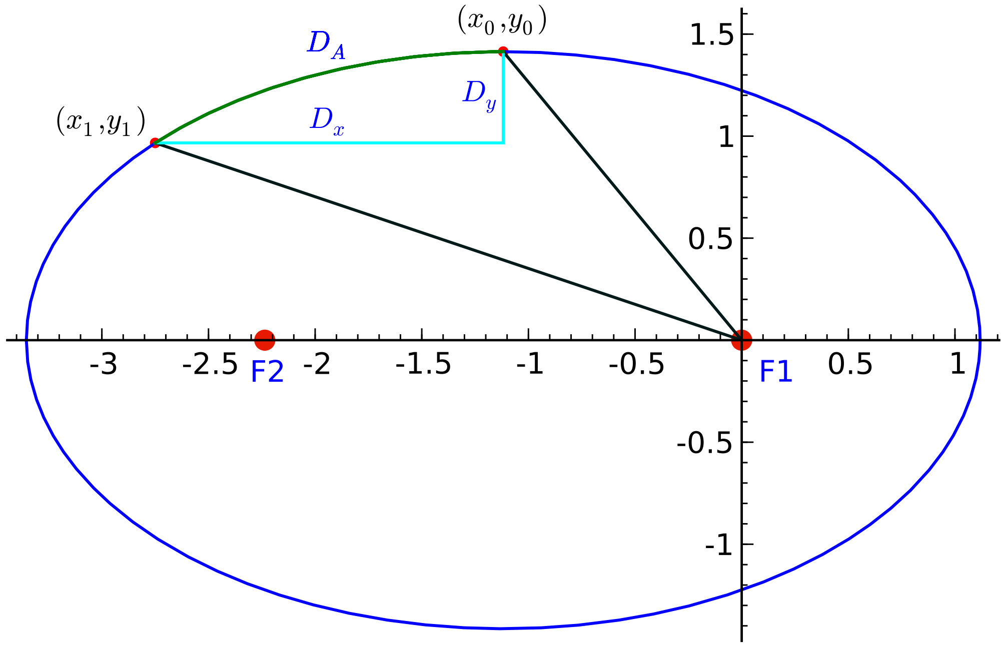

Given t_0, t_1, and (x_0, y_0) on our ellipse (ε is eccentricity). We wish to use polar coordinates with the origin at one focus such that

$$\begin{aligned} r(\theta) &= \frac{a(1-\varepsilon^2)}{1 \pm \varepsilon \cos\theta} \\ x(\theta) &= r(\theta) \cdot \cos(\theta) \\ y(\theta) &= r(\theta) \cdot \sin(\theta) \end{aligned}$$

where at time t_0 we are at the point (x_0, y_0) on the ellipse. We want to be able to determine the exact position of (x_1, y_1) on the ellipse at time t_1.

I need help with finding this solution. I figured if we can find D_x, D_y as pictured then, we can construct

$$(x_1, y_1) = (x_0, y_0) - (D_x, D_y)$$

Unfortunately, I've only been able to calculate the arc length, D_A, as follows:

$$\begin{aligned} x(\theta) &= r(\theta) \cdot \cos(\theta) \\ y(\theta) &= r(\theta) \cdot \sin(\theta) \end{aligned}$$

then

$$\begin{aligned} \frac{dx}{d\theta}(\theta) &= \frac{-a(1-\varepsilon^2) \cdot \varepsilon \cdot \sin\theta \cdot \cos\theta}{(1+\varepsilon \cdot \cos\theta)^2} - \frac{a(1-\varepsilon^2)}{1+\varepsilon \cdot \cos\theta} \sin\theta \\ \frac{dy}{d\theta}(\theta) &= \frac{-a(1-\varepsilon^2) \cdot \varepsilon \cdot \sin^2\theta}{(1+\varepsilon \cdot \cos\theta)^2} + \frac{a(1-\varepsilon^2)}{1+\varepsilon \cdot \cos\theta} \cos\theta \end{aligned}$$

and the arc length distance is:

$$\begin{aligned} D(t) &= \left| \left(\frac{dx}{d\theta}, \frac{dy}{d\theta} \right) \cdot t \right| \\ &= t \cdot \sqrt{\left(\frac{dx}{d\theta} \right)^2 + \left(\frac{dy}{d\theta} \right)^2} \end{aligned}$$

And so at t_0 we're at (x_0, y_0) on our ellipse and at t_1 we are at (x_1, y_1), we then have $\Delta t = t_1 - t_0$ and

$$D_A = D(\Delta t) = \Delta t \cdot \sqrt{\left(\frac{dx}{d\theta} \right)^2 + \left(\frac{dy}{d\theta} \right)^2}$$

I am still unsure as to how to find (D_x, D_y). My ultimate goal is to find (x_1, y_1) and this was the approach I wanted to take however, I am not sure what my next step is or if this is the correct approach.

Any and all advice, suggestions, etc. are appreciated.

NOTE: one can assume that as we 'walk' on the ellipse, our 'velocity' is constant.