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$\vec{F}(x,y,z) = y\hat{j} - z\hat{k}$

Let S_1 be the surface of the paraboloid: $y = x^2 + z^2$, $0 \le y \le 1$

S_1 can be parameterized as follows

$\vec{r}(u,v) = \langle u, u^2+v^2, v \rangle$

convert to polar coordinates:

$$x = r\cos\theta, \quad y = r^2, \quad z = r\sin\theta$$

$\vec{r}(r,\theta) = \langle r\cos\theta, r^2, r\sin\theta \rangle$

compute $\vec{r}_r \times \vec{r}_\theta$

$\vec{r}_r = \langle \cos\theta, 2r, \sin\theta \rangle$

$\vec{r}_r = \langle \cos\theta, 2r, \sin\theta \rangle$

$\vec{r}_\theta = \langle -r\sin\theta, 0, r\cos\theta \rangle$

$\vec{r}_r \times \vec{r}_\theta = \langle 2r^2\cos\theta, -r, 2r^2\sin\theta \rangle$

$\vec{F}(x,y,z)$ in terms of r and θ

Should be: $y\hat{j} - z\hat{k} = \boxed{r^2\hat{j} - r\sin\theta\,\hat{k}}$

$\vec{F} \cdot (\vec{r}_r \times \vec{r}_\theta) = \langle 0, r^2, -r\sin\theta \rangle \cdot \langle 2r^2\cos\theta, -r, 2r^2\sin\theta \rangle$

$= -r^3 - 2r^3\sin^2\theta$

Knowing that $0 \leq r \leq 1$

and $0 \leq \theta \leq 2\pi$:

$$\int_0^{2\pi}\int_0^1 (-r^3 - 2r^3\sin^2\theta)\, r\,dr\,d\theta = \boxed{-\frac{4\pi}{5}}$$

$\int_0 \int_0 \qquad \boxed{5}$

however I know the answer should be $-\pi$. I noticed if i express $\vec{F}_{(r,\theta)}$ as $r\hat{j} - \sin\theta \hat{k}$, rather than: $\vec{F}_{(r,\theta)} = r^2\hat{j} - r\sin\theta \hat{k}$, then i get the right answer, but don't understand why.