MATH 19A - CALCULUS I: WORKSHEET #1

To enhance your learning experience of the material in this class, weekly worksheets will be given (either on Tuesdays or Thursdays). The goal is to provide you with additional practice in addition to trying to understand things at a deeper level. A calculator may be necessary at times, especially if you're doing problems involving approximations. The number of questions on the worksheets will vary from week to week; For instance, I don't want to have you guys working on a lot of worksheet problems if you have a huge assignment due that week.

Problem 1 (Average and Instantaneous Velocity)**.** Let us consider a real valued function $f(x)$ over the interval $[t_0, t_1]$. In class, the average rate of change of $f(x)$ over $[t_0, t_1]$ was given by the formula

$$f_{\text{avg}} = \frac{\Delta f}{\Delta t} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}.$$

Suppose that Jill dropped a ball from rest from the top of a building that is 200m tall for her friend Jack to catch (see Figure 1). The ball falls under the influence of gravity, obeying the equation $f(t) = 4.9t^2$.

(a) How long does it take the ball to reach Jack (who is 2m tall)? Denote the time found by t_0 and round your answer to two decimal places.

(b) Calculate the average velocity of the ball over the time interval $[2, t_0]$. Round this value to two decimal places.

(c) Approximately how fast is the ball falling at time $t = t_0$? Fill out the following table to help you estimate the instantaneous velocity at $t = t_0$ to two decimal places.

interval	$[t_0 - 0.001, t_0]$	$[t_0 - 0.0001, t_0]$	$[t_0, t_0 + 0.0001]$	$[t_0, t_0 + .001]$
average velocity				

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(d) From Jill's point of view, the equation $f(t) = 4.9t^2$ represents how far the ball has fallen from the top of the 200m building. What would be the equation that represents the position of the same falling ball relative to Jack's position on the ground?

(e) At what time is the ball 40m from Jill? 50m from Jack? 75m from the ground? Round your answers to two decimal places.

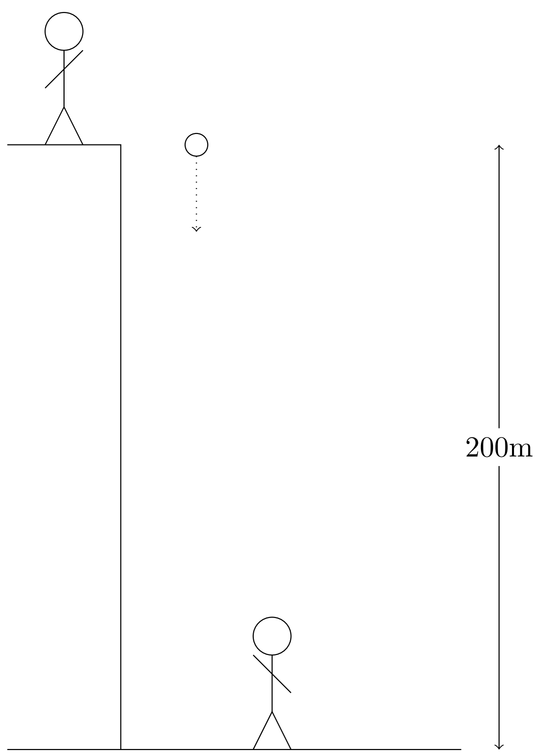

FIGURE 1. Jill dropping a ball to Jack (not to scale).

Problem 2 (Computing Limits Graphically and Continuity)**.** Consider the following graph of the function $y = f(x)$:

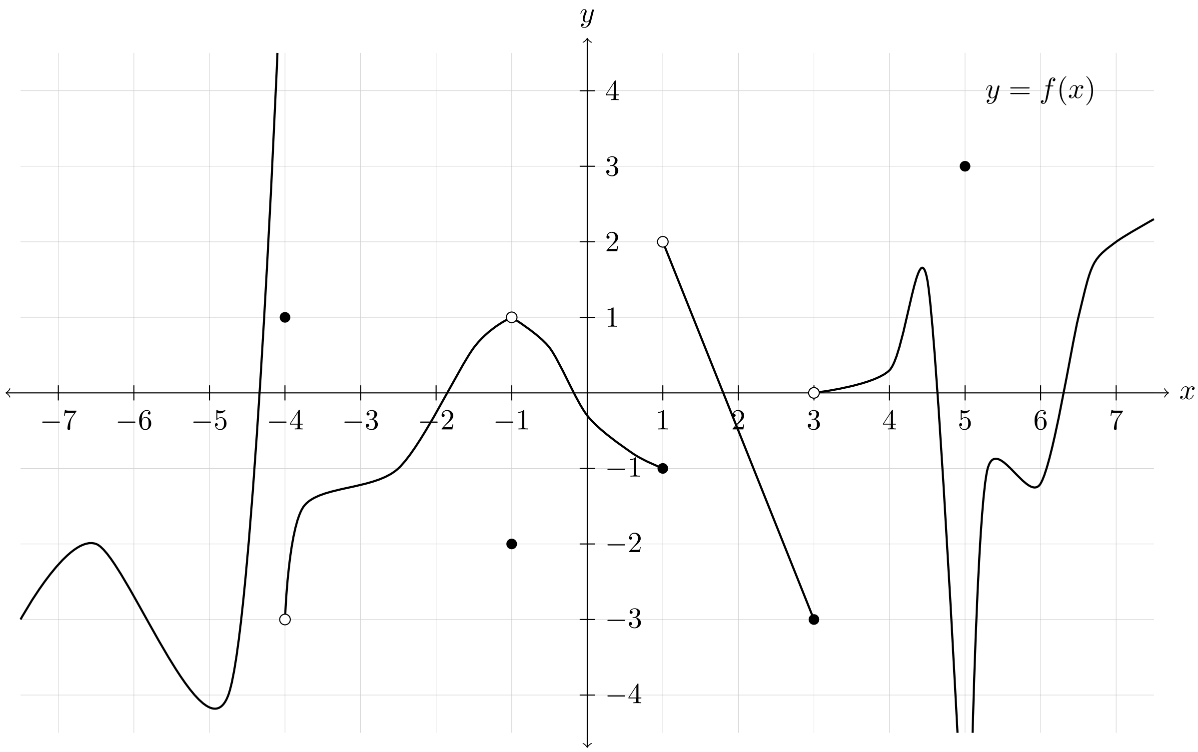

FIGURE 2. Graph of $f(x)$ used in Problem 2.

