higher period
  - (d) remains periodic with a lower period

7. A planet orbits a massive star in a highly elliptical orbit, i.e., the total orbital energy  $E$  is close to zero. The initial distance of closest approach is  $R_0$ . Energy is dissipated through tidal motions until the orbit is circularized with a final radius of  $R_f$ . Assume that orbital angular momentum is conserved during the circularization process. Then

- (a)  $R_f = R_0/2$
- (b)  $R_f = R_0$
- (c)  $R_f = \sqrt{2}R_0$
- (d)  $R_f = 2R_0$

8. A binary system consists of two stars of equal mass  $m$  orbiting each other in a circular orbit under the influence of gravitational forces. The period of the orbit is  $\tau$ . At  $t = 0$ , the motion is stopped and the stars are allowed to fall towards each other. After what time  $t$ , expressed in terms of  $\tau$ , do they collide? The following integral may be useful ( $x = r^{1/2}$ )

$$\int \frac{x^2 dx}{\sqrt{\alpha - x^2}} = -\frac{x}{2} \sqrt{\alpha - x^2} + \frac{\alpha}{2} \sin^{-1} \left( \frac{x}{\sqrt{\alpha}} \right).$$

- (a)  $\sqrt{2}\tau$
- (b)  $\frac{\tau}{\sqrt{2}}$
- (c)  $\frac{\tau}{2\sqrt{2}}$
- (d)  $\frac{\tau}{4\sqrt{2}}$

9. Consider a system maintained at temperature  $T$ , with two available energy states  $E_1$  and  $E_2$  each with degeneracies  $g_1$  and  $g_2$ . If  $p_1$  and  $p_2$  are probabilities of occupancy of the two energy states, what is the entropy of the system?

- (a)  $S = -k_B [p_1 \ln(p_1/g_1) + p_2 \ln(p_2/g_2)]$
- (b)  $S = -k_B [p_1 \ln(p_1 g_1) + p_2 \ln(p_2 g_2)]$
- (c)  $S = -k_B [p_1 \ln(p_1^{g_1}) + p_2 \ln(p_2^{g_2})]$
- (d)  $S = -k_B [(1/p_1) \ln(p_1/g_1) + (1/p_2) \ln(p_2/g_2)]$

10. Consider the Bohr model of the hydrogen atom. If  $\alpha$  is the fine-structure constant, the velocity of the electron in its lowest orbit is

- (a)  $\frac{c}{1 + \alpha}$
- (b)  $\frac{c}{1 + \alpha^2}$  or  $(1 - \alpha)c$
- (c)  $\alpha^2 c$
- (d)  $\alpha c$

11. Consider a particle of mass  $m$  moving inside a two-dimensional square box whose sides are described by the equations  $x = 0, x = L, y = 0, y = L$ . What is the lowest eigenvalue of an eigenstate which changes sign under the exchange of  $x$  and  $y$ ?

- (a)  $\hbar^2/(mL^2)$
- (b)  $3\hbar^2/(2mL^2)$
- (c)  $5\hbar^2/(2mL^2)$
- (d)  $7\hbar^2/(2mL^2)$

