Photoelectric Effect and Compton Effect

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Photoelectric Effect

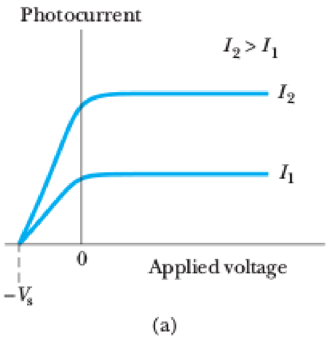
Hertz, the discoverer of the radio waves, also observed that when ultraviolet light falls on metal, charged particles are emitted from the metal.

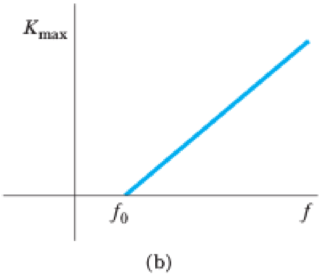
Further studies showed that these charges are electrons and they are called photo-electrons.

Lenard studied photoelectric effect with sources of different frequencies and also different intensities. He observed four important facts:

1. Photoelectrons are emitted only when the frequency of the incident light is above critical frequency, for a given metal.
2. For a given frequency, the number of photoelectrons emitted, which form a photocurrent, is larger for larger intensity.
3. The stopping potential, that is the potential needed to stop the photocurrent, is **independent** of the intensity for a given frequency. It also larger for larger frequency.
4. There is no time delay in the emission of photoelectrons. As soon as the light source is turned on, the emission of photoelectrons starts.

Photoelectric Effect





Failure of Wave Theory

Lenard's observations are inexplicable in wave theory of light. In the wave theory, the intensity of light is a measure of the energy carried by the wave.

The larger the intensity, the more energy the wave carries and it should be able to transfer that energy even to a tightly bound electron and free it.

But it does not seem to happen!

According to the wave theory, energy absorbed by the atom is Intensity $\times$ Area of the atom $\times$ Time interval of absorbtion. The electron is emitted only when this quantity is greater than the work-function.

Failure of Wave Theory

Sodium has a work-function of 2.28 eV and it is observed that a light of intensity of 1.00×10^{-7} mW/cm^2 produces measurable photocurrent in sodium.

We assume the radius of sodium atom is about an Angstrom $= 10^{-8}$ cm.

We calculate the time interval of absorption as

$$t = \frac{2.28 \text{ eV} \times 1.6 \times 10^{-16} \text{ mJ/eV}}{3 \times 10^{-16} \text{ cm}^2 \times 10^{-7} \text{ mW/cm}^2} \approx 10^7 \text{ sec} \approx 100 \text{ days}.$$

According to the wave theory, photoelectrons should appear only after a time delay of more than 100 days after the light is turned on.

According to the experiment, there is no measureable time delay between turning on the light and emission of photoelectrons.

Planck's Picture of Quantum

Planck, in explaining the blackbody radiation, said that the energy of the oscillators in the blackbody is quantized and hence the energy of the emitted radiation is quantized.

In Planck's picture, radiation of frequency ν can exist as a quantum of energy $h\nu$ or $2h\nu$ or $3h\nu$ or ...

If an energy of $2h\nu$ is emitted, the following question arises:
Is it emitted as one quantum of $2h\nu$? OR
Are two quanta of energy $h\nu$ emitted?

The derivation of Planck's Blackbody radiation formula makes no distinction between the above two cases. We get the same answer in both cases.

Einstein's Picture of Quantum

Einstein pictured the radiation in the blackbody to be an Ideal gas of light quanta.

Suppose you have a gas of mass $M = N * m$. We can consider two possibilities:
There N molecules, each of mass m OR there are $N/2$ molecules, each of mass $2m$.

From your study of the thermodynamic properties of ideal gases, you know that the two cases behave very differently.

Einstein used the same thermodynamic arguments and argued that the behaviour of one quantum of energy $2h\nu$ is very different from the behaviour of two quanta of energy $h\nu$.

He also argued that the data indicate that radiation of frequency ν **MUST** exist in the form of quanta of energy $h\nu$. It can't be in quanta of $2h\nu$ or $3h\nu$ etc. Einstein labelled these radiation quanta as photons.

What is light?

That is the question which bothered scientists during Newton's time. A study of light gave the empirical laws of reflection and refraction.

