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LWI in a driven Lambda three-level atom and effects of the probe laser on EIT

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Abstract

Steady-state dynamics of a Lambda atom driven by two coherent fields is studied for arbitrary detunings, arbitrary incoherent pumping and coherent driving intensities. Effects of a strong probe on EIT are worked out. Features of EIT and LWI are presented in a new physical picture on the basis of the concepts of quantum interference and coherence. In the weak field regime, an absorption-gain profile is the *subtraction* of two Lorentzians with the *same* central frequency but different widths and heights. This case is mostly the result of quantum interference and coherence rather than the result of Autler–Townes level splitting. In a strong field regime, however, the profile is the *addition* of two Lorentzian-like components with *different* central frequencies but the same width, where Autler–Townes splitting and quantum interference are equally important. © 1999 Published by Elsevier Science B.V. All rights reserved.

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1. Introduction

In recent years, there has been considerable interest in quantum interference and coherence effects in a multilevel atom system induced by coherent electromagnetic field(s). Many related phenomena such as electromagnetically induced transparency (EIT) [1–9], lasing without inversion (LWI) [10–18] and others [19–31], have been predicted and subse-

quently experimentally demonstrated. In almost all work done, the configuration of a stronger driving field and a weaker probe field with a first-order approximation in terms of the probe field are generally used as the standard procedure. Though most particular interesting situations in LWI and EIT were theoretically analyzed in detail earlier (see Refs. [1–18] and [19,20]), the most general case, in fact, has not been considered. Though the importance of quantum interference and coherence in the limit of a not-too-strong drive is well known qualitatively, the relative role of quantum coherence and Stark splitting in the formation of an absorption-gain profile has not been worked out *quantitatively*. In this pa-

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per, the emphasis is focused on: (1) a unified treatment including arbitrary incoherent pumping configuration and any detuning; and (2) an absorption-gain profile analyzed on the basis of quantum interference and coherence. In the weak field regime, this profile is the *subtraction* of two Lorentzians with the *same* central frequency but different widths and heights, which mostly is like the result of quantum interference and coherence rather than that simply due to Autler–Townes level splitting. In the strong field regime, this profile is the *addition* of two Lorentzian-like components with *different* central frequencies, but the same width, which shows the equal importance of Autler–Townes splitting and quantum interference. (3) The effects of a strong probe on EIT are also studied in this paper.

2. Model

For maximizing the utility of the theory given here, we start from one general model of a driven Lambda atom (see Fig. 1) with an arbitrary incoherent pump scheme and any detuning. The equations for density-matrix elements [32] are

$$\begin{aligned}\frac{\partial \rho_{bb}}{\partial t} &= R_{bb} \rho_{bb} + R_{ba} \rho_{aa} + R_{bc} \rho_{cc} \\ &\quad + i \frac{\Omega_p}{2} (\rho_{ab} e^{i\phi_p} - \rho_{ba} e^{-i\phi_p}) \\ \frac{\partial \rho_{cc}}{\partial t} &= R_{cb} \rho_{bb} + R_{ca} \rho_{aa} + R_{cc} \rho_{cc} \\ &\quad - i \frac{\Omega_c}{2} (\rho_{ca} e^{-i\phi_c} - \rho_{ac} e^{i\phi_c}) \\ &\quad \times \rho_{bb} + \rho_{aa} + \rho_{cc} = 1 \\ \frac{\partial \rho_{ba}}{\partial t} &= -\Gamma_{ba} \rho_{ba} + i \frac{\Omega_p}{2} e^{i\phi_p} (\rho_{aa} - \rho_{bb}) \\ &\quad - i \frac{\Omega_c}{2} e^{i\phi_c} \rho_{bc}\end{aligned}$$

$$\begin{aligned}\frac{\partial \rho_{bc}}{\partial t} &= -\Gamma_{bc} \rho_{bc} + i \frac{\Omega_p}{2} e^{i\phi_p} \rho_{ac} - i \frac{\Omega_c}{2} e^{-i\phi_c} \rho_{ba} \\ \frac{\partial \rho_{ac}}{\partial t} &= -\Gamma_{ac} \rho_{ac} + i \frac{\Omega_c}{2} e^{-i\phi_c} (\rho_{cc} - \rho_{aa}) \\ &\quad + i \frac{\Omega_p}{2} e^{-i\phi_p} \rho_{bc}\end{aligned}\quad (1)$$

if the coherent interacting Hamiltonian under the rotating wave approximation is

$$\begin{aligned}\frac{H}{\hbar} &= \Delta_c \sigma_{cc} + \Delta_p \sigma_{bb} \\ &\quad - \left[\frac{\Omega_c}{2} \sigma_{ac} e^{-i\phi_c} + \frac{\Omega_p}{2} \sigma_{ab} e^{-i\phi_p} + H.C. \right]\end{aligned}\quad (2)$$

and the dipole moment decay rates are

$$\begin{aligned}\Gamma_{ba} &= \gamma_{ba} + i \Delta_p, \quad \Gamma_{bc} = \gamma_{bc} + i(\Delta_p - \Delta_c), \\ \Gamma_{ac} &= \gamma_{ac} - i \Delta_c\end{aligned}\quad (3)$$

where R_{ij} represents a repopulating rate due to spontaneous emission or incoherent pump from $|j\rangle$ to $|i\rangle$, and Γ_{ij} is a complex relaxation rate of the corresponding dipole moment ρ_{ij} . Ω_c , Ω_p and ϕ_c , ϕ_p are referred to as Rabi frequencies and phases of an atom interacting with two laser fields at frequencies ω_c and ω_p , respectively. Notice that detunings are commonly defined as $\Delta_c = \omega_c - \omega_{ac}$ and $\Delta_p = \omega_p - \omega_{ab}$, respectively.

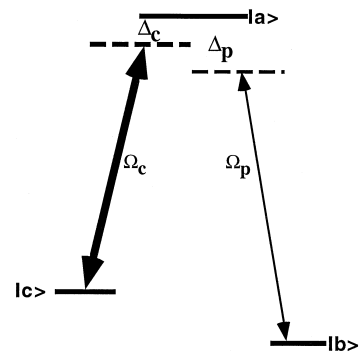


Fig. 1. The atom level scheme under consideration for EIT and LWI.

