Luminosity Calculation in Crab Crossing Simulation

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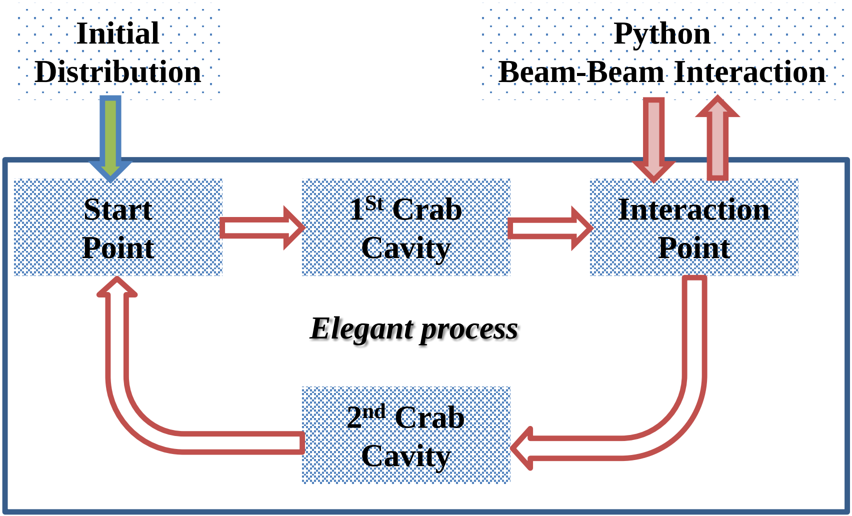
Outline

- Introduction
- Analytic Methods
- Numerical Calculations
- Benchmark Result
- Future Plans

Introduction

- Crab dynamics simulations are in progress using Elegant without Beam-Beam effect
- Beam-Beam effect is important for crab dynamics, e.g. due to synchro-betatron resonances (Yves had shown it with BeamBeam3D)
- BeamBeam3D has simplified beam dynamics. We would like to combine accurate dynamics simulation with an adequate beam-beam model.
- We develop a code in Python for simulating beam-beam effect and calculation of luminosity during the simulation process.

Introduction



Analytic Methods

1) Direct Integral Method (J. E. Augustin, A. W. Chao, …) calculates luminosity for any collision configurations.

2) Bassetti-Erskine-based technique extended to finite bunch length, crossing angle and bunch offset; it is straightforward to use it to apply beam-beam kick to individual particles.

Direct Integral Method (in the lab frame)

$$\mathcal{L} = N_1 N_2 f_c N_B \int\int\int\int \rho_1 \rho_2 dxdydsdt$$

$$\rho_i(x_i, y_i, s_i, t) = \rho_{ix}(x_i)\rho_{iy}(y_i)\rho_{is}(s_i - ct), \quad i = 1, 2$$

For the Gaussian beam

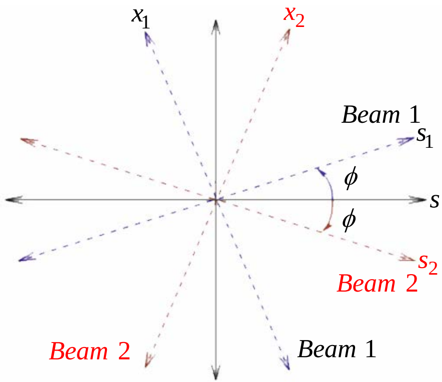
$$\rho_{iw}(w) = \frac{1}{\sigma_{iw}\sqrt{2\pi}} \exp\left(-\frac{w^2}{2\sigma_{iw}^{\ 2}}\right), \quad w = x, y$$

$$\rho_{is}(s - s_0) = \frac{1}{\sigma_{is}\sqrt{2\pi}} \exp\left(-\frac{(s - s_0)^2}{2\sigma_{is}^{\ 2}}\right)$$

$$\sigma_{iw} = \sigma_{iw}^{*}\sqrt{1 + \frac{s_i^2}{\beta_{iw}^{*\ 2}}}, \quad s_0 = ct \text{ (the beam center to the colliding point)}$$

Direct Integral Method (Coordinates transformation)

