MATH 19A - CALCULUS I: WORKSHEET #2

To enhance your learning experience of the material in this class, weekly worksheets will be given (either on Tuesdays or Thursdays). The goal is to provide you with additional practice in addition to trying to understand things at a deeper level. A calculator may be necessary at times, especially if you're doing problems involving approximations. The number of questions on the worksheets will vary from week to week; For instance, I don't want to have you guys working on a lot of worksheet problems if you have a huge assignment due that week.

Problem 1 (The Indeterminate Cases $0/0$, ∞/∞, and $\infty - \infty$)**.** Each of the following limits, when evaluated initially by substitution fall into one of the three indeterminate cases: $\frac{0}{0}$, $\frac{\infty}{\infty}$ or $\infty - \infty$. For each part, identify what indeterminate case occurs, transform the limit algebraically, and then use continuity to evaluate the limit.

(a) $\lim_{x \to 2} \dfrac{x^2 - 6x + 8}{x^3 - 8}$

(b) $\lim_{x \to 2^+} \dfrac{1}{x - 2} - \dfrac{4}{x^2 - 4}$

(c) $\lim_{x \to 0} \dfrac{e^{2x} - 1}{e^x - 1}$

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(d) $\lim_{x\to 1^+} \dfrac{\ln(x^2-1)}{\ln(x-1)}$

(e) $\lim_{x\to \frac{\pi}{2}^-} \tan x - \sec x$ (*Hint*: To evaluate, rewrite in terms of $\sin x$ and $\cos x$.)

(f) $\lim_{x\to \pi} \dfrac{\csc x}{\cot x}$

Problem 2 (Evaluating Trigonometric Limits)**.** Evaluate the following limits. The limit identities

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \text{and} \qquad \lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

and the trigonometric identities

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \text{and} \qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta$$

may come in handy. Note that there may be more than one way to compute each limit.

(a) $\lim_{\theta\to 0} \dfrac{2\sin\theta - \sin(2\theta)}{2\theta^2}$

(b) $\lim_{z\to\frac{\pi}{2}} \dfrac{\sin\left(z-\frac{\pi}{2}\right)}{z^2-\frac{\pi^2}{4}}$

(c) $\lim_{x\to 0} \dfrac{\tan(3x)+\tan(4x)}{\tan(7x)}$

(d) $\lim_{s\to 2} \dfrac{1-\cos^2(2s-4)+\sin^2(2s-4)-2\sin(2s-4)\cos(2s-4)}{4s-8}$

Problem 3 (Evaluating Limits Using the Squeeze Theorem)**.** Find suitable functions $l(x)$ and $u(x)$ that squeeze the function $f(x) = x^2 \cos\left(\frac{1}{x}\right)$ at $x = 0$ on the open interval $(-1, 1)$ (i.e. find $l(x)$ and $u(x)$ such that $l(x) \leq f(x) \leq u(x)$ for $x \neq 0$ on $(-1, 1)$). Then apply the Squeeze Theorem to compute $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$.