2.1. Steady-state analysis with master equation

By setting all time derivatives equal to zero, we solve Eq. (1) in the steady state as follows

$$\begin{aligned}
 0 &= R_{bb} \rho_{bb} + R_{ba} \rho_{aa} + R_{bc} \rho_{cc} \\
 &\quad + i \frac{\Omega_p}{2} (\rho_{ab} e^{i\phi_p} - \rho_{ba} e^{-i\phi_p}) \\
 0 &= R_{cb} \rho_{bb} + R_{ca} \rho_{aa} + R_{cc} \rho_{cc} \\
 &\quad - i \frac{\Omega_c}{2} (\rho_{ca} e^{-i\phi_c} - \rho_{ac} e^{i\phi_c}) \\
 1 &= \rho_{bb} + \rho_{aa} + \rho_{cc} \\
 0 &= -\Gamma_{ba} \rho_{ba} + i \frac{\Omega_p}{2} e^{i\phi_p} (\rho_{aa} - \rho_{bb}) \\
 &\quad - i \frac{\Omega_c}{2} e^{i\phi_c} \rho_{bc} \\
 0 &= -\Gamma_{bc} \rho_{bc} + i \frac{\Omega_p}{2} e^{i\phi_p} \rho_{ac} \\
 &\quad - i \frac{\Omega_c}{2} e^{-i\phi_c} \rho_{ba} \\
 0 &= -\Gamma_{ac} \rho_{ac} + i \frac{\Omega_c}{2} e^{-i\phi_c} (\rho_{cc} - \rho_{aa}) \\
 &\quad + i \frac{\Omega_p}{2} e^{-i\phi_p} \rho_{bc}
 \end{aligned} \tag{4}$$

After defining

$$D = \Gamma_{ba} \Gamma_{ac} \Gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \Gamma_{ba} + \left(\frac{\Omega_c}{2} \right)^2 \Gamma_{ac} \tag{5a}$$

$$\begin{aligned}
 M &= \Gamma_{bc} \Gamma_{ac} + \left(\frac{\Omega_p}{2} \right)^2 - \left(\frac{\Omega_c}{2} \right)^2 \\
 &= M_0 + \left(\frac{\Omega_p}{2} \right)^2 - \left(\frac{\Omega_c}{2} \right)^2
 \end{aligned} \tag{5b}$$

$$\begin{aligned}
 N &= \Gamma_{ba} \Gamma_{bc} + \left(\frac{\Omega_c}{2} \right)^2 - \left(\frac{\Omega_p}{2} \right)^2 \\
 &= N_0 + \left(\frac{\Omega_c}{2} \right)^2 - \left(\frac{\Omega_p}{2} \right)^2
 \end{aligned} \tag{5c}$$

we have from the latter three equations in Eq. (4)

$$\frac{\rho_{ba} e^{-i\phi_p}}{i \Omega_p / 2} = \frac{(\rho_{aa} - \rho_{bb}) M + (\rho_{cc} - \rho_{bb}) (\Omega_c / 2)^2}{D} \tag{6}$$

$$\frac{\rho_{ac} e^{i\phi_c}}{i \Omega_c / 2} = \frac{(\rho_{cc} - \rho_{aa}) N + (\rho_{cc} - \rho_{bb}) (\Omega_p / 2)^2}{D} \tag{7}$$

where $\rho_{cc} - \rho_{bb}$ is the Raman inversion for a probe laser field. For the transition $a-b$, an amplitude gain/absorption of a probe laser and a driving laser are directly proportional to

$$\begin{aligned}
 G_{b-a} &= \text{Re} \left(\frac{\rho_{ba} e^{-i\phi_p}}{i \Omega_p / 2} \right) \\
 &= (\rho_{aa} - \rho_{bb}) \cdot \text{Re}(M/D) \\
 &\quad + (\rho_{cc} - \rho_{bb}) (\Omega_c / 2)^2 \cdot \text{Re}(1/D)
 \end{aligned} \tag{8a}$$

$$\begin{aligned}
 G_{c-a} &= \text{Re} \left(\frac{\rho_{ca} e^{-i\phi_c}}{i \Omega_c / 2} \right) \\
 &= (\rho_{aa} - \rho_{cc}) \cdot \text{Re}(N/D) \\
 &\quad - (\rho_{cc} - \rho_{bb}) (\Omega_p / 2)^2 \cdot \text{Re}(1/D)
 \end{aligned} \tag{8b}$$

Now, we go to the calculation of steady-state population inversions. From Eq. (4), it is direct to get

$$\begin{aligned}
 0 &= R_{bb} \rho_{bb} + R_{ba} \rho_{aa} + R_{bc} \rho_{cc} \\
 &\quad + P_{b-a} (\rho_{aa} - \rho_{bb}) + P_r (\rho_{cc} - \rho_{bb}) \\
 0 &= R_{cb} \rho_{bb} + R_{ca} \rho_{aa} + R_{cc} \rho_{cc} \\
 &\quad - P_{a-c} (\rho_{cc} - \rho_{aa}) - P_r (\rho_{cc} - \rho_{bb}) \\
 1 &= \rho_{bb} + \rho_{aa} + \rho_{cc}
 \end{aligned} \tag{9}$$

where efficient pumping rates by the coherent fields are

$$\begin{aligned}
 P_{b-a} &= \left(\frac{\Omega_p}{2} \right)^2 2 \text{Re} \left(\frac{M}{D} \right) \\
 P_{a-c} &= \left(\frac{\Omega_c}{2} \right)^2 2 \text{Re} \left(\frac{N}{D} \right) \\
 P_r &= \left(\frac{\Omega_p}{2} \right)^2 \left(\frac{\Omega_c}{2} \right)^2 2 \text{Re} \left(\frac{1}{D} \right)
 \end{aligned} \tag{10}$$