What picture of light can explain these laws? Newton advocated corpuscular theory which postulated that light consists of a large number of tiny particles. Huygens favoured wave theory.

Both theories could explain laws of reflection and refraction. How to distinguish between the two?

An Italian physicist of that time, XXX, realized that waves should exhibit interference. He tried doing such an experiment but his experiments were inconclusive because his equipment was not sensitive enough.

Because of Newton's prestige, people generally favoured corposcular theory.

What is light?

But Young's double slit experiment (in 1800) and a detailed study of diffraction firmly established wave theory. Maxwell equations provided further proof by showing that light is an electromagnetic wave.

But blackbody radiation and photoelectric effect led to a more complicated picture of light.

Planck said that light is emitted in quanta. He did not say anything further about how light behaves once it is emiited.

Einstein went further. He said that light of frequency ν can only be in the form of a quantum of energy (photon) of energy $h\nu$. This quantum is pictured as a localized object.

The physical waves we see are extended objects. So we pictured light waves also as extended objects. All the optical phenomena, explained using wave theory, use the same picture.

Photon explanation of Photoelectric Effect

How should we picture photon? It is very difficult because it seems to have both wave as well as particle properties.

It is a wave with frequency ν but is a localized bundle with energy $h\nu$. This picture explains all the puzzling features of photoelectric effect seen in Lenard's experiments.

When a photon hits a bound electron in a metal, the electron absorbs the photon and the energy of the electron is increased by an amount equal to the energy of the photon.

If the photon energy is greater than binding energy of the electron, the electron comes out of the metal and becomes "photoelectron". If the photon energy is less than the binding energy, the electron remains bound in the metal. This explains the existence of critical frequency.

The time interval for an electron to absorb a photon is less than a nano-second. So there is no time delay in the emission of the electron, if the energy of the photon is large enough.

Photon explanation of Photoelectric Effect

The photoelectrons have maximum kinetic energy given by $K_{\max} = h\nu - \Phi$, where Φ is the work-function. The stopping potential is the potential difference needed to convert this kinetic energy into electrostatic potential energy. It is proportional to $K_{\max}$ and is linearly dependent on ν and is independent of intensity of the light.

Millikan did an experiment where he measured the stopping potential as a function of frequency for various metals. He found all the plots are straight lines with the same slope $= h/e$.

Intensity is indeed the measure of the energy carried by the light. Larger intensity means there more photons. If the frequency is high enough, each of these photons can free an electron hence for larger intensity more photoelectrons are emitted.

Question: In the case of low frequencies and high intensities, why does not the electron absorb two photons and become free?

A Bit of Special Relativity

Newton's first Law may be stated the following way: Physics Equations should be invariant under Galilean transformations $\vec{x}' = \vec{x} - \vec{v}t$ and $t' = t$.

Maxwell's equations, which describe all the electrical and magnetic phenomena, are not invariant under the above transformations.

Do we give up on Newton's first Law? Or on Maxwell's Equations?

Einstein's Answer: Neither! Change the transformation equations from Galilean transformations to Lorentz transformations.

Maxwell's equations are Covariant under Lorentz transformations. Covariant means: LHS changes, RHS changes but both change the same way so that LHS = RHS is retained.

A Bit of Special Relativity

This change from Galilean transformations to Lorentz transformations meant that a lot of other things need to changed.

Most Important: $t' \neq t$. Time does not flow at the same rate in all intertial frames!

We also need to change our notions about mass, momentum and energy.

A object has an intrinsic mass (also called rest mass) m. Such an object is said to possess a rest-mass energy $E = mc^2$.

If an object has momentum p, then its energy is given by

$$E = \sqrt{p^2c^2 + m^2c^4} = mc^2 + \frac{p^2}{2m} \text{ for } p \ll mc.$$

Momentum p is related to the velocity by $\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = \gamma m\vec{v}$. From this, we can derive $E = \gamma mc^2$ and $v/c = pc/E$.

Photon in Relativity

In Einstein's view light is a stream of "particles" called photons. What can one say about photons from the point of view of relativity?

In relativity we have $v/c = pc/E$. For photons, which necessarily travel with speed of light, $v = c$ and hence $E = pc$.