If there is a crossing angle 2ϕ in horizontal (x, s)-plane



$$x_1 = x \cdot \cos\phi - s \cdot \sin\phi$$
$$s_1 = s \cdot \cos\phi + x \cdot \sin\phi$$

$$x_2 = -x \cdot \cos\phi - s \cdot \sin\phi$$
$$s_2 = -s \cdot \cos\phi + x \cdot \sin\phi$$

Direct Integral Method (after integration over y, s_0, x)

$$\mathcal{L} = \frac{\cos\phi N_1 N_2 f_{rev} N_B}{\sqrt{2}\pi^{3/2}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}} \int \frac{\exp\left[s^2\left(-\frac{1+\cos 2\phi}{\sigma_{1s}^2+\sigma_{2s}^2}+\frac{\cos 2\phi}{s_{1x}+s_{2x}}\right)\right]}{\sqrt{s_{1x}+s_{2x}}\sqrt{s_{1y}+s_{2y}}} ds$$

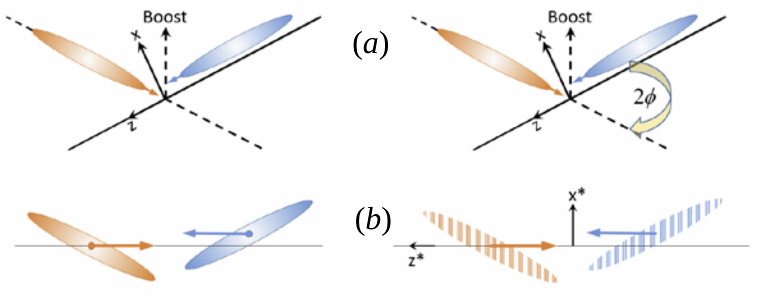
where $$s_{iw} = \frac{\left(s^2\cos^2\phi+\beta_{iw}^{*\,2}\right)\sigma_{iw}^{*\,2}}{\beta_{iw}^{*\,2}}$$

Comparing with the "Head on" $$\mathcal{L}_0 = \frac{N_1 N_2 f_{rev} N_B}{2\pi\sqrt{\sigma_{1x}^{*\,2}+\sigma_{2x}^{*\,2}}\sqrt{\sigma_{1y}^{*\,2}+\sigma_{2y}^{*\,2}}}$$

The luminosity reduction factor with hourglass effect and crossing angle for two bunches

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \frac{\sqrt{2}\cos\phi\sqrt{\sigma_{1x}^{*\,2}+\sigma_{2x}^{*\,2}}\sqrt{\sigma_{1y}^{*\,2}+\sigma_{2y}^{*\,2}}}{\sqrt{\pi}\sqrt{\sigma_{1s}^2+\sigma_{2s}^2}} \int \frac{\exp\left[s2\left(-\frac{1+\cos 2\phi}{\sigma_{1s}^2+\sigma_{2s}^2}+\frac{\cos 2\phi}{s_{1x}+s_{2x}}\right)\right]}{\sqrt{s_{1x}+s_{2x}}\sqrt{s_{1y}+s_{2y}}} ds$$

Model for Lorentz Boost Method



Beams colliding at an angle:
in the original frame (a) and in the boosted frame (b).

The direction of the Lorentz boost is indicated by a dished line.

Assume one IP (the interaction point) in a ring located at *s*=0, where *s* is the azimuthal coordinate.

At the IP, based on Lorentz transformation, the coordinates of a particle are boosted so that the collision becomes head on, then the particle interacts with the other beam in this boosted frame.

Finally, the luminosity calculation for the head-on is carry out in this frame.

Accelerator frame → Lab frame → Boosted frame → Luminosity

The coordinate system $\mathbf{x} = (x, p_x, y, p_y, z, p_z; h, s)$ is called the accelerator coordinate. Here x and y are horizontal and vertical coordinates, p_x, p_y, p_z are three-momentums, $z = s - ct$ (c is the light velocity, t is the arrival time at the position s), and h is the "Hamiltonian" (Details can be found in our technical report)

Accelerator frame → Lab frame

For the laboratory frame, the Cartesian system is given by

$$\mathbf{X} = (X, Y, Z, P_X, P_Y, P_Z; H, T) \quad \text{and} \quad s = \frac{z_+ - z_-}{2}$$

where H is the true Hamiltonian, which is the energy, and T is the time. The relations between the accelerator coordinates are

$$\begin{pmatrix} cT \\ X \\ Z \\ Y \end{pmatrix} = \underbrace{\begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{A} \begin{pmatrix} z \\ x \\ s \\ y \end{pmatrix} \quad \text{and} \quad P_0 \begin{pmatrix} p_z \\ p_x \\ h \\ p_y \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{B} \begin{pmatrix} H/c - P_0 \\ P_X \\ P_Z - P_0 \\ P_Y \end{pmatrix}$$

where P_0 is the absolute value of the three-momentum P of the reference particle.

