$$Determine\ whether\ the\ following\ series\ is\ convergent\ or\ divergent$$

$$\sum_{n=1}^{\infty} \frac{1}{3^{ln(n)}}$$

$$Consider\ the\ following:$$

$$\sum_{n=1}^{\infty} \frac{1}{e^{ln(n)}} = \sum_{n=1}^{\infty} \frac{1}{n}, which\ diverges$$

$$e\ <\ 3, which\ means\ there\ must\ be\ some\ value\ k\ >\ 1, such\ that$$

$$e < e^k < 3$$

$$\frac{1}{e} > \frac{1}{e^k} > \frac{1}{3}$$

$$\frac{1}{e^{ln(n)}} > \frac{1}{(e^k)^{ln(n)}} > \frac{1}{3^{ln(n)}}, for\ all\ n\ \geq\ 1$$

$$So, consider\ \sum_{n=1}^{\infty} \frac{1}{(e^k)^{ln(n)}} = \sum_{n=1}^{\infty} \frac{1}{e^{kln(n)}} = \sum_{n=1}^{\infty} \frac{1}{e^{ln(n^k)}} = \sum_{n=1}^{\infty} \frac{1}{n^k}$$

$$Because\ k\ >\ 1, \sum_{n=1}^{\infty} \frac{1}{n^k}\ \ converges\ by\ the\ p - test$$

$$Thus, because\ \frac{1}{(e^k)^{ln(n)}} > \frac{1}{3^{ln(n)}},$$

$$\frac{1}{3^{ln(n)}}\ also\ converges\ by\ the\ direct\ comparison\ test$$