12. Consider an ideal gas of mass  $m$  at temperature  $T_1$  which is mixed isobarically (i.e. at constant pressure) with an equal mass of same gas at temperature  $T_2$  in a thermally insulated container. What is the change of entropy of the universe?
- $2mC_p \ln \left( \frac{T_1+T_2}{2\sqrt{T_1T_2}} \right)$
  - $2mC_p \ln \left( \frac{T_1-T_2}{2\sqrt{T_1T_2}} \right)$
  - $2mC_p \ln \left( \frac{T_1+T_2}{2T_1T_2} \right)$
  - $2mC_p \ln \left( \frac{T_1-T_2}{2\sqrt{T_1T_2}} \right)$
13. In a certain inertial frame two light pulses are emitted at points 5 km apart and separated in time by  $5 \mu\text{s}$ . An observer moving at a speed  $V$  along the line joining these points notes that the pulses are simultaneous. Therefore  $V$  is
- $0.7c$
  - $0.8c$
  - $0.3c$
  - $0.9c$
14. A circular conducting ring of radius  $R$  rotates with constant angular velocity  $\omega$  about its diameter placed along the  $x$ -axis. A uniform magnetic field  $B$  is applied along the  $y$ -axis. If at time  $t = 0$  the ring is entirely in the  $xy$ -plane, the emf induced in the ring at time  $t > 0$  is
- $B\omega^2\pi R^2 t$
  - $B\omega\pi R^2 \tan(\omega t)$
  - $B\omega\pi R^2 \sin(\omega t)$
  - $B\omega\pi R^2 \cos(\omega t)$
15. Define  $\sigma_x = (f^\dagger + f)$ , and  $\sigma_y = -i(f^\dagger - f)$ , where the  $\sigma$ 's are Pauli spin matrices and  $f, f^\dagger$  obey anticommutation relations  $\{f, f\} = 0, \{f, f^\dagger\} = 1$ . Then  $\sigma_z$  is given by
- $f^\dagger f - 1$
  - $2f^\dagger f - 1$
  - $2f^\dagger f + 1$
  - $f^\dagger f$
16. The value of the integral  $\int_0^\infty \frac{\ln x}{(x^2+4)^2} dx$  is
- 0
  - $-\pi/4$
  - $-\pi/2$
  - $\pi/2$
17. An electric field in a region is given by  $\vec{E}(x, y, z) = ax\hat{i} + cz\hat{j} + 6by\hat{k}$ . For which values of  $a, b, c$  does this represent an electrostatic field?
- 13, 1, 12
  - 17, 6, 1
  - 13, 1, 6
  - 45, 6, 1

18. If  $[x]$  denotes the greatest integer not exceeding  $x$ , then  $\int_0^\infty [x]e^{-x}dx$  is

- (a)  $\frac{1}{e-1}$
- (b) 1
- (c)  $\frac{e-1}{e}$
- (d)  $\frac{e}{e^2-1}$

19. Consider a system of two spin-1/2 particles with total spin  $\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2$ , where  $\mathbf{s}_1$  and  $\mathbf{s}_2$  are in terms of Pauli matrices  $\sigma_i$ . The spin triplet projection operator is

- (a)  $\frac{1}{4} + \mathbf{s}_1 \cdot \mathbf{s}_2$
- (b)  $\frac{3}{4} - \mathbf{s}_1 \cdot \mathbf{s}_2$
- (c)  $\frac{3}{4} + \mathbf{s}_1 \cdot \mathbf{s}_2$
- (d)  $\frac{1}{4} - \mathbf{s}_1 \cdot \mathbf{s}_2$

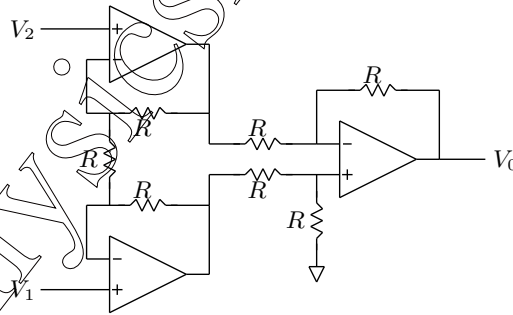
20. A collection of  $N$  two-level systems with energies 0 and  $E > 0$  is in thermal equilibrium at temperature  $T$ . For  $T \rightarrow \infty$ , the specific heat approaches

- (a) 0
- (b)  $Nk_B$
- (c)  $3Nk_B/2$
- (d)  $\infty$

21. Consider a spin-1/2 particle in the presence of a homogeneous magnetic field of magnitude  $B$  along  $z$ -axis which is prepared initially in a state  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$  at time  $t = 0$ . At what time  $t$  will the particle be in the state  $-\lvert\Psi\rangle$  ( $\mu_B$  is Bohr magneton)?