Based on this model, after Lorentz boost, the collision becomes head-on in other words, in Lorentz Boost Frame, we can easily calculate the luminosity via the head-on formula, which is well studied. Without offset, the Lorentz boost yields

$$\begin{pmatrix} cT^* \\ X^* \\ Z^* \\ Y^* \end{pmatrix} = L \begin{pmatrix} cT \\ X \\ Z \\ Y \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} H^*/c \\ P_X{}^* \\ P_Z{}^* \\ P_Y{}^* \end{pmatrix} = L \begin{pmatrix} H/c \\ P_X \\ P_Z \\ P_Y \end{pmatrix}$$

where

$$L = \begin{pmatrix} 1/\cos\phi & -\sin\phi & -\tan\phi\sin\phi & 0 \\ -\tan\phi & 1 & \tan\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Accelerator → Lorentz Boosted frame

$$\begin{pmatrix} z^*(s) \\ x^*(s) \\ s^* \\ y^*(s) \end{pmatrix} = A^{-1}LA \begin{pmatrix} z(0) \\ x(0) \\ 0 \\ y(0) \end{pmatrix} = \begin{pmatrix} 1/\cos\phi & 0 & 0 & 0 \\ -\tan\phi & 1 & 0 & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z(0) \\ x(0) \\ 0 \\ y(0) \end{pmatrix}$$

and

$$\begin{pmatrix} p_z^* \\ p_x^* \\ h^* \\ p_y^* \end{pmatrix} = B^{-1}LB \begin{pmatrix} p_z \\ p_x \\ h \\ p_y \end{pmatrix} = \begin{pmatrix} 1 & -\tan\phi & -\tan^2\phi & 0 \\ 0 & 1/\cos\phi & -\tan\phi/\cos\phi & 0 \\ 0 & 0 & 1/\cos^2\phi & 0 \\ 0 & 0 & 0 & 1/\cos\phi \end{pmatrix} \begin{pmatrix} p_z \\ p_x \\ h \\ p_y \end{pmatrix}$$

where $h_i^* = \frac{\partial h^*}{\partial p_i^*}$ and $h^*(p_x^*, p_y^*, p_z^*; P_0^*) = \frac{1}{\cos^2\phi} h(p_x, p_y, p_z; P) = h(p_x^*, p_y^*, p_z^*; P_0^*)$

Accelerator → Lorentz Boosted frame (continue…)

Finally,

$$x^* = \tan\phi z + [1 + h_x^* \sin\phi] x$$

$$y^* = y + \sin\phi\, h_y^*\, x$$

$$z^* = z / \cos\phi + h_z^* \sin\phi\, x$$

$$p_x{}^* = (p_x - \tan\phi\, h) / \cos\phi$$

$$p_y{}^* = p_y / \cos\phi$$

$$p_z{}^* = p_z - \tan\phi\, p_x + \tan^2\phi h$$

and

$$\left(\sigma_x^*\right)^2 = \sigma_x{}^2 + \sigma_z{}^2 \tan^2\phi$$

$$\left(\sigma_y^*\right)^2 = \sigma_y{}^2$$

$$\left(\sigma_z^*\right)^2 = \sigma_z{}^2 / \cos^2\phi$$

e.g., the effective beam size $\sigma_{x\_eff} = \sqrt{\sigma_{1x}{}^2 + \sigma_{2x}{}^2}$ $\sigma_{y\_eff} = \sqrt{\sigma_{1y}{}^2 + \sigma_{2y}{}^2}$

