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Elastic Plate Theories and Their Governing Differential Equations

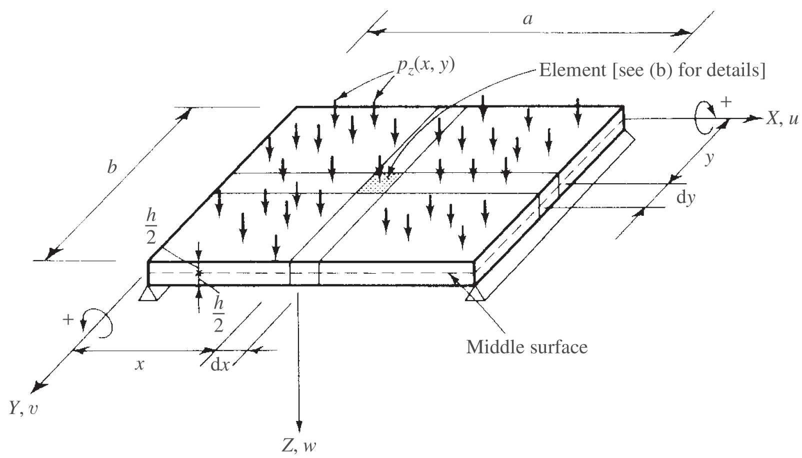
1.1 Classical Small-Deflection Theory of Thin Plates

A mathematically exact stress analysis of a thin plate—subjected to loads acting normal to its surface—requires solution of the differential equations of three-dimensional elasticity [1.1.1]. In most cases, however, such an approach would encounter insurmountable mathematical difficulties. Yet, for the vast majority of technical applications Kirchhoff's classical theory of thin plates† yields sufficiently accurate results without the need of carrying out a full three-dimensional stress analysis. Consequently, classical plate theory occupies a unique position on this subject. It is formulated in terms of transverse deflections $w(x, y)$ for which the governing differential equation is of fourth order, requiring only two boundary conditions to be satisfied at each edge. The simplifications used in the derivation of the plate equation are based on the following assumptions:

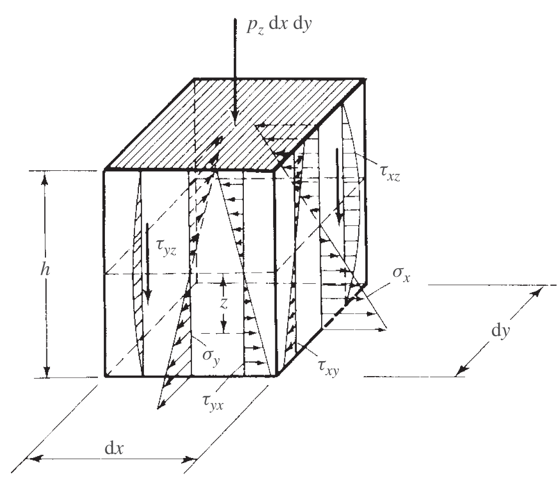
1. The material is homogeneous, isotropic and linear elastic; that is, it follows Hooke's law.
2. The plate is initially flat.
3. The middle surface‡ of the plate remains unstrained during bending.
4. The *constant* thickness of the plate, h, is small compared to its other dimensions; that is, the smallest lateral dimension of the plate is at least 10 times larger than its thickness.

† As already mentioned under Historical Background, other contributors to classical plate theory include Bernoulli, Navier, Saint-Venant and Lagrange.
‡ Equivalent to the neutral axis in elementary beam theory.

5. The transverse deflections $w(x, y)$ are small compared to the plate thickness. A maximum deflection of one-tenth of the thickness is considered the limit of the small-deflection theory.
6. Slopes of the defected middle surface are small compared to unity.
7. Sections taken normal to the middle surface before deformation remain plane and normal to the deflected middle surface. Consequently, shear deformations are neglected. This assumption represents an extension of Bernoulli's hypothesis for beams to plates.
8. The normal stress σ_z in the direction transverse to the plate surface can be neglected.



(a)
Laterally loaded rectangular plate



(b)
Stress components on plate element

Figure 1.1.1 Laterally loaded rectangular plate.