MATH 19A - CALCULUS I: WORKSHEET #4

To enhance your learning experience of the material in this class, weekly worksheets will be given (either on Tuesdays or Thursdays). The goal is to provide you with additional practice in addition to trying to understand things at a deeper level. A calculator may be necessary at times, especially if you're doing problems involving approximations. The number of questions on the worksheets will vary from week to week; For instance, I don't want to have you guys working on a lot of worksheet problems if you have a huge assignment due that week.

Problem 1 (The Derivative of a Function; Part 1)**.** Use the limit definition to compute $f'(x)$.

(a) $f(x) = x^2 - 2x$

(b) $f(x) = x + \sqrt{x}$

(c) $f(x) = \dfrac{1}{\sqrt{2x}}$

Date: February 4, 2013.

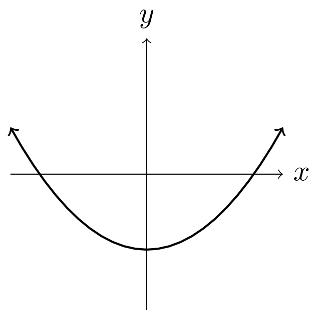
Problem 2 (The Derivative of a Function; Part 2)**.** Use the power rule to compute $f'(x)$. Then find the equation of the tangent line of $f(x)$ at $x = a$.

(a) $f(x) = x^2 - 2x$; $x = 1$

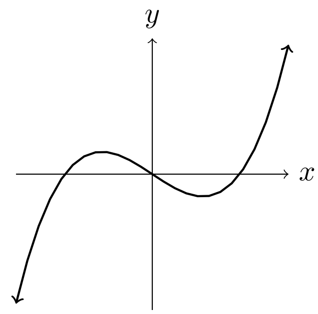
(b) $f(x) = x + \sqrt{x}$; $x = 4$

(c) $f(x) = \dfrac{1}{\sqrt{2x}}$; $x = 2$ $\left(\textit{Note}: \dfrac{1}{\sqrt{2x}} = \dfrac{1}{\sqrt{2}\sqrt{x}} \right)$

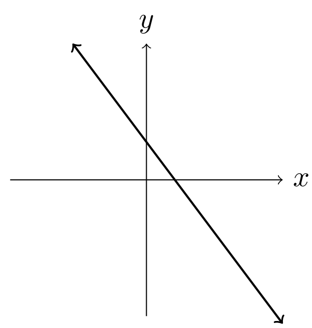
Problem 3 (The Derivative of a Function; Part 3)**.** Match the graphs (A)–(D) of $f(x)$ with their derivatives (I)–(III). Which two functions from (A)–(D) have the same derivative?



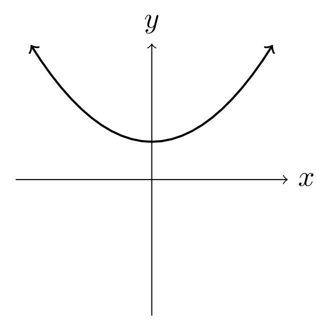
(A)



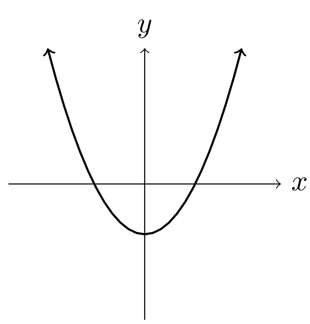
(B)



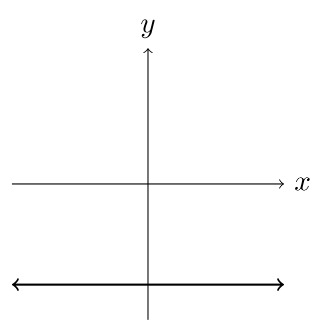
(C)



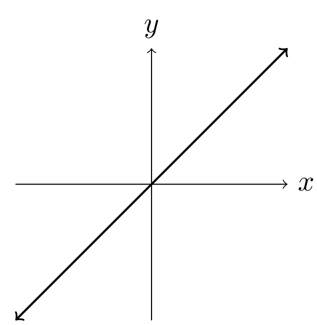
(D)



(I)



(II)



(III)

Problem 4 (The Product and Quotient Rule for Derivatives; Part 1)**.** Below are proofs for the product and quotient rules of differentiation with some of the pieces left out. Fill in the boxes with the missing parts to complete the proofs.

Theorem (Product Rule for Derivatives)**.** Let $f(x)$ and $g(x)$ be differentiable functions. Then,

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + f'(x)g(x).$$

Proof. By the definition of the derivative, we have that

$$\begin{aligned}
\frac{d}{dx}\left[f(x)g(x)\right] &= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x+h) - \boxed{\qquad\qquad\qquad} + \boxed{\qquad\qquad\qquad} - f(x)g(x)}{h} \\
&= \lim_{h\to 0} \frac{\boxed{\qquad\quad}(g(x+h) - g(x)) + \boxed{\qquad\quad}(f(x+h) - f(x))}{h} \\
&= \lim_{h\to 0}\boxed{\qquad\quad}\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} + \lim_{h\to 0}\boxed{\qquad\quad}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\
&= f(x)g'(x) + f'(x)g(x).
\end{aligned}$$

$\square$

Theorem (Quotient Rule for Derivatives)**.** Let $f(x)$ and $g(x)$ be differentiable functions. Then,

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$$

Proof. By the definition of the derivative, we have that

$$\begin{aligned}
\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] &= \lim_{h\to 0} \frac{\dfrac{f(x+h)}{g(x+h)} - \dfrac{f(x)}{g(x)}}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{hg(x)g(x+h)} \\
&= \lim_{h\to 0} \frac{\dfrac{f(x+h)g(x) - \boxed{\qquad\qquad\qquad} + \boxed{\qquad\qquad\qquad} - f(x)g(x+h)}{h}}{g(x)g(x+h)} \\
&= \lim_{h\to 0} \frac{\dfrac{\boxed{\qquad\quad}(f(x+h) - f(x)) - \boxed{\qquad\quad}(g(x+h) - g(x))}{h}}{g(x)g(x+h)} \\
&= \lim_{h\to 0} \frac{\boxed{\qquad\quad}\dfrac{f(x+h)-f(x)}{h} - \boxed{\qquad\quad}\dfrac{g(x+h)-g(x)}{h}}{g(x)g(x+h)} \\
&= \frac{\lim_{h\to 0}\boxed{\qquad\quad}\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h} - \lim_{h\to 0}\boxed{\qquad\quad}\lim_{h\to 0}\dfrac{g(x+h)-g(x)}{h}}{\lim_{h\to 0} g(x)g(x+h)} \\
&= \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.
\end{aligned}$$

$\square$

Problem 5 (The Product and Quotient Rule for Derivatives; Part 2). Use the product or quotient rule to find $f'(x)$.

(a) $f(x) = \dfrac{x^2 + 2x}{x - 5}$

(b) $f(x) = (x^2 + 1)e^x$

(c) $f(x) = \dfrac{e^x}{e^x + 1}$

(d) $f(x) = (2x^{3/2} + 3x^{4/3})(x^2 - 1)$