

Physicsforums problem - sara_87

1st December 2008

$$\lim_{x \rightarrow 1} \frac{x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + 1}{(x - 1)^2}$$

$$= \lim_{x \rightarrow 1} \frac{(x^{\frac{1}{3}} - 1)^2}{(x - 1)^2} = \lim_{x \rightarrow 1} \frac{(x^{\frac{1}{3}} - 1)^2}{((x^{\frac{1}{3}})^3 - 1)^2}$$

Now we apply the fact that the difference of two cubes a and b is given by the simple formula:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Note: You probably should also know that:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

And more generally that:

$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + ab^{n-2} + b^{n-1})$, which holds true for both even and odd n

And:

$a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \dots - ab^{n-2} + b^{n-1})$, which holds true only for odd n

Applying this to the problem,

$$\lim_{x \rightarrow 1} \frac{(x^{\frac{1}{3}} - 1)^2}{((x^{\frac{1}{3}})^3 - 1)^2} = \lim_{x \rightarrow 1} \frac{(x^{\frac{1}{3}} - 1)^2}{(x^{\frac{1}{3}} - 1)^2 (x^{\frac{2}{3}} + x^{\frac{1}{3}} + 1)^2} = \lim_{x \rightarrow 1} \frac{1}{(x^{\frac{2}{3}} + x^{\frac{1}{3}} + 1)^2} = \frac{1}{3^2} = \frac{1}{9} \#$$

- Solution by wimma