By solving Eq. (9), we get

$$\rho_{bb} = \frac{1}{D_0} [R_{ca}R_{bc} - R_{cc}R_{ba} - P_{b-a}R_{cc} + P_{a-c}(R_b - R_{bb}) + P_r(R_{ba} + R_{ca}) + (P_{b-a}P_{a-c} + P_{b-a}P_r + P_rP_{a-c})] \quad (11)$$

$$\rho_{aa} = \frac{1}{D_0} [R_{cc}R_{bb} - R_{cb}R_{bc} - P_{b-a}R_{cc} - P_{a-c}R_{bb} + P_r[(R_{ba} + R_{ca}) - (R_b + R_c)] + (P_{b-a}P_{a-c} + P_{b-a}P_r + P_rP_{a-c})] \quad (12)$$

$$\rho_{cc} = \frac{1}{D_0} [R_{cb}R_{ba} - R_{ca}R_{bb} + P_{b-a}(R_c - R_{cc}) - P_{a-c}R_{bb} + P_r(R_{ba} + R_{ca}) + (P_{b-a}P_{a-c} + P_{b-a}P_r + P_rP_{a-c})] \quad (13)$$

$$D_0 = D_c + P_{b-a}(R_c - 3R_{cc}) + P_{a-c}(R_b - 3R_{bb}) + P_r[3(R_{ba} + R_{ca}) - (R_b + R_c)] + 3(P_{b-a}P_{a-c} + P_{b-a}P_r + P_rP_{a-c}) \quad (14)$$

where

$$D_c = R_{ca}(R_{bc} - R_{bb}) + R_{cb}(R_{ba} - R_{bc}) + R_{cc}(R_{bb} - R_{ba}) \quad (15a)$$

$$R_b = R_{bb} + R_{ba} + R_{bc} \\ R_c = R_{cb} + R_{ca} + R_{cc} \quad (15b)$$

The population inversions for the transitions $a-b$, $a-c$ and $b-c$ are respectively

$$\rho_{aa} - \rho_{bb} = \frac{1}{D_0} [T_{a-b} - P_{a-c}R_b - P_r(R_b + R_c)] \quad (16)$$

$$\rho_{aa} - \rho_{cc} = \frac{1}{D_0} [T_{a-c} - P_{b-a}R_c - P_r(R_b + R_c)] \quad (17)$$

$$\rho_{cc} - \rho_{bb} = \frac{1}{D_0} [T_{c-b} + P_{b-a}R_c - P_{a-c}R_b] \quad (18)$$

where

$$T_{a-b} = R_{cc}R_b - R_cR_{bc}, \\ T_{a-c} = R_cR_{bb} - R_{cb}R_b, \\ T_{c-b} = T_{a-b} - T_{a-c} \quad (19)$$

and T_{i-j} represents the effects of incoherent processes in a population inversion between states $|i\rangle$ and $|j\rangle$. Substituting Eqs. (16)–(18) into Eqs. (8a) and (8b), probe and driving laser absorptions for transitions $a-b$ and $a-c$ become

$$G_{b-a} = \left\{ -T_{a-c} \left(\frac{\Omega_c}{c} \right)^2 \text{Re}(D) + T_{a-b} \left[\text{Re}(M_0^* D) + \left(\frac{\Omega_p}{2} \right)^2 \text{Re}(D) \right] - 2 \left(\frac{\Omega_c}{2} \right)^2 R_b \left[\text{Re}(M_0^* D) \text{Re}(N_0^* D) + \left(\frac{\Omega_p}{2} \right)^2 \text{Re}(N_0^* D) \text{Re}(D) + \left(\frac{\Omega_c}{2} \right)^2 \times \text{Re}(M_0^* D) \text{Re}(D) \right] \frac{1}{DD^*} \right\} \frac{1}{D_0 DD^*} \quad (20a)$$

$$G_{c-a} = \left\{ -T_{a-b} \left(\frac{\Omega_p}{c} \right)^2 \text{Re}(D) + T_{a-c} \left[\text{Re}(N_0^* D) + \left(\frac{\Omega_c}{2} \right)^2 \text{Re}(D) \right] - 2 \left(\frac{\Omega_p}{2} \right)^2 R_c \left[\text{Re}(M_0^* D) \text{Re}(N_0^* D) + \left(\frac{\Omega_p}{2} \right)^2 \text{Re}(N_0^* D) \text{Re}(D) + \left(\frac{\Omega_c}{2} \right)^2 \times \text{Re}(M_0^* D) \text{Re}(D) \right] \frac{1}{DD^*} \right\} \frac{1}{D_0 DD^*} \quad (20b)$$

First, suppose $G_{i-j} = G_{i-j}^n / G_0$ and $G_0 = D_0 DD^*$. Then, notice that

$$\begin{aligned} & \left[\text{Re}(M_0^* D) + \left(\frac{\Omega_p}{2} \right)^2 \text{Re}(D) \right] \\ &= \left[\gamma_{ac} \gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] D_r^0 + \gamma_{ba} \Delta_c^2 (\Delta_p - \Delta_c)^2 \\ & \quad + \gamma_{ba} \gamma_{ac}^2 (\Delta_p - \Delta_c)^2 + \gamma_{bc} \left[\gamma_{ba} \gamma_{bc} + \left(\frac{\Omega_c}{2} \right)^2 \right] \\ & \quad \times \Delta_c^2 + 2 \left(\frac{\Omega_p}{2} \right)^2 \gamma_{ba} \Delta_c (\Delta_p - \Delta_c) \end{aligned} \quad (21a)$$

$$\begin{aligned} & \left[\text{Re}(N_0^* D) + \left(\frac{\Omega_c}{2} \right)^2 \text{Re}(D) \right] \\ &= \left[\gamma_{ba} \gamma_{bc} + \left(\frac{\Omega_c}{2} \right)^2 \right] D_r^0 + \gamma_{ac} \Delta_p^2 (\Delta_p - \Delta_c)^2 \\ & \quad + \gamma_{ba}^2 \gamma_{ac} (\Delta_p - \Delta_c)^2 \\ & \quad + \gamma_{bc} \left[\gamma_{ac} \gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] \Delta_p^2 \\ & \quad - 2 \left(\frac{\Omega_c}{2} \right)^2 \gamma_{ac} \Delta_p (\Delta_p - \Delta_c) \end{aligned} \quad (21b)$$