then

$$\sqrt{\left(\sigma_{1x}^*\right)^2 + \left(\sigma_{2x}^*\right)^2} = \sqrt{\sigma_{1x}{}^2 + \sigma_{2x}{}^2 + \left(\sigma_{1z}{}^2 + \sigma_{2z}{}^2\right)\tan^2\phi}$$

$$= \sqrt{\sigma_{1x}{}^2 + \sigma_{2x}{}^2} \Big/ \sqrt{1 + \frac{\sigma_{1z}^2 + \sigma_{2z}^2}{\sigma_{1x}^2 + \sigma_{2x}^2}\tan^2\phi}$$

$$= \sigma_{x\_eff} \cdot \mathcal{S}_x$$

$$\sqrt{\left(\sigma_{1y}^*\right)^2 + \left(\sigma_{2y}^*\right)^2} = \sqrt{\sigma_{1y}{}^2 + \sigma_{2y}{}^2} = \sigma_{y\_eff}$$

where, $\mathcal{S}_x$ is the luminosity reduction factor due to the crossing angle.

After Lorentz boost, we can calculate the luminosity by using "head-on" formula,

$$\mathcal{L} = \frac{N_1 N_2 f_c N_B}{2\pi\sqrt{\left(\sigma_{1x}^*\right)^2 + \left(\sigma_{2x}^*\right)^2}\sqrt{\left(\sigma_{1y}^*\right)^2 + \left(\sigma_{2y}^*\right)^2}} = \mathcal{L}_0 \cdot \mathcal{S}_x$$

In general, following the integration strategy, consider the luminosity reduction effects caused by crossing angle and offset

Luminosity $\mathcal{L} = \mathcal{L}_0 \cdot \mathcal{S} \cdot \mathcal{T} \cdot \mathcal{U}$

where
$$\mathcal{S} = \frac{1}{\sqrt{1 + \dfrac{\sigma_{1z}^2 + \sigma_{2z}^2}{\sigma_{1x}^2 + \sigma_{2x}^2} \tan^2 \phi_x + \dfrac{\sigma_{1z}^2 + \sigma_{2z}^2}{\sigma_{1y}^2 + \sigma_{2y}^2} \tan^2 \phi_y}}$$

$$\mathcal{T} = \exp\left[-\frac{{\delta_x}^2}{2\left(\sigma_{1x}^2 + \sigma_{2x}^2\right)} - \frac{{\delta_y}^2}{2\left(\sigma_{1y}^2 + \sigma_{2y}^2\right)} \right]$$

$$\mathcal{U} = \exp\left[\mathcal{S}^2 \frac{\sigma_{1z}^2 + \sigma_{2z}^2}{2} \left(\frac{\delta_x \tan \phi_x}{\sigma_{1x}^2 + \sigma_{2x}^2} + \frac{\delta_y \tan \phi_y}{\sigma_{1y}^2 + \sigma_{2y}^2} \right)^2 \right]$$

Hourglass Effect: For real case, the β-function in a drift space varies and depends on:

$$\beta(s) = \beta^* \left(1 + \frac{s^2}{\beta^{*2}}\right)$$

where β^* is the β-function at the interaction point

The name "hourglass effect" comes from the shape of $\beta(s)$.

$$\sigma(s) = \sqrt{\beta(s)\varepsilon} = \sigma^* \sqrt{1 + \frac{s^2}{\beta^{*2}}}$$

Analytic solution: symmetric-collider case with "**Flat Beam**"

For the symmetric-collider and the two identical flat beams,

$$\sigma_{1x}^{*} = \sigma_{2x}^{*} = \sigma_{x}^{*}, \quad \sigma_{1y}^{*} = \sigma_{2y}^{*} = \sigma_{y}^{*}, \quad \sigma_{1s} = \sigma_{2s} = \sigma_{s}, \quad \text{and} \quad \sigma_{y}^{*} << \sigma_{x}^{*}$$

Including the hourglass effect and the beam-tilt effect, K. Hirata and A. W. Chao suggested using following approximate solution

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \sqrt{\frac{2}{\pi}} a e^{b} K_0(b) \quad \text{where} \quad a = \frac{\beta_y^{*}}{\sqrt{2}\sigma_z^{*}} \quad \text{and} \quad b = a^2 \left[1 + \left(\frac{\sigma_z^{*}}{\sigma_x^{*}} \tan\phi \right)^2 \right]$$