- (a)  $t = \frac{\pi\hbar}{\mu_B B}$
- (b)  $t = \frac{2\pi\hbar}{\mu_B B}$
- (c)  $t = \frac{\pi\hbar}{2\mu_B B}$
- (d) Never

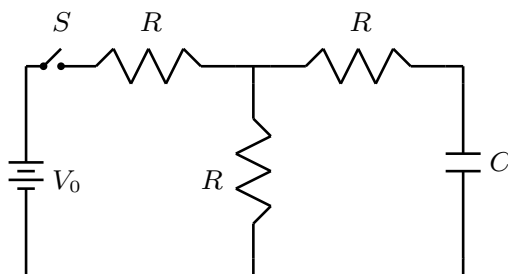
22. The classic three op-amp instrumentation amplifier configuration is shown below:



The op-amps are ideal and all resistors are of equal value  $R$ . The gain, defined as the output voltage  $V_0$  divided by the differential input voltage  $V_1 - V_2$ , is equal to

- (a) 2
- (b) 3
- (c) 4
- (d) 6

23. A capacitor  $C$  is connected to a battery  $V_0$  through three equal resistors  $R$  and a switch  $S$  as shown below:



The capacitor is initially uncharged. At time  $t = 0$ , the switch  $S$  is closed. The voltage across the capacitor as a function of time  $t$  for  $t > 0$  is given by

- (a)  $(V_0/2)(1 - \exp(-t/2RC))$
  - (b)  $(V_0/3)(1 - \exp(-t/3RC))$
  - (c)  $(V_0/3)(1 - \exp(-3t/2RC))$
  - (d)  $(V_0/2)(1 - \exp(-2t/3RC))$
24. As  $x \rightarrow 1$ , the infinite series  $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots$
- (a) diverges
  - (b) converges to unity
  - (c) converges to  $\pi/4$
  - (d) none of the above
25. A beam of X-rays is incident on a BCC crystal. If the difference between the incident and the scattered wavevectors is  $\vec{K} = h\hat{x} + k\hat{y} + l\hat{z}$  where  $\hat{x}, \hat{y}, \hat{z}$  are the unit vectors of the associated cubic lattice, the necessary condition for the scattered beam to give a Laue maximum is
- (a)  $h + k + l = \text{even}$
  - (b)  $h = k = l$
  - (c)  $h, k, l$  are all distinct
  - (d)  $h + k + l = \text{odd}$

## PART-B: ONE MARK QUESTIONS

26. A small magnet is dropped down a long vertical copper tube in a uniform gravitational field. After a long time, the magnet
- (a) attains a constant velocity
  - (b) moves with a constant acceleration
  - (c) moves with a constant deceleration
  - (d) executes simple harmonic motion
27. The ground state energy of 5 identical spin-1/2 particles which are subject to a one-dimensional simple harmonic oscillator potential of frequency  $\omega$  is
- (a)  $(15/2)\hbar\omega$
  - (b)  $(13/2)\hbar\omega$
  - (c)  $(1/2)\hbar\omega$
  - (d)  $5\hbar\omega$
28. Efficiency of a perfectly reversible (Carnot) heat engine operating between absolute temperatures  $T$  and zero is equal to
- (a) 0
  - (b) 0.5
  - (c) 0.75
  - (d) 1
29. Consider a particle of electric charge  $e$  and mass  $m$  moving under the influence of a constant horizontal electric field  $E$  and constant vertical gravitational field described by acceleration due to gravity  $g$ . If the particle starts from rest, what will be its trajectory?
- (a) parabolic
  - (b) elliptic
  - (c) straight line
  - (d) circular
30. A thermally insulated ideal gas of volume  $V_1$  and temperature  $T$  expands into another enclosure of volume  $V_2$  through a porous plug. What is the change in the temperature of the gas?
- (a) 0
  - (b)  $T \ln(V_1/V_2)$
  - (c)  $T \ln(V_2/V_1)$
  - (d)  $T \ln[(V_2 - V_1)/V_2]$