$$\begin{aligned} \text{Re}(D) &= D_r^0 + [\gamma_{ba} \Delta_c (\Delta_p - \Delta_c) \\ & \quad - \gamma_{ac} \Delta_p (\Delta_p - \Delta_c) + \gamma_{bc} \Delta_c \Delta_p] \\ &= D_r^0 + [-\gamma_{ac} (\Delta_p - \Delta_c)^2 \\ & \quad + (\gamma_{ba} - \gamma_{ac} + \gamma_{bc}) \\ & \quad \times \Delta_c (\Delta_p - \Delta_c) + \gamma_{bc} \Delta_c^2] \end{aligned} \quad (21c)$$

$$\begin{aligned} & \left[\text{Re}(M_0^* D) \text{Re}(N_0^* D) + \left(\frac{\Omega_p}{2} \right)^2 \text{Re}(N_0^* D) \text{Re}(D) \right. \\ & \quad \left. + \left(\frac{\Omega_c}{2} \right)^2 \text{Re}(M_0^* D) \text{Re}(D) \right] \frac{1}{DD^*} \\ &= [\gamma_{bc} D_r^0 + [\gamma_{ba} (\Delta_p - \Delta_c) + \gamma_{bc} \Delta_p] \\ & \quad \times [\gamma_{ac} (\Delta_p - \Delta_c) - \gamma_{bc} \Delta_c]] \end{aligned} \quad (21d)$$

where $D_r^0 = \gamma_{ba} \gamma_{ac} \gamma_{bc} + \gamma_{ba} (\Omega_p/2)^2 + \gamma_{ac} (\Omega_c/2)^2$. Finally, we have

$$\begin{aligned} G_{b-a}^n &= G_{b-a}^0 D_r^0 + \Delta_c^2 (\Delta_p - \Delta_c)^2 T_{a-b} \gamma_{ba} + \Delta_c^2 \gamma_{bc} \\ & \quad \cdot \left[T_{a-b} \gamma_{ba} \gamma_{bc} + \left(\frac{\Omega_c}{2} \right)^2 T_{c-b} \right] \\ & \quad + (\Delta_p - \Delta_c)^2 \gamma_{ac} \left[T_{a-b} \gamma_{ba} \gamma_{ac} \right. \\ & \quad \left. + \left(\frac{\Omega_c}{2} \right)^2 (T_{a-c} - 2\gamma_{ba} R_b) \right] \\ & \quad + \Delta_c (\Delta_p - \Delta_c) \left[\left(\frac{\Omega_p}{2} \right)^2 2T_{a-b} \gamma_{ba} \right. \\ & \quad \left. - \left(\frac{\Omega_c}{2} \right)^2 T_{a-c} (\gamma_{ba} - \gamma_{ac} + \gamma_{bc}) \right] \end{aligned} \quad (22)$$

where

$$\begin{aligned} G_{b-a}^0 &= T_{a-b} \left[\gamma_{ac} \gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] \\ & \quad - \left(\frac{\Omega_c}{2} \right)^2 (T_{a-c} + 2\gamma_{bc} R_b) \end{aligned} \quad (23)$$

and

$$\begin{aligned} G_{a-c}^n &= G_{a-c}^0 D_r^0 + \Delta_p^2 (\Delta_p - \Delta_c)^2 T_{a-c} \gamma_{ac} \\ & \quad + \Delta_p^2 \gamma_{bc} \left[T_{a-c} \gamma_{ac} \gamma_{bc} - \left(\frac{\Omega_p}{2} \right)^2 T_{c-b} \right] \\ & \quad + (\Delta_p - \Delta_c)^2 \gamma_{ba} \left[T_{a-c} \gamma_{ba} \gamma_{ac} \right. \\ & \quad \left. + \left(\frac{\Omega_p}{2} \right)^2 (T_{a-b} - 2\gamma_{ac} R_c) \right] \\ & \quad \times \Delta_p (\Delta_p - \Delta_c) \left[- \left(\frac{\Omega_c}{2} \right)^2 2T_{a-c} \gamma_{ac} \right. \\ & \quad \left. - \left(\frac{\Omega_p}{2} \right)^2 T_{a-b} (\gamma_{ba} - \gamma_{ac} - \gamma_{bc}) \right] \end{aligned} \quad (24)$$

where

$$G_{a-c}^0 = T_{a-c} \left[\gamma_{ba} \gamma_{bc} + \left(\frac{\Omega_c}{2} \right)^2 \right] - \left(\frac{\Omega_p}{2} \right)^2 (T_{a-b} + 2\gamma_{bc} R_c) \quad (25)$$

When $G_{b-a}^0 = 0$, a perfect EIT should be observed. When $G_{b-a}^0 > 0$, a probe laser is amplified under an all-resonance condition. We use the following equation for verifying the absorption-gain spectroscopic profile. The denominator of the gain is

$$\begin{aligned} G_0 &= G_0^0 + G_0^A + G_0^B \\ &= DD^* \{ D_c + P_{b-a} (R_c - 3R_{cc}) \\ &\quad + P_{a-c} (R_b - 3R_{bb}) + P_r [3(R_{ba} + R_{ca}) \\ &\quad - (R_b + R_c)] + 3(P_{b-a} P_{a-c} + P_{b-a} P_r \\ &\quad + P_r P_{a-c}) \} \\ &= 12 \left(\frac{\Omega_p}{2} \right)^2 \left(\frac{\Omega_c}{2} \right)^2 \left[\gamma_{bc} D_r^0 + \gamma_{ba} \gamma_{ac} (\Delta_p - \Delta_c) \right] \\ &\quad + 2 \left(\frac{\Omega_p}{2} \right)^2 (R_c - 3R_{cc}) \left[\text{Re}(M_0^* D) \right. \\ &\quad \left. + \left(\frac{\Omega_p}{2} \right)^2 \text{Re}(D) \right] + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \\ &\quad \times \left[\text{Re}(N_0^* D) + \left(\frac{\Omega_c}{2} \right)^2 \text{Re}(D) \right] + 2 \left(\frac{\Omega_p}{2} \right)^2 \\ &\quad \times \left(\frac{\Omega_c}{2} \right)^2 [R_b + R_c - 3(R_{bc} + R_{cb})] (\text{Re}(D) \\ &\quad + DD^* D_c) \end{aligned} \quad (26)$$

