••39. In Fig. 29-49, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a = 13.5\,\mathrm{cm}$. Each wire carries 7.50 A, and the currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3. In unit-vector notation, what is the net magnetic force *per meter of wire length* on wire 4?

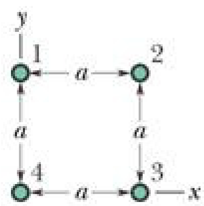

FIGURE 29-49 Problems 19, 36, and 39.

$I = 7.50\mathrm{A}$ $\quad a = 13.5\mathrm{cm}$ $\quad \mu T = T \cdot 10^{-6}$ $\quad \mu N = N \cdot 10^{-6}$

Describe each current as a vector. "Out of the page" means +z direction:

$$i_1 = I \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad i_2 = I \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad i_3 = I \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad i_4 = I \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Place wire 4 at the origin. The vectors from the origin to each of the other wires are then:

$$r_1 = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \qquad r_2 = \begin{pmatrix} a \\ a \\ 0 \end{pmatrix} \qquad r_3 = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$$

The net magnetic field from the other wires at the origin is then:

$$B = \frac{-\mu_0}{2\pi} \cdot \left[\frac{i_1 \times r_1}{(|r_1|)^2} + \frac{i_2 \times r_2}{(|r_2|)^2} + \frac{i_3 \times r_3}{(|r_3|)^2} \right] \qquad B = \begin{pmatrix} 5.556 \\ 16.667 \\ 0 \end{pmatrix} \mu T$$

<u>Note:</u> The initial negative sign accounts for the fact that our radii are leading from the destination back to the sources of the fields rather than vice-versa.

<u>Note:</u> One can also write the square of the magnitude of the r vector as a dot product, $r \cdot r$

The force on wire 4 due to the magnetic fields will be:

$$F_{net} = i_4 \times B \qquad F_{net} = \begin{pmatrix} -125 \\ 41.7 \\ 0 \end{pmatrix} \frac{\mu N}{m}$$

The Cross Product

Suppose we have vectors i and r. The cross product i x r can be computed as a determinant:

$$i \times r = \left| \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ ix & iy & iz \\ rx & ry & rz \end{pmatrix} \right| \qquad \text{and} \qquad \left| \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ ix & iy & iz \\ rx & ry & rz \end{pmatrix} \right| \rightarrow \begin{pmatrix} iy \cdot rz - iz \cdot ry \\ -ix \cdot rz + rx \cdot iz \\ ix \cdot ry - rx \cdot iy \end{pmatrix}$$