1. Initial Angular Velocity: $\omega a_i := 200 \cdot \frac{rad}{s}$ $\omega b_i := 200 \cdot \frac{rad}{s}$

2. Initial Inertia: $Ia_i := .5 \cdot m^2 \cdot kg$ $Ia_e := .4 \cdot m^2 \cdot kg$ $Ib_i := .5 \cdot m^2 \cdot kg$ $Ib_e := .6 \cdot m^2 \cdot kg$

3. Output Inertia (fixed): $Ic := 5 \cdot m^2 \cdot kg$

4. Initial Momentum: $La_i := Ia_i \cdot \omega a_i$ $La_i = 100 \frac{m^2 \cdot kg}{s}$ $Lb_i := Ib_i \cdot \omega b_i$ $Lb_i = 100 \frac{m^2 \cdot kg}{s}$

5. Set the desired ending angular velocity and calculate the output momentum (Lc). Ending ang. vel. for output (manually entered at step 14) will be when the torque on FW A & FW B are equal:

$\omega c = 20.408 \frac{1}{s}$

$Lc := Ic \cdot \omega c$ $Lc = 102.041 \frac{m^2 \cdot kg}{s}$

6. Find ending momentum that satisfies ending ωc and ending Lc:

6.1. Start with the eq. for a differential:

$$\omega c := \frac{\omega a_e - \omega b_e}{2}$$

6.2. Since ending angular velocities (ωa,b) aren't known, substitute the momentum/inertia eq.:

$$\omega c := \frac{\frac{La_e}{Ia_e} - \frac{Lb_e}{Ib_e}}{2}$$

$\omega c = 20.408 \frac{1}{s}$

6.3. Eliminate La_e by substituting the total initial momentum (La_i+Lb_i) minus Lc-Lb_e so that the only unknown is Lb_e:

$$\omega c := \frac{(La_i + Lb_i) - Lc - Lb_e}{2 \cdot Ia_e} - \frac{Lb_e}{2 \cdot Ib_e}$$

$\omega c = 20.408 \frac{1}{s}$

6.4. Move terms so that you can solve for Lb_e:

$$Lb_e := \frac{Ib_e \cdot \left[(2 \cdot \omega c \cdot Ia_e) - (La_i + Lb_i) + Lc\right]}{-Ib_e - Ia_e}$$

$Lb_e = 48.979 \frac{m^2 \cdot kg}{s}$

6.5. Knowing Lb_e allows computation of La_e

$$La_e := (La_i + Lb_i) - Lc - Lb_e \qquad La_e = 48.979 \frac{m^2 \cdot kg}{s}$$

6.6. Knowing La_e and Lb_e allows computation of both FW ending velocities:

$$\omega a_e := \frac{La_e}{Ia_e} \qquad \omega a_e = 122.448 \frac{1}{s}$$

$$\omega b_e := \frac{Lb_e}{Ib_e} \qquad \omega b_e = 81.632 \frac{1}{s}$$

6.7. All of the computed data:

$$\omega a_i = 200 \frac{1}{s} \qquad \omega a_e = 122.448 \frac{1}{s} \qquad La_i = 100 \frac{m^2 \cdot kg}{s} \qquad La_e = 48.979 \frac{m^2 \cdot kg}{s}$$

$$\omega b_i = 200 \frac{1}{s} \qquad \omega b_e = 81.632 \frac{1}{s} \qquad Lb_i = 100 \frac{m^2 \cdot kg}{s} \qquad Lb_e = 48.979 \frac{m^2 \cdot kg}{s}$$

$$\omega c_i := 0 \qquad \omega c = 20.408 \frac{1}{s} \qquad Lc = 102.041 \frac{m^2 \cdot kg}{s}$$

7. Conservation of Momentum:

$$L_i := La_i + Lb_i \qquad L_i = 200 \frac{m^2 \cdot kg}{s}$$

$$L_e := La_e + Lb_e + Lc \qquad L_e = 200 \frac{m^2 \cdot kg}{s}$$

8. Kinetic Energy:

$$E_{kai} := \frac{1}{2} \cdot Ia_i \cdot \omega a_i^2 \qquad E_{kai} = 1 \times 10^4 \text{ J} \qquad E_{kae} := \frac{1}{2} \cdot Ia_e \cdot \omega a_e^2 \qquad E_{kae} = 2.999 \times 10^3 \text{ J}$$

$$E_{kbi} := \frac{1}{2} \cdot Ib_i \cdot \omega b_i^2 \qquad E_{kbi} = 1 \times 10^4 \text{ J} \qquad E_{kbe} := \frac{1}{2} \cdot Ib_e \cdot \omega a_e^2 \qquad E_{kbe} = 4.498 \times 10^3 \text{ J}$$

9. Delta Kinetic Energy:

$$E_{ka} := E_{kai} - E_{kae} \qquad E_{ka} = 7.001 \times 10^3 \text{ J}$$

$$E_{kb} := E_{kbi} - E_{kbe} \qquad E_{kb} = 5.502 \times 10^3 \text{ J}$$

$$E_{kc} := \frac{1}{2} \cdot Ic \cdot \omega c^2 \qquad E_{kc} = 1.041 \times 10^3 \text{ J}$$

10. Time to Change Moments: $tr := 10 \cdot s$ $\quad t := 0 \cdot s, .0001 \cdot s .. tr$

11. Inertia as a function of time: $Ia(t) := Ia_i + t \cdot \frac{Ia_e - Ia_i}{tr}$ $\quad Ib(t) := Ib_i + t \cdot \frac{Ib_e - Ib_i}{tr}$

12. Momentum as a function of time: $La(t) := La_i + t \cdot \frac{La_e - La_i}{tr}$ $\quad Lb(t) := Lb_i + t \cdot \frac{Lb_e - Lb_i}{tr}$

13. Velocity as a function of time: $\omega a(t) := \frac{La(t)}{Ia(t)}$ $\quad \omega b(t) := \frac{Lb(t)}{Ib(t)}$

$\omega c(t) := \frac{\omega a(t) - \omega b(t)}{2}$ $\quad Lc(t) := Ic \cdot \omega c(t)$

14. Find ωc that satisfies equal torque on FW A & FW B: $\omega c \equiv 20.4083 \cdot \frac{rad}{s}$

15. Calculate torque: $tqa(t) := \frac{d}{dt} La(t)$ $\quad tqb(t) := \frac{d}{dt} Lb(t)$

$tqa(.0001 \cdot s) = -45.157\ in \cdot lbf$ $\quad tqb(.0001 \cdot s) = -45.157\ in \cdot lbf$

$tqa(tr) = -45.157\ in \cdot lbf$ $\quad tqb(tr) = -45.157\ in \cdot lbf$

$tqa := \frac{La_e - La_i}{tr}$ $\quad tqa = -45.157\ in \cdot lbf$

$tqb := \frac{Lb_e - Lb_i}{tr}$ $\quad tqb = -45.157\ in \cdot lbf$

$tqc := \frac{Lc(tr)}{tr}$ $\quad tqc = 90.314\ in \cdot lbf$