where

$$\begin{aligned} G_0^0 &= D_r^0 \left\{ D_c D_r^0 + 2 \left(\frac{\Omega_p}{2} \right)^2 (R_c - 3R_{cc}) \right. \\ &\quad \times \left[\gamma_{ac} \gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] \\ &\quad + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \\ &\quad \cdot \left[\gamma_{ba} \gamma_{bc} + \left(\frac{\Omega_c}{2} \right)^2 \right] + 2 \left(\frac{\Omega_p}{2} \right)^2 \left(\frac{\Omega_c}{2} \right)^2 \\ &\quad \left. \times [R_b + R_c + 6\gamma_{bc} - 3(R_{bc} + R_{cb})] \right\} \end{aligned} \quad (27)$$

$$\begin{aligned} G_0^A &= (\Delta_p - \Delta_c)^2 (\Delta_p \Delta_c)^2 D_c + (\Delta_p - \Delta_c)^2 (\Delta_p)^2 \gamma_{ac} \\ &\quad \times \left[D_c \gamma_{ac} + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \right] \\ &\quad + (\Delta_p - \Delta_c)^2 (\Delta_c)^2 \gamma_{ba} \left[D_c \gamma_{ba} \right. \\ &\quad \left. + 2 \left(\frac{\Omega_p}{2} \right)^2 (R_c - 3R_{cc}) \right] + \Delta_p \Delta_c D_c \\ &\quad \times \left[\gamma_{bc}^2 \Delta_p \Delta_c + 2 \left(\frac{\Omega_p}{2} \right)^2 \Delta_p (\Delta_p - \Delta_c) \right. \\ &\quad \left. - 2 \left(\frac{\Omega_c}{2} \right)^2 \Delta_c (\Delta_p - \Delta_c) \right] \end{aligned} \quad (28)$$

$$\begin{aligned} G_0^B &= (\Delta_p - \Delta_c)^2 \gamma_{ba} \gamma_{ac} \left\{ D_c \gamma_{ba} \gamma_{ac} \right. \\ &\quad + 2 \left(\frac{\Omega_p}{2} \right)^2 (R_c - 3R_{cc}) \gamma_{ac} \\ &\quad + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \gamma_{ba} \\ &\quad + 12 \left(\frac{\Omega_p}{2} \right)^2 \left(\frac{\Omega_c}{2} \right)^2 \left. \right\} + (\Delta_p)^2 \\ &\quad \times \left[\gamma_{ac} \gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] \left\{ D_c \left[\gamma_{ac} \gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] \right. \\ &\quad + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \gamma_{bc} \left. \right\} \\ &\quad + (\Delta_c)^2 \left[\gamma_{ba} \gamma_{bc} + \left(\frac{\Omega_c}{2} \right)^2 \right] \left\{ D_c \left[\gamma_{ba} \gamma_{bc} \right. \right. \\ &\quad \left. + \left(\frac{\Omega_c}{2} \right)^2 \right] + 2 \left(\frac{\Omega_p}{2} \right)^2 (R_c - 3R_{cc}) \gamma_{bc} \left. \right\} \\ &\quad - \Delta_p (\Delta_p - \Delta_c) \gamma_{ac} \cdot 2 \left(\frac{\Omega_c}{2} \right)^2 \left\{ D_c \gamma_{ac} \right. \\ &\quad \left. + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \right\} \end{aligned}$$

$$\begin{aligned}
& + \left(\frac{\Omega_p}{2} \right)^2 \left[R_b + R_c - 3(R_{bc} - R_{cb}) \right] \Big\} \\
& + \Delta_c (\Delta_p - \Delta_c) \gamma_{ba} \cdot 2 \left(\frac{\Omega_p}{2} \right)^2 \\
& \times \left\{ D_c \gamma_{ba} + 2 \left(\frac{\Omega_p}{2} \right)^2 (R_c - 3R_{cc}) \right. \\
& + \left(\frac{\Omega_c}{2} \right)^2 \left[R_b + R_c - 3(R_{bc} + R_{cb}) \right] \Big\} \\
& + \Delta_p \Delta_c 2 \left(\frac{\Omega_p}{2} \right)^2 \left(\frac{\Omega_c}{2} \right)^2 \{ -D_c + \gamma_{bc} \\
& + [R_b + R_c - 3(R_{bc} + R_{cb})] \} \quad (29)
\end{aligned}$$

3. Discussion

If $\Delta_c = 0$, we scan only the probe laser with the result being

$$G_{b-a}(\Delta_p) = \frac{G_{b-a}^0 D_r^0 + A \cdot \Delta_p^2}{G_0^0 + B \cdot \Delta_p^2 + C \cdot \Delta_p^4} \quad (30)$$

$$A = \left[T_{a-b} \gamma_{ba} \gamma_{ac} + \left(\frac{\Omega_c}{2} \right)^2 (T_{a-c} - 2\gamma_{ba} R_b) \right] \gamma_{ac} \quad (31)$$

$$\begin{aligned}
B = & \gamma_{ba} \gamma_{ac} \left\{ D_c \gamma_{ba} \gamma_{ac} + 2 \left(\frac{\Omega_p}{2} \right)^2 (R_c - 3R_{cc}) \gamma_{ac} \right. \\
& + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \gamma_{ba} \\
& + 12 \left(\frac{\Omega_p}{2} \right)^2 \left(\frac{\Omega_c}{2} \right)^2 \Big\} + \left[\gamma_{ac} \gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] \\
& \times \left\{ D_c \left[\gamma_{ac} \gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] \right. \\
& + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \gamma_{bc} \Big\} \\
& - \gamma_{ac} 2 \left(\frac{\Omega_c}{2} \right)^2 \left\{ D_c \gamma_{ac} + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \right. \\
& + \left(\frac{\Omega_p}{2} \right)^2 [R_b + R_c - 3(R_{bc} + R_{cb})] \Big\} \quad (32)
\end{aligned}$$

$$C = \gamma_{ac} \left[D_c \gamma_{ac} + 2 \left(\frac{\Omega_c}{2} \right)^2 (R_b - 3R_{bb}) \right] \quad (33)$$

Now, a discussion of the above results indicates the following: (1) when $\Delta_p \rightarrow \pm\infty$, $G_{b-a}(\pm\infty) = 0$ which is the baseline of the absorption profile; and (2) when $\Delta_p = 0$, $G_{b-a}(0) = G_{b-a}^0 D_r^0 / G_0^0$ and its polarity is totally dependent on G_{b-a}^0 . Furthermore, if the intensity of a driving coherent field Ω_c is much larger than other parameters, (3) we have

$$G_{b-a}(0) \rightarrow -\frac{T_{a-c} + 2\gamma_{bc} R_b}{2(\Omega_c/2)^2 (R_b - 3R_{bb})} \rightarrow 0 \quad (34)$$

This means that the atomic medium is coherently transparent at the frequency center of the probe laser when the coherent field is strong enough. The physics here is that almost all of the atoms in this case is prepared in the dark state. Naturally, we can design other EIT or LWI schemes just following Eqs. (30)–(34).