If the beams are, in addition, offset transversely,

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \sqrt{\frac{2}{\pi}} a e^{b} K_0(b) \cdot \mathcal{R}_0 \quad \text{where} \quad \mathcal{R}_0 = \exp\left[-\frac{\left(\delta_x / 2 \right)^2}{\sigma_x^{*2} \cos^2\phi + \sigma_z^{*2} \sin^2\phi} - \left(\frac{\delta_y}{2\sigma_y^{*}} \right)^2 \right]$$

In the numerical calculation processes, two colliding bunched beams are cut into many slices whose normal direction is parallel to the longitudinal direction. the luminosity will be calculated by the summation of each individual discrete part.

$$\mathcal{L} = \frac{N_B f_c}{2\pi} \sum_{i,j} \mathcal{L}_{0_{i,j}} \mathcal{S}_{i,j} \mathcal{T}_{i,j} \mathcal{U}_{i,j}$$

where,

$$\mathcal{L}_{0_{i,j}} = \frac{N_{1,i} N_{2,j}}{\sqrt{\sigma_{x1,i}^2 + \sigma_{x2,j}^2}\sqrt{\sigma_{y1,i}^2 + \sigma_{y2.j}^2}}$$

$$\mathcal{S}_{i,j} = \frac{1}{\sqrt{1 + \frac{\sigma_{1z}^2 + \sigma_{2z}^2}{\sigma_{x1,j}^2 + \sigma_{x2,j}^2} \tan^2 \phi_x + \frac{\sigma_{1z}^2 + \sigma_{2z}^2}{\sigma_{y1,i}^2 + \sigma_{y2,j}^2} \tan^2 \phi_y}}$$

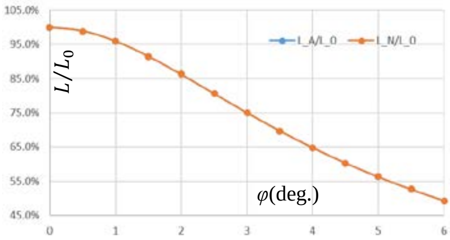
$$\mathcal{T}_{i,j} = \exp\left[-\frac{\delta_x^{\ 2}}{2\left(\sigma_{x1,j}^2 + \sigma_{x2,j}^2\right)} - \frac{\delta_y^{\ 2}}{2\left(\sigma_{y1,i}^2 + \sigma_{y2,j}^2\right)} \right]$$

$$\mathcal{U}_{i,j} = \exp\left[\mathcal{S}_{i,j}^{\ 2} \frac{\sigma_{1z}^2 + \sigma_{2z}^2}{2} \left(\frac{\delta_x \tan \phi_x}{\sigma_{x1,j}^2 + \sigma_{x2,j}^2} + \frac{\delta_y \tan \phi_y}{\sigma_{y1,i}^2 + \sigma_{y2,j}^2} \right)^2 \right]$$

Case 1: Test of the Crossing Angle Factor (Symmetric Collider, Flat Beam without Hourglass Effect and offset)

Electron_Mass	0.000511	Gev
Electron_Beam_Energy	10	GeV
Collision_Frequency	1.19E+08	Hz
Electron_Bunch_Length_RMS	1.0	cm
Number_of_Electron_per_bunch	3.70E+10	
Number_of_Slices_for_e_bunch	20	
Normalized_emittance_x_for_e	4.32E-02	cm
Normalized_emittance_y_for_e	1.72E-05	cm
beta_star_x_for_e	4.00E+02	cm
beta_star_y_for_e	8.00E+01	cm
Crossing_Angle_x	0.0~6.0	Degree
Crossing_Angle_y	0.0	Degree
Offset_x	0.00E+00	cm
Offset_y	0.00E+00	cm

$\sigma_x = 50\sigma_y$ $\sigma_z << \beta_x^*$ $\sigma_z << \beta_y^*$

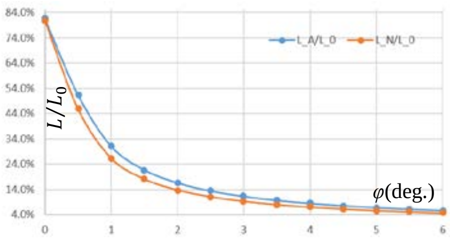


The analytic results vs. the numerical results for the case 1. The *x*-axis means the crossing angle which unit is degree, the *y*-axis shows the reduction factor, where L_0 is the luminosity without hourglass and tilt effect, L_A and L_N are the luminosities of the analytic and the numerical respectively. The numerical result matches the analytic result very well.

