

the total *electromotive force*, or *emf*, \mathcal{E} of the circuit. The field \mathbf{E}_e is produced, in the present example, by chemical action in the battery. If it were absent, no current would flow since an electric field \mathbf{E}_e due to charges is not able to maintain a steady current. The right-hand side of (3) equals the total IR drop around the circuit. Hence (3) becomes

$$\mathcal{E} = IR_T \quad (4)$$

where R_T is the total resistance of the circuit (equals R_0 if the internal resistance of the battery is zero).

In general, for a closed circuit containing many resistors and sources of emf,

$$\sum \mathcal{E} = I \sum R \quad (5)$$

This is *Kirchhoff's voltage law*. In words it states that *the algebraic sum of the emfs around a closed circuit equals the algebraic sum of the ohmic, or IR , drops around the circuit*.† Kirchhoff's voltage law applies not only to an isolated electric circuit as in Fig. 4-9 but to any single mesh (closed path) of a network.

To distinguish emf from the scalar potential V , the symbol \mathcal{E} (script V) is used for emf. Both V and \mathcal{E} are expressed in volts, so that either may be referred to as a voltage if one does not wish to make a distinction between potential and emf.

It is to be noted that the scalar potential V is equal to the negative of the line integral of the static field \mathbf{E}_e , while the emf \mathcal{E} equals the line integral of \mathbf{E}_e . Thus, between any two points a and b ,‡

$$V_{ab} = V_b - V_a = - \int_a^b \mathbf{E}_e \cdot d\mathbf{l} \quad (6)$$

$$\text{and} \quad \mathcal{E}_{ab} = \int_a^b \mathbf{E}_e \cdot d\mathbf{l} \quad (7)$$

In (6) V_{ab} is independent of the path of integration between a and b , but \mathcal{E}_{ab} , in (7), is not.

For closed paths, $\oint \mathbf{E}_e \cdot d\mathbf{l} = 0$ and $\oint \mathbf{E}_e \cdot d\mathbf{l} = \mathcal{E}_T$, where \mathcal{E}_T is the total emf around the circuit.

Example Let the circuit of Fig. 4-9 be redrawn as in Fig. 4-10*a*. The battery has an internal resistance R_1 , and it will be convenient in this example to assume that the field \mathbf{E}_e in the battery is uniform between the terminals

† In time-varying situations, where the circuit dimensions are small compared with the wavelength, Kirchhoff's law is modified: The algebraic sum of the *instantaneous* emfs around a closed circuit equals the algebraic sum of the *instantaneous* ohmic drops around the circuit.

‡ An open-circuited battery (no current flowing) has a terminal potential difference V equal to its emf \mathcal{E} . The potential V is as given by (6). As explained in the examples that follow, \mathbf{E}_i and \mathbf{E}_e have opposite directions in the battery. Therefore, in order that $V_{ab} = \mathcal{E}_{ab}$ for an open-circuited battery, (7) has no negative sign.