4. Effects of the probe field on EIT with $\gamma_{bc} = 0$ and $\Delta_c = 0$

When $(R_{bb}, R_{ba}, R_{bc}) \rightarrow (0, \gamma_b, 0)$, $(R_{cb}, R_{ca}, R_{cc}) \rightarrow (0, \gamma_c, 0)$ and $\Delta_c = 0$, we have $\gamma_{bc} = 0$ and $\gamma_{ba} = \gamma_{ac}$ if atomic collision is neglected. In this case, we have

$$G_{b-a}^n = -\Delta_p^2 \gamma_{ac} \gamma_{ba} \gamma_b 2 \left(\frac{\Omega_c}{2} \right)^2 \quad (35)$$

$$\begin{aligned}
G_0^0 = & 2 \left[\left(\frac{\Omega_p}{2} \right)^2 \gamma_{ba} + \left(\frac{\Omega_c}{2} \right)^2 \gamma_{ac} \right] \left[\left(\frac{\Omega_p}{2} \right)^4 \gamma_c \right. \\
& + \left(\frac{\Omega_c}{2} \right)^4 \gamma_b + \left(\frac{\Omega_p}{2} \right)^2 \left(\frac{\Omega_c}{2} \right)^2 (\gamma_b + \gamma_c) \Big] \quad (36)
\end{aligned}$$

$$G_0^A = 2 \Delta_p^4 \gamma_{ac} \left(\frac{\Omega_c}{2} \right)^2 \gamma_b \quad (37)$$

$$\begin{aligned}
G_0^B = & \Delta_p^2 \gamma_{ac} 2 \left\{ \gamma_{ba} \left[\left(\frac{\Omega_p}{2} \right)^2 \gamma_c \gamma_{ac} + \left(\frac{\Omega_c}{2} \right)^2 \gamma_b \gamma_{ba} \right. \right. \\
& + 6 \left(\frac{\Omega_p}{2} \right)^2 \left(\frac{\Omega_c}{2} \right)^2 \Big] - \left(\frac{\Omega_c}{2} \right)^2 \left[2 \left(\frac{\Omega_c}{2} \right)^2 \gamma_b \right. \\
& + \left(\frac{\Omega_p}{2} \right)^2 (\gamma_b + \gamma_c) \Big] \Big\} \quad (38)
\end{aligned}$$

Therefore, after letting $\alpha = (\gamma_c \Omega_p^2)/(\gamma_b \Omega_c^2)$ and the total Rabi frequency $(\Omega/2)^2 = (\Omega_c/2)^2 + (\Omega_p/2)^2$, we have

$$G_{b-a}(\Delta_p) = \frac{-\gamma_{ba}\Delta_p^2}{\left[\Delta_p^2 - \left(\frac{\Omega}{2}\right)^2 \sqrt{1+\alpha}\right]^2 + (\Delta_p)^2 (2W_{dip})^2} \quad (39)$$

$$(2W_{dip})^2 = 2\left(\frac{\Omega}{2}\right)^2 (\sqrt{1+\alpha} - 1) + 4\left(\frac{\Omega_p}{2}\right)^2 + 2\alpha\left(\frac{\Omega_c}{2}\right)^2 + \gamma_{ba}^2(1+\alpha) \quad (40)$$

It is clear that G_{b-a} only has its maximum value 0 at $\Delta_p = 0$, and its minimum value is $-(\gamma_{ba})/((2W_{dip})^2)$ at $\Delta_p = \pm(\Omega/2)\sqrt{1+\alpha}$. This represents 100% EIT at the probe frequency center. The physical explanation is: at two-photon resonance and at infinite time, an atom gradually falls down into the dark state, because this state does not decay. Thus, no interaction among the atom, laser fields and vacuum fields can survive. As a consequence, 100% EIT should be observed. Notice that no matter if the coherent field is stronger than the probe field, the conclusion is still valid.

From Eq. (39), the influence of the probe laser on the width of EIT resonance can be immediately determined as follows. (1) By increasing the EIT transparency window; because $G_{b-a} = 0$ at both $\Delta_p = \pm\infty$ and $\Delta_p = 0$, it is easy to define the half-value width of the transparency window and the half-value width of the absorption doublet. The former is exactly calculated as

$$2\left[\sqrt{\left(\frac{\Omega}{2}\right)^2 \sqrt{1+\alpha} + (W_{dip})^2} - W_{dip}\right] \quad (41)$$

If the probe intensity α is large enough, the transparency window will increase substantially. However, we can also select a suitable atomic species so that γ_c is much smaller than γ_b , which results in

suppression of the window broadening due to an increase in probe field intensity. But notice that this broadening due to the probe field cannot be completely avoided. (2) By increasing the widths of the two absorption dips at $\Delta_p = \pm(\Omega/2)\sqrt{1+\alpha}$, the half-value of the absorption dips is exactly equal to $2W_{dip}$. When $\Omega_c \gg \Omega_p$,

$$(2W_{dip})^2 \approx (4 + 3\gamma_c/\gamma_b)\left(\frac{\Omega_p}{2}\right)^2 + \gamma_{ba}^2 + (1+\alpha) \quad (42)$$

Furthermore, when the probe Rabi frequency is much smaller than γ_{ba} , the width of the absorption dip has its minimum value at about $2W_{dip} \approx \gamma_{ba}$.