Comparing this with $E = \sqrt{p^2c^2 + m^2c^4}$, we get $m = 0$ for photons. That is photons are taken to be massless particles with energy E and momentum $p = E/c$. To describe the polarization of light, we also have to assume that they carry spin $= 1$ (as opposed to electrons which carry spin $1/2$).

In describing photoelectric effect, we assumed a photon collides an electron, gets absorbed and transfers its energy to the electron.

We just saw that the photon carries a momentum also. What about momentum conservation?

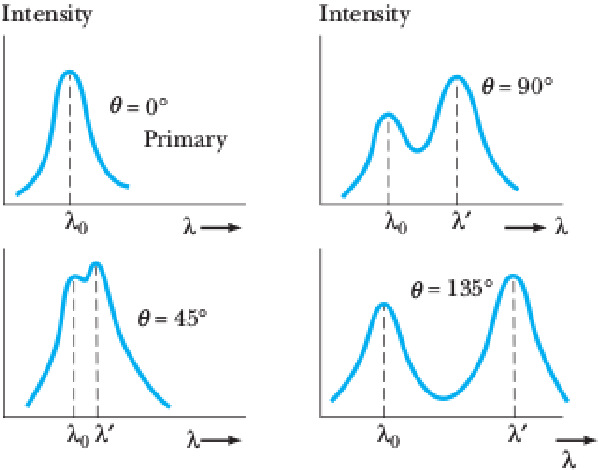
Compton Effect

Compton observed the scattering of X-rays (energy $\sim$ 10 keV) off electrons in a thin sheet of metal. He measured the intensity of scattered X-rays as a function of the scattering angle θ.

He observed the relation

$$\lambda' - \lambda = \lambda_c(1 - \cos\theta),$$

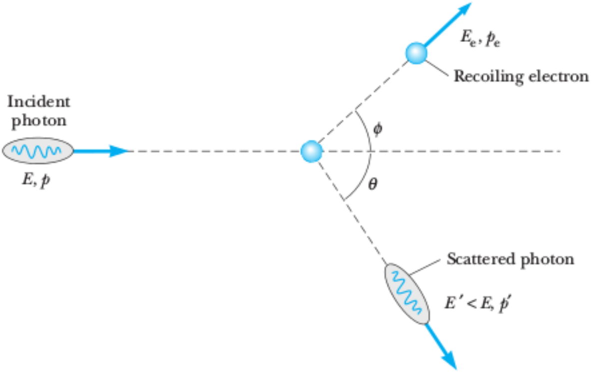
where λ is the wavelength of the incident X-ray, λ' is the wavelength of the scattered X-ray and λ_c is a fundamental constant and can be shown to be $h/m_e c$. It is called the Compton wavelength of electron.


Intensity
θ = 0°
Primary
λ0
λ
θ = 45°
λ0 λ′
λ
Intensity
θ = 90°
λ0
λ′
λ
θ = 135°
λ0
λ′
λ

(b)

Compton Effect

Compton relation can be understood by assuming the scattering to be a collision of the X-ray photon with electron, where some of the energy of the photon is transferred to the electron.



Compton Effect

Before the collision, the energy and the momentum of the photon are $(h\nu, 0, 0, h\nu/c)$ and for electron they are $(m_e c^2, 0, 0, 0)$.

After the collision, the energy and the momentum of the photon are $(h\nu', h\nu' \sin\theta/c, 0, h\nu' \cos\theta/c)$ and for electron they are $(E_e, p_e \sin\phi, 0, p_e \cos\phi)$.

Conservation of energy and momentum give the following equations:

$$
\begin{aligned}
h\nu + m_e c^2 &= h\nu' + E_e \\
0 &= h\nu' \sin\theta/c + p_e \sin\phi \\
h\nu/c &= h\nu' \cos\theta/c + p_e \cos\phi.
\end{aligned}
$$

Compton Effect

From these equations we get

$$
\begin{aligned}
E_e^2 &= (h\nu - h\nu' - m_e c^2)^2 \\
p_e^2 c^2 &= \left(\frac{h}{c}\right)^2 \left(\nu^2 + \nu'^2 - 2\nu\nu' \cos\theta\right)
\end{aligned}
$$

The energy and momentum of the final state electron are constrained by $E_e^2 = p_e^2 c^2 + m_e^2 c^4$. Substituting for E_e and m_e and doing some algebra, we get the Compton relation $\lambda' - \lambda = \lambda_c(1 - \cos\theta)$.