31. The spatial part of a two-electron state is symmetric under exchange. If  $|\uparrow\rangle$  and  $|\downarrow\rangle$  represent the spin-up and spin-down states respectively of each particle, the spin-part of the two-particle state is
- $|\uparrow\rangle|\uparrow\rangle$
  - $|\uparrow\rangle|\downarrow\rangle$
  - $(|\downarrow\rangle|\uparrow\rangle - |\uparrow\rangle|\downarrow\rangle)/\sqrt{2}$
  - $(|\downarrow\rangle|\uparrow\rangle + |\uparrow\rangle|\downarrow\rangle)/\sqrt{2}$
32. The ratio of maximum to minimum resistance that can be obtained with  $N$   $1\text{-}\Omega$  resistors is
- $N$
  - $N^2$
  - 1
  - $\infty$
33. The wave function of a free particle in one dimension is given by  $\psi(x) = A \sin x + B \sin 3x$ . Then  $\psi(x)$  is an eigenstate of
- the position operator
  - the Hamiltonian
  - the momentum operator
  - the parity operator
34. A point charge  $+q$  is placed at  $(0, 0, d)$  above a grounded infinite conducting plane defined by  $z = 0$ . There are no charges present anywhere else. What is the magnitude of electric field at  $(0, 0, -d)$ ?
- $q/(8\pi\epsilon_0 d^2)$
  - $-\infty$
  - 0
  - $q/(16\pi\epsilon_0 d^2)$
35. What is the value of the following series?
- $$\left(1 + \frac{1}{2!} + \frac{1}{4!} + \dots\right)^2 - \left(1 + \frac{1}{3!} + \frac{1}{5!} + \dots\right)^2 \quad (1)$$
- 0
  - $e$
  - $e^2$
  - 1
36. The second order maximum in the diffraction of X-rays of 0.20 nanometer wavelength from a simple cubic crystal is found to occur at an angle of thirty degrees with respect to the crystal plane. The distance between the lattice planes is
- 1 Angstrom
  - 2 Angstroms
  - 4 Angstroms
  - 8 Angstroms



37. A jet of gas consists of molecules of mass  $m$ , speed  $v$  and number density  $n$  all moving co-linearly. This jet hits a wall at an angle  $\theta$  to the normal. The pressure exerted on the wall by the jet assuming elastic collision will be
- $p = 2mnv^2 \cos^2 \theta$
  - $p = 2mnv^2 \cos \theta$
  - $p = \sqrt{3/2} mnv \cos^2 \theta$
  - $p = mnv^2$
38. A time-dependent magnetic field  $\vec{B}(t)$  is produced in a circular region of space, infinitely long and of radius  $R$ . The magnetic field is given as  $\vec{B} = B_0 t \hat{z}$  for  $0 \leq r < R$  and is zero for  $r > R$ , where  $B_0$  is a positive constant. The electric field for  $r > R$  is
- $\frac{B_0 R^2}{r} \hat{r}$
  - $\frac{B_0 R^2}{2r} \hat{\theta}$
  - $-\frac{B_0 R^2}{r} \hat{r}$
  - $-\frac{B_0 R^2}{2r} \hat{\theta}$
39. If the coordinate  $q$  and the momentum  $p$  form a canonical pair  $(q, p)$ , which one of the sets given below also forms a canonical pair?
- $(q, -p)$
  - $(q^2, p^2)$
  - $(p, -q)$
  - $(q^2, -p^2)$
40. The net charge of an  $n$ -type semiconductor is
- positive
  - zero
  - negative
  - dependent on the dopant density
41. The length and radius of a perfect cylinder are each measured with an RMS error of 1%. The RMS error on the inferred volume of the cylinder is roughly
- 1.7%
  - 3.3%
  - 0.5%
  - 1%.
42. When unpolarised light is incident on a glass plate at a particular angle, it is observed that the reflected beam is linearly polarised. What is the angle of the refracted beam with respect to the surface normal? Refractive index of glass is 1.52
- $56.7^\circ$
  - $33.4^\circ$
  - $23.3^\circ$
  - The light is completely reflected and there is no refracted beam.