On the other hand, Eq. (39) can be rewritten as

$$G_{b-a}(\Delta_p) = \frac{\gamma_{ba}}{4W_{dip}\sqrt{W_{dip}^2 - \left(\frac{\Omega}{2}\right)^2 \sqrt{1+\alpha}}} \times \left[\frac{(L_-)^2}{(\Delta_p)^2 + (L_-)^2} - \frac{(L_+)^2}{(\Delta_p)^2 + (L_+)^2} \right] \quad (43)$$

Then if

$$W_{dip} > \frac{\Omega}{2}\sqrt{1+\alpha}, \quad L_{\pm} = W_{dip} \pm \sqrt{W_{dip}^2 - \left(\frac{\Omega}{2}\right)^2 \sqrt{1+\alpha}} \quad (44)$$

The absorption spectrum is the subtraction of two Lorentzians with different widths, but same heights and the same frequency center. Apparently, this result cannot be explained simply on the basis of Autler–Townes level splitting.

$$\text{If } W_{dip} < \frac{\Omega}{2}\sqrt{1+\alpha},$$

$$\Omega_0 = \sqrt{\left(\frac{\Omega}{2}\right)^2 \sqrt{1+\alpha} - W_{dip}^2} \quad (45)$$

Eq. (39) then can be rewritten as

$$G_{b-a}(\Delta_p) = \frac{\gamma_{ba}}{4\sqrt{\left(\frac{\Omega}{2}\right)^2 \sqrt{1+\alpha} - W_{dip}^2}} \times \left[\frac{-\Delta_p}{(\Delta_p - \Omega_0)^2 + (W_{dip})^2} + \frac{\Delta_p}{(\Delta_p + \Omega_0)^2 + (W_{dip})^2} \right] \quad (46)$$

Thus, the profile of EIT in this case can be viewed as the addition of two mirror-symmetrical Lorentzian-like components with the same width and different central frequencies, which corresponds in similarity to Autler–Townes level splitting. In other words, EIT is the consequence of quantum coherence and AC-Stark effect. In the weak-field regime (Eq. (44)), quantum interference and coherence is dominant while in the strong-field regime, AC-Stark effect and quantum coherence are equally important.

The above results concerning EIT are focused *only* on the derivation of a non-absorption condition and the explanation of its physical origin. We will not discuss the problem of nonlinear propagation of driving and probing fields, which is in the area of resonant driving fields and was studied earlier in Refs. [33,34].

Fig. 2 shows the probe effects on EIT.

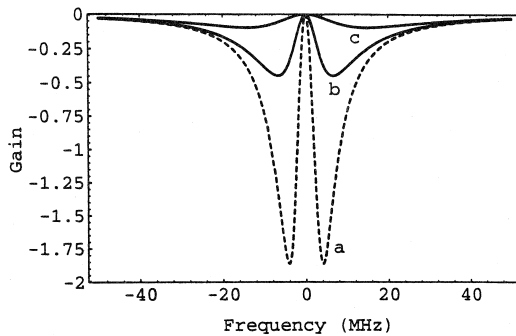


Fig. 2. The effects of a probe laser on EIT with $\gamma_b = 1.65$ MHz, $\gamma_c = 8.25$ MHz and $\Omega_c = 6.0$ MHz and $\Omega_p = 3.0$ MHz in curve (a), $\Omega_p = 6.0$ MHz in curve (b) and $\Omega_p = 12.0$ MHz in curve (c).

5. LWI with $\Delta_c = 0$

We now consider gain of the probe laser. From Eq. (23), probe gain at resonance can be achieved if

$$T_{a-b} > \frac{(\Omega_c/2)^2}{[\gamma_{ac}\gamma_{bc} + (\Omega_p/2)^2]} (T_{a-c} + 2\gamma_{bc}R_b) \quad (47)$$

Then, we consider the condition of no inversion in the probing transition. From Eq. (16), we let $\rho_{aa} - \rho_{bb} < 0$ and have

$$\begin{aligned} & [T_{a-b}\gamma_{ac} - 2(\Omega_c/2)^2 R_b] \\ & < \frac{(\Omega_p/2)^2}{[\gamma_{ba}\gamma_{bc} + (\Omega_c/2)^2]} \\ & \times [-T_{a-b}\gamma_{ba} + 2(\Omega_c/2)^2 R_c] \end{aligned} \quad (48)$$

Finally, the condition of no Raman inversion comes from Eq. (18) if $\rho_{cc} - \rho_{bb} < 0$

$$\begin{aligned} & T_{c-b}D_r^0 + 2\left(\frac{\Omega_p}{2}\right)^2 \left[\gamma_{ac}\gamma_{bc} + \left(\frac{\Omega_p}{2}\right)^2 - \left(\frac{\Omega_c}{2}\right)^2 \right] R_c \\ & < 2\left(\frac{\Omega_c}{2}\right)^2 \left[\gamma_{ba}\gamma_{bc} + \left(\frac{\Omega_c}{2}\right)^2 - \left(\frac{\Omega_p}{2}\right)^2 \right] R_b \end{aligned} \quad (49)$$

For example, if $(R_{bb}, R_{ba}, R_{bc}) \rightarrow (-W, \gamma_b, 0)$ and $(R_{cb}, R_{ca}, R_{cc}) \rightarrow (0, \gamma_c, 0)$ where W is an incoherent pump rate, we reach

$$(\gamma_b - W) < \frac{W}{2\gamma_{bc}} \gamma_c \quad (50)$$

$$(\gamma_b - W) > - \frac{(\Omega_p/2)^2}{[\gamma_{ba}\gamma_{bc} + (\Omega_c/2)^2]} \gamma_c \quad (51)$$

$$\begin{aligned} & \left[\gamma_{ba}\gamma_{bc} + \left(\frac{\Omega_c}{2}\right)^2 - \left(\frac{\Omega_p}{2}\right)^2 \right] (\gamma_b - W) \\ & > \frac{W\gamma_c D_r^0}{2(\Omega_c/2)^2} + \left(\frac{\Omega_p}{\Omega_c}\right)^2 \left[\gamma_{ac}\gamma_{bc} + \left(\frac{\Omega_p}{2}\right)^2 - \left(\frac{\Omega_c}{2}\right)^2 \right] \gamma_c \end{aligned} \quad (52)$$

