Find the Laplace transform of the fuction F(t) = sin(wt)/(1 + cos^2(wt)).
1+cos^2(u) = 1 + (1/2)[cos(2u)+1] = (1/2)(3+cos(2u)),
so 1/(1+cos^2(wt)) = 2/(3 + cos(2wt))

exp(-st) sin(wt) = 1/(2i) exp(-st) [exp(i wt) - exp(-iwt)] = 1/(2i) [exp(-(s-iw)t) - exp(-(s+iw)t)], so if
L0(S) = laplace(2/(3 + cos(2wt)),t,S),
then L(f)(s) = 1/(2i) [L0(s-iw) - L0(s+iw)]

L0 is expressible in terms of the so-called "lerchPhi" function--- see below.

```
> restart;
> with(inttrans);
```

$$[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable]$$

```
> f:=2/(3 + cos(2*w*t));
```

$$f := \frac{2}{3+\cos(2\,w\,t)}$$

```
> laplace(f,t ,S);
```

$$-\frac{\sqrt{2}}{8\,S\,w^2}+\frac{1}{8}I\left(-\mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{\frac{-1}{4}I\,S}{w}\right)+\mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{\frac{1}{4}I\,S}{w}\right) - \mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{1}{2}-\frac{\frac{1}{4}I\,S}{w}\right)(-3+2\sqrt{2}) + \mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{1}{2}+\frac{\frac{1}{4}I\,S}{w}\right)(-3+2\sqrt{2})\right)\sqrt{2}\,/\,w$$

```
> Lf:=unapply(%,S):
> L:=1/2/I *(Lf(s-I*w) - Lf(s+I*w));
```

$$L := \frac{-1}{2}I\left(-\frac{\sqrt{2}}{8\,(s-w\,I)\,w^2}+\frac{1}{8}I\left(-\mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{\frac{-1}{4}I\,(s-w\,I)}{w}\right) + \mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{\frac{1}{4}I\,(s-w\,I)}{w}\right)\right.\right.$$

$$- \mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{1}{2} - \frac{\frac{1}{4} I (s - w I)}{w}\right) (-3+2\sqrt{2})$$

$$+ \mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{1}{2} + \frac{\frac{1}{4} I (s - w I)}{w}\right) (-3+2\sqrt{2}) \Bigg) \sqrt{2} / w + \frac{\sqrt{2}}{8 (s + w I) w^2} - \frac{1}{8} I \Bigg($$

$$-\mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{\frac{-1}{4} I (s + w I)}{w}\right) + \mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{\frac{1}{4} I (s + w I)}{w}\right)$$

$$- \mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{1}{2} - \frac{\frac{1}{4} I (s + w I)}{w}\right) (-3+2\sqrt{2})$$

$$+ \mathrm{LerchPhi}\left(\frac{1}{(3+2\sqrt{2})^2}, 1, \frac{1}{2} + \frac{\frac{1}{4} I (s + w I)}{w}\right) (-3+2\sqrt{2}) \Bigg) \sqrt{2} / w \Bigg)$$

The Lerch Phi function is defined as follows:

LerchPhi(z,a,v) = sum_{n=0..infinity} z^n /(v+n)^a

This definition is valid for |z| < 1or |z| = 1 and Re(a) > 1.

By analytic continuation, it is extended to the whole complex
z-plane for each value of a and v

If v and a are are positive integers, LerchPhi(z, a, v) has a branch cut in the z-plane along the real axis to the right of
z = 1, with a branch point at z=1.

If a is a non-positive integer, LerchPhi(z, a, v) is a rational function of z with a pole of order 1 - a at z = 1

LerchPhi(1,a,v) = Zeta(0,a,v). If 1 < Re(a), it is also true that limit(LerchPhi(z,a,v),z=1) = Zeta(0,a,v).
If Re(a) <= 1, this limit does not exist.

If 0 <= Re(a), LerchPhi(z, a, v) has an infinite singularity at each non-positive integer v.

If the coefficients of the series representation of a hypergeometric function are rational functions of the summation indices, then the hypergeometric function can be expressed as a linear sum of Lerch Phi functions. If the parameters of the hypergeometric functions are rational, we can express the hypergeometric function as a linear sum of polylog functions.

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