Case 2: Test of Crossing and Hourglass Effect together (Symmetric Collider, Flat Beam without offset)

Electron_Mass	0.000511	Gev
Electron_Beam_Energy	10	GeV
Collision_Frequency	1.19E+08	Hz
Electron_Bunch_Length_RMS	**1.0**	cm
Number_of_Electron_per_bunch	3.70E+10	
Number_of_Slices_for_e_bunch	20	
Normalized_emittance_x_for_e	4.32E-02	cm
Normalized_emittance_y_for_e	1.72E-05	cm
beta_star_x_for_e	**4.0**	cm
beta_star_y_for_e	**0.8**	cm
Crossing_Angle_x	**0.0~6.0**	Degree
Crossing_Angle_y	0.0	Degree
Offset_x	0.00E+00	cm
Offset_y	0.00E+00	cm

$\sigma_x = 50\sigma_y$

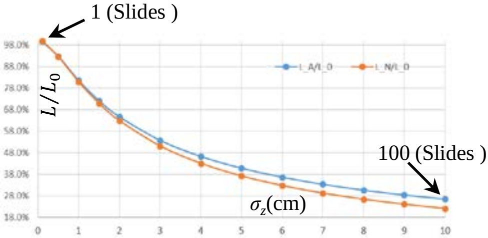


The analytic results vs. the numerical results for the case 2. The *x*-axis means the crossing angle which unit is degree, the *y*-axis shows the reduction factor, where L_0 is the luminosity without hourglass and tilt effect, L_A and L_N are the luminosities of the analytic and the numerical respectively.

Case 3: Test of Hourglass Effect (Symmetric Collider, Head-on Flat Beams without offset)

Electron_Mass	0.000511	Gev
Electron_Beam_Energy	10	GeV
Collision_Frequency	1.19E+08	Hz
Electron_Bunch_Length_RMS	0.1~10.0	cm
Number_of_Electron_per_bunch	3.70E+10	
Number_of_Slices_for_e_bunch	1~100	
Normalized_emittance_x_for_e	4.32E-02	cm
Normalized_emittance_y_for_e	1.72E-05	cm
beta_star_x_for_e	4.0	cm
beta_star_y_for_e	0.8	cm
Crossing_Angle_x	0.0	Degree
Crossing_Angle_y	0.0	Degree
Offset_x	0.00E+00	cm
Offset_y	0.00E+00	cm

$$\sigma_x = 50\sigma_y$$

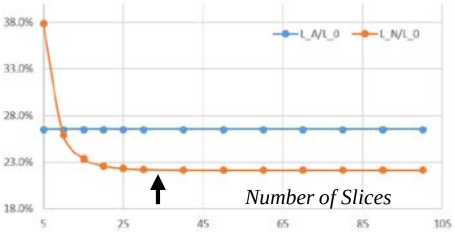


The analytic results vs. the numerical results for the case 3. The *x*-axis means the electron bunch length (RMS), the *y*-axis shows the reduction factor, where L_0 is the luminosity without hourglass and tilt effect, L_A and L_N are the luminosities of the analytic and the numerical respectively.

Case 4: Test of Hourglass Effect (Symmetric Collider, Head-on Flat Beams without offset)

Electron_Mass	0.000511	Gev
Electron_Beam_Energy	10	GeV
Collision_Frequency	1.19E+08	Hz
Electron_Bunch_Length_RMS	10.0	cm
Number_of_Electron_per_bunch	3.70E+10	
Number_of_Slices_for_e_bunch	5~100	
Normalized_emittance_x_for_e	4.32E-02	cm
Normalized_emittance_y_for_e	1.72E-05	cm
beta_star_x_for_e	4.0	cm
beta_star_y_for_e	0.8	cm
Crossing_Angle_x	0.0	Degree
Crossing_Angle_y	0.0	Degree
Offset_x	0.00E+00	cm
Offset_y	0.00E+00	cm