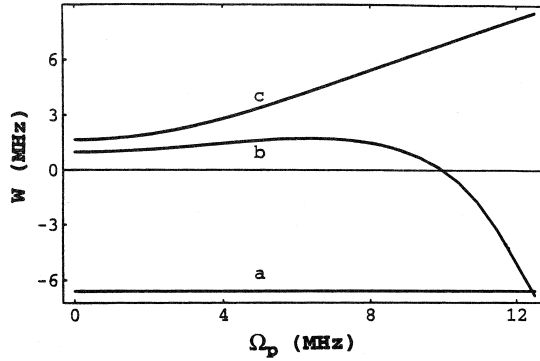


Fig. 3. The operating regime for LWI, located in the intersection between the transverse axis and curve (b). The regime above curve (a) is designated for the gain condition and the regime below curve (c) is for lasing without one-photon inversion while the regime below curve (b) is for the condition of lasing without two-photon inversion. The parameters used here are $\gamma_b = 1.65$ MHz, $\gamma_c = 8.25$ MHz and $\Omega_c = 10.0$ MHz.

We see that an incoherent pumping rate providing the probe amplification without one-photon and two-photon inversions is limited in the special region. Furthermore, when $\Omega_p \ll \Omega_c$ (such as in the case for the onset of the probing lasing) and atomic collision is neglected, Eqs. (50)–(52) become

$$(\gamma_b - W) < \gamma_c \quad (53)$$

$$(\gamma_b - W) > 0 \quad (54)$$

$$(\gamma_b - W) > \frac{W\gamma_{ac}}{2(\Omega_c/2)^2} \gamma_c \quad (55)$$

Thus, the final condition for lasing without inversion in a Lambda scheme is

$$\gamma_c > (\gamma_b - W) > \frac{W\gamma_{ac}}{2(\Omega_c/2)^2} \gamma_c \quad (56)$$

That is, LWI can be observed only if Eq. (56) is obeyed and if $W\gamma_{ac} < 2(\Omega_c/2)^2$.

Fig. 3 shows the operation regime of LWI, located in the intersection between the transverse axis and curve (b). Lasing without one-photon inversion may operate in the regime between the transverse axis and curve (c). If the intensity of the driving laser is fixed, the condition for lasing without two-photon inversion will limit the selection of an incoherent pump in the transition $a-b$ and the probe intensity, which is consistent with the results in Refs. [35–38].

The most important conclusion from Eq. (56) is that we may have LWI even when γ_b is larger than γ_c . This point is a supplement or extension to the condition of LWI in Refs. [1–9], which is $\gamma_c > \gamma_b$. The difference in the two conclusions exists, whether or not the first-order approximation is used.

From Eqs. (30)–(33), we know that the gain profile of LWI is basically the same as the absorption profile of EIT discussed in Section 4. In the weak-field regime, the gain profile also can be decomposed into the subtraction of two Lorentzians with the same central frequency but different widths and heights. In the strong-field regime, it can be decomposed into the addition of two Lorentzian-like components with the same width and different central frequencies. To this end we see

$$G_{b-a}(\Delta_p) = \frac{G_{b-a}^0 D_r^0 + A \cdot \Delta_p^2}{G_0^0 + B \cdot \Delta_p^2 + C \cdot \Delta_p^4} \quad (57)$$

$$G_{b-a}^0 = \left(\frac{\Omega_c}{2} \right)^2 [W\gamma_c - 2\gamma_{bc}(\gamma_b - W)], \quad (58)$$

where

$$A = - \left(\frac{\Omega_c}{2} \right)^2 [W\gamma_c + 2\gamma_{ba}(\gamma_b - W)] \gamma_{ac} \quad (59)$$

$$\begin{aligned} B = & \gamma_{ba}\gamma_{ac} \left[W\gamma_c\gamma_{ba}\gamma_{ac} + 2 \left(\frac{\Omega_p}{2} \right)^2 \gamma_c\gamma_{ac} \right. \\ & + 2 \left(\frac{\Omega_c}{2} \right)^2 (\gamma_b + 2W)\gamma_{ba} + 12 \left(\frac{\Omega_p}{2} \right)^2 \left(\frac{\Omega_c}{2} \right)^2 \\ & + \left[\gamma_{ac}\gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] \left\{ W\gamma_c \left[\gamma_{ac}\gamma_{bc} + \left(\frac{\Omega_p}{2} \right)^2 \right] \right. \\ & + 2 \left(\frac{\Omega_c}{2} \right)^2 (\gamma_b + 2W)\gamma_{bc} \left. \right\} - \gamma_{ac} 2 \left(\frac{\Omega_c}{2} \right)^2 \\ & \times \left\{ W\gamma_c\gamma_{ac} + 2 \left(\frac{\Omega_c}{2} \right)^2 (\gamma_b + 2W) \right. \\ & + \left. \left. \left(\frac{\Omega_p}{2} \right)^2 (\gamma_b + \gamma_c - W) \right\} \right] \quad (60) \end{aligned}$$

$$C = \gamma_{ac} \left[W\gamma_c\gamma_{ac} + 2 \left(\frac{\Omega_c}{2} \right)^2 (\gamma_b + 2W) \right] \quad (61)$$

Because G_{b-a}^0 is no longer 0, the decomposition of Eq. (57) will be more complex than before, but the basic conclusion is the same as that in Section 4.

6. Conclusion

In this paper, a general treatment including an arbitrary incoherent pumping configuration, arbitrary coherent field and detuning is used so that we can discuss any possible case for EIT and LWI. The resulting absorption-gain profile for EIT and LWI is analyzed on the basis of the concepts of quantum interference and coherence. In the weak field regime, this profile is the *subtraction* of two Lorentzians with the same central frequency but different widths and heights, and is more like the result of quantum interference and coherence than the result of Autler–Townes level splitting. In the strong field regime, this profile will be the *addition* of two Lorentzian-like components with *different* central frequencies but the same width, which is primarily the result of the combination of quantum coherence and the AC-Stark effect. Effects of a strong probe on EIT is discussed subsequently. If the probe intensity α is large enough, the transparency window increases substantially. But if $\gamma_c \ll \gamma_b$, the broadening of the transparency window can be partially controlled.

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