Use the above graph to answer the following:

(a) Evaluate $\lim_{x \to -4^-} f(x)$, $\lim_{x \to -4^+} f(x)$, and $f(-4)$. Does $\lim_{x \to -4} f(x)$ exist? Is $f(x)$ left continuous, right continuous, both or neither at $x = -4$?

(b) Evaluate $\lim_{x \to -1^-} f(x)$, $\lim_{x \to -1^+} f(x)$, and $f(-1)$. Does $\lim_{x \to -1} f(x)$ exist? Is $f(x)$ left continuous, right continuous, both or neither at $x = -1$?

(c) Evaluate $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$, and $f(1)$. Does $\lim_{x\to 1} f(x)$ exist? Is $f(x)$ left continuous, right continuous, both or neither at $x = 1$?

(d) Evaluate $\lim_{x\to 3^-} f(x)$, $\lim_{x\to 3^+} f(x)$, and $f(3)$. Does $\lim_{x\to 3} f(x)$ exist? Is $f(x)$ left continuous, right continuous, both or neither at $x = 3$?

(e) Evaluate $\lim_{x\to 5^-} f(x)$, $\lim_{x\to 5^+} f(x)$, and $f(5)$. Does $\lim_{x\to 5} f(x)$ exist? Is $f(x)$ left continuous, right continuous, both or neither at $x = 5$?

(f) Evaluate $\lim_{x\to 7^-} f(x)$, $\lim_{x\to 7^+} f(x)$, and $f(7)$. Does $\lim_{x\to 7} f(x)$ exist? Is $f(x)$ left continuous, right continuous, both or neither at $x = 7$?

Problem 3 (The Limit Laws)**.** Given below are calculations performed to evaluate the limits

$$\lim_{x\to 3} \frac{x^2+2x}{3x^2}, \quad \lim_{x\to \frac{\pi}{2}} \left(x - \frac{\pi}{2}\right)\tan x, \quad \lim_{x\to 1} \frac{x^3-1}{x-1}, \quad \text{and } \lim_{x\to 1} \sqrt{x^2+3x+5}.$$

Identify the limit rules that were used to do each limit computation. Which of the following computations were done correctly? For the computations that are incorrect, identify the rule(s) that were misused (if any), explain why they failed, and rework the problem (if possible).

(a) We have that

$$\begin{aligned}
\lim_{x\to 3}\frac{x^2+2x}{3x^2} &= \frac{\lim\limits_{x\to 3}(x^2+2x)}{\lim\limits_{x\to 3}3x^2} \\
&= \frac{\lim\limits_{x\to 3}x^2+\lim\limits_{x\to 3}2x}{3\lim\limits_{x\to 3}x^2} \\
&= \frac{\left(\lim\limits_{x\to 3}x\right)^2+2\lim\limits_{x\to 3}x}{3\left(\lim\limits_{x\to 3}x\right)^2} \\
&= \frac{(3)^2+2(3)}{3(3)^2} \\
&= \frac{5}{9}.
\end{aligned}$$

(b) We have that

$$\begin{aligned}
\lim_{x\to\frac{\pi}{2}}\left(x-\frac{\pi}{2}\right)\tan x &= \left[\lim_{x\to\frac{\pi}{2}}\left(x-\frac{\pi}{2}\right)\right]\cdot\left[\lim_{x\to\frac{\pi}{2}}\tan x\right] \\
&= 0\cdot\lim_{x\to\frac{\pi}{2}}\tan x \\
&= 0.
\end{aligned}$$

(c) We have that

$$\begin{aligned}
\lim_{x\to 1} \frac{x^3-1}{x-1} &= \frac{\lim\limits_{x\to 1}(x^3-1)}{\lim\limits_{x\to 1}(x-1)} \\
&= \frac{\lim\limits_{x\to 1} x^3 - \lim\limits_{x\to 1} 1}{\lim\limits_{x\to 1} x - \lim\limits_{x\to 1} 1} \\
&= \frac{\left(\lim\limits_{x\to 1} x\right)^3 - \lim\limits_{x\to 1} 1}{\lim\limits_{x\to 1} x - \lim\limits_{x\to 1} 1} \\
&= \frac{(1)^3-1}{1-1} \\
&= \frac{0}{0}.
\end{aligned}$$

Since $\lim\limits_{x\to 1} \frac{x^3-1}{x-1} = \frac{0}{0}$, the limit does not exist.

(d) We have that

$$\begin{aligned}
\lim_{x\to 1} \sqrt{x^2+3x+5} &= \sqrt{\lim_{x\to 1}(x^2+3x+5)} \\
&= \sqrt{\lim_{x\to 1} x^2 + \lim_{x\to 1} 3x + \lim_{x\to 1} 5} \\
&= \sqrt{\left(\lim_{x\to 1} x\right)^2 + 3\lim_{x\to 1} x + \lim_{x\to 1} 5} \\
&= \sqrt{(1)^2+3(1)+5} \\
&= \sqrt{9} \\
&= 3.
\end{aligned}$$