$$\sigma_x = 50\sigma_y$$

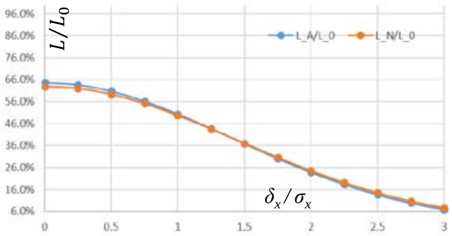


The analytic results vs. the numerical results for the case 4. The *x*-axis means the number of slices for electron bunch, the *y*-axis shows the reduction factor, where L_0 is the luminosity without hourglass and tilt effect, L_A and L_N are the luminosities of the analytic and the numerical respectively.

Case 4: Test of Hourglass Effect (Symmetric Collider, Head-on Flat Beams with offset)

Electron_Mass	0.000511	Gev
Electron_Beam_Energy	10	GeV
Collision_Frequency	1.19E+08	Hz
Electron_Bunch_Length_RMS	10.0	cm
Number_of_Electron_per_bunch	3.70E+10	
Number_of_Slices_for_e_bunch	10	
Normalized_emittance_x_for_e	4.32E-02	cm
Normalized_emittance_y_for_e	1.72E-05	cm
beta_star_x_for_e	4.0	cm
beta_star_y_for_e	0.8	cm
Crossing_Angle_x	0.0	Degree
Crossing_Angle_y	0	Degree
Offset_x	0.0~3.0	σ_x
Offset_y	0.00E+00	cm

$$\sigma_x = 50\sigma_y$$



The analytic results vs. the numerical results for the case 4. The *x*-axis means the offset on *x*-direction which unit is σx, the *y*-axis shows the reduction factor, where L_0 is the luminosity without hourglass and tilt effect, L_A and L_N are the luminosities of the analytic and the numerical respectively.

Case 6: Head-on collision with Hourglass Effect for JLEIC

CM energy	GeV	21.9 (low)		44.7 (medium)		63.3 (high)	
		p	e	p	e	p	e
Beam energy	GeV	40	3	100	5	100	10
Collision frequency	MHz	476		476		476/4=119	
Particles per bunch	10^{10}	0.98	3.7	0.98	3.7	3.9	3.7
Beam current	A	0.75	2.8	0.75	2.8	0.75	0.71
Polarization	%	**80%**	**80%**	**80%**	**80%**	**80%**	**75%**
Bunch length, RMS	µm	30000	10000	10000	10000	22000	10000
Norm. emittance, hor / ver	µm	0.3/0.3	24/24	0.5/0.1	54/10.8	0.9/0.18	432/86.4
Horizontal & vertical β*	cm	8/8	13.5/13.5	6/1.2	5.1/1.0	10.5/2.1	4/0.8
Ver. beam-beam parameter		0.015	0.092	0.015	0.068	0.008	0.034
Laslett tune-shift		0.06	7×10^{-4}	0.055	6×10^{-4}	0.056	7×10^{-5}
Detector space, up/down	m	3.6/7	3.2/3	3.6/7	3.2/3	3.6/7	3.2/3
Hourglass(HG) reduction		1		0.87		0.75	
Luminosity/IP, w/HG, 10^{33}	$cm^{-2}s^{-1}$	**2.5**		**21.4**		5.9	

Crossing_Angle_x	0	Degree
Crossing_Angle_y	0	Degree
Offset_x	0	cm
Offset_y	0	cm
Luminosity	5.75	$(\times10^{33})\ cm^{-2}s^{-1}$
Hourglass reduction	0.75	

Future Plans

- Extract all parameters from tracking data
- Apply Bassetti-Erskine kick to individual particles and feed it back into the simulation
- Do crab dynamics studies with beam-beam interaction


謝謝
BIYAN
SHUKRIA
DANKSCHEEN
TINGKI
YAQHANYELAY
TASHAKKUR ATU
THANK
GRACIAS
SUKSAMA
EKHMET
ARIGATO
SHUKURIA
GOZAIMASHITA
EFCHARISTO
JUSPAXAR
KOMAPSUMNIDA
MAAKE
GRAZIE
MEHRBANI
PALDIES
YOU
BOLZÏN
MERCI