43. The Dulong-Petit law fails near room temperature (300 K) for many light elements (such as boron and beryllium) because their Debye temperature is
- $\gg 300$  K
  - $\sim 300$  K
  - $\ll 300$  K
  - 0 K
44. An unbiased die is cast twice. The probability that the positive difference (bigger – smaller) between the two numbers is 2 is
- 1/9
  - 2/9
  - 1/6
  - 1/3
45. A cube has a constant electric potential  $V$  on its surface. If there are no charges inside the cube, the potential at the center of the cube is
- $V$
  - $V/8$
  - 0
  - $V/6$
46. A girl measures the period of a simple pendulum inside a stationary lift and finds it to be  $T$  seconds. If the lift accelerates upward with an acceleration  $g/4$ , then the period of the pendulum will be
- $T$
  - $T/4$
  - $2T/\sqrt{5}$
  - $2T\sqrt{5}$
47. The quantum state  $\sin x |\uparrow\rangle + \exp(i\phi) \cos x |\downarrow\rangle$ , where  $\langle \uparrow | \downarrow \rangle = 0$  and  $x, \phi$  are real, is orthogonal to:
- $\sin x |\uparrow\rangle$
  - $\cos x |\uparrow\rangle + \exp(i\phi) \sin x |\downarrow\rangle$
  - $-\cos x |\uparrow\rangle - \exp(i\phi) \sin x |\downarrow\rangle$
  - $-\exp(-i\phi) \cos x |\uparrow\rangle + \sin x |\downarrow\rangle$
48. The binding energy of the hydrogen atom (electron bound to proton) is 13.6 eV. The binding energy of positronium (electron bound to positron) is
- 13.6/2 eV
  - 13.6/1810 eV
  - $13.6 \times 1810$  eV
  - $13.6 \times 2$  eV
49. For an  $N \times N$  matrix consisting of all ones,
- all eigenvalues = 1
  - all eigenvalues = 0
  - the eigenvalues are  $1, 2, \dots, N$
  - one eigenvalue =  $N$ , the others = 0

50. A sodium atom in the first excited  $3P$  state has a lifetime of 16 ns for decaying to the ground  $3S$  state. The wavelength of the emitted photon is 589 nm. The corresponding linewidth of the transition (in frequency units) is about

- (a)  $1.7 \times 10^6$  Hz
- (b)  $1 \times 10^7$  Hz
- (c)  $6.3 \times 10^7$  Hz
- (d)  $5 \times 10^{14}$  Hz

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Physics JEST 2012 Answer Key: Booklet Series **P**

**PART A: THREE MARK QUESTIONS**

1. (d)
2. (b)
3. (a)
4. (b)
5. (d)
6. (c)
7. (d)
8. (d)
9. (a)
10. (d)
11. (c)
12. (a)
13. (c)
14. (c)
15. (b)
16. (b)
17. (c)
18. (a)
19. (c)
20. (a)
21. (b)
22. (b)
23. (d)
24. (c)
25. (a)

**PART-B: ONE MARK QUESTIONS**

26. (a)
27. (b)
28. (d)
29. (c)
30. (a)
31. (c)
32. (b)
33. (d)
34. (c)
35. (d)
36. (c)
37. (a)
38. (d)
39. (c)
40. (b)
41. (a)
42. (b)
43. (a)
44. (b)
45. (a)
46. (c)
47. (d)
48. (a)
49. (d)
50. (b)

—END—