velocity. That this would imply that terrestrial electrical machinery would behave differently in winter and summer does not appear to have raised any doubts!

After Michelson and Morley's experiment, a long controversy ensued and, though this is of great historical interest, it will not be recounted in this book. The special principle is now firmly established and is accepted on the grounds that the conclusions which may be deduced from it are everywhere found to be in conformity with experiment and also because it is felt to possess *a priori* a high degree of plausibility. A description of the steps by which it ultimately came to be appreciated that the principle was of quite general application would therefore be superfluous in an introductory text. It is, however, essential for our future development of the theory to understand the prime difficulty preventing an early acceptance of the idea that the electromagnetic laws are in conformity with the special principle.

Consider the two inertial frames S, $\bar{S}$. Suppose that an observer employing S measures the velocity of a light pulse and finds it to be $\mathbf{c}$. If the velocity of the same light pulse is measured by an observer employing the frame $\bar{S}$, let this be $\bar{\mathbf{c}}$. Then, by equation (1.1),

$$\bar{\mathbf{c}} = \mathbf{c} - \mathbf{u} \tag{3.5}$$

and it is clear that, in general, the magnitudes of the vectors $\bar{\mathbf{c}}$, $\mathbf{c}$ will be different. It appears to follow, therefore, that either Maxwell's equations (3.1)–(3.4) must be modified, or the special principle of relativity abandoned for electromagnetic phenomena. Attempts were made (e.g. by Ritz) to modify Maxwell's equations, but certain consequences of the modified equations could not be confirmed experimentally. Since the special principle was always found to be valid, the only remaining alternative was to reject equation (1.1) and to replace it by another in conformity with the experimental result that the speed of light is the same in all inertial frames. As will be shown in the next section, this can only be done at the expense of a radical revision of our intuitive ideas concerning the nature of space and time and this was very understandably strongly resisted.

4. Lorentz transformations. Minkowski space–time

The argument of this section will be founded on the following three postulates:

Postulate 1. A particle free to move under no forces has constant velocity in any inertial frame.

Postulate 2. The speed of light relative to any inertial frame is c in all directions.

Postulate 3. The geometry of space is Euclidean in any inertial frame.

Let the reference frame S comprise rectangular Cartesian axes $Oxyz$. We shall assume that the coordinates of a point relative to this frame are measured by the usual procedure and employing a measuring scale which is stationary in S (it is necessary to state this precaution, since it will be shown later that the length of a bar is not independent of its motion). It will also be supposed that standard

atomic clocks, stationary relative to S, are distributed throughout space and are all synchronized with a master-clock at O. A satisfactory synchronization procedure would be as follows: Warn observers at all clocks that a light source at O will commence radiating at $t = t_0$. When an observer at a point P first receives light from this source, he is to set the clock at P to read $t_0 + \mathrm{OP}/c$, i.e. it is assumed that light travels with a speed c relative to S, as found by experiment. The position and time of an event can now be specified relative to S by four coordinates (x, y, z, t), t being the time shown on the clock which is contiguous to the event. We shall often refer to the four numbers (x, y, z, t) as an *event.*

Let $\bar{O}\bar{x}\bar{y}\bar{z}$ be rectangular Cartesian axes determining the frame $\bar{S}$ (to be precise, these are rectangular as seen by an observer stationary in $\bar{S}$) and suppose that clocks at rest relative to this frame are synchronized with a master at $\bar{O}$. Any event can now be fixed relative to $\bar{S}$ by four coordinates $(\bar{x}, \bar{y}, \bar{z}, \bar{t})$, the space coordinates being measured by scales which are at rest in $\bar{S}$ and the time coordinate by the contiguous clock at rest in $\bar{S}$. If (x, y, z, t), $(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ relate to the same event, in this section we are concerned to find the equations relating these corresponding coordinates. It is helpful to think of these transformation equations as a dictionary which enables us to translate a statement relating to any set of events from the S-language to the $\bar{S}$-language (or vice versa).

The possibility that the length of a scale and the rate of a clock might be affected by uniform motion relative to a reference frame was ignored in early physical theories. Velocity measurements were agreed to be dependent upon the reference frame, but lengths and time measurements were thought to be absolute. In relativity theory, as will appear, very few quantities are absolute, i.e. are independent of the frame in which the measuring instruments are at rest.

To comply with Postulate 1, we shall assume that each of the coordinates $(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ is a linear function of the coordinates (x, y, z, t). The inverse relationship is then of the same type. A particle moving uniformly in S with velocity (v_x, v_y, v_z) will have space coordinates (x, y, z) such that

$$x = x_0 + v_x t, \quad y = y_0 + v_y t, \quad z = z_0 + v_z t \tag{4.1}$$

If linear expressions in the coordinates $(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ are now substituted for (x, y, z, t), it will be found on solving for $(\bar{x}, \bar{y}, \bar{z},)$ that these quantities are linear in $\bar{t}$ and hence that the particle's motion is uniform relative to $\bar{S}$. In fact, it may be proved that only a linear transformation can satisfy the Postulate 1.

Now suppose that at the instant $t = t_0$ a light source situated at the point P_0 (x_0, y_0, z_0) in S radiates a pulse of short duration. At any later instant t, the wavefront will occupy the sphere whose centre is P_0 and radius $c(t - t_0)$. This has equation

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = c^2(t - t_0)^2 \tag{4.2}$$

Let $(\bar{x}_0, \bar{y}_0, \bar{z}_0)$ be the coordinates of the light source as observed from $\bar{S}$ at the instant $\bar{t} = \bar{t}_0$ the short pulse is radiated. At any later instant $\bar{t}$, in accordance with Postulate 2, the wavefront must also appear from $\bar{S}$ to occupy a sphere of radius