HW 10

Problem 1 :

Calculate the transmission and reflection coefficients as functions of the incident angle for right handed circularly polarized light.

Are they the same or different for left handed circularly polarized light?

Transmission = $\dfrac{energy\ flux\ of\ transited\ wave}{energy\ flux\ of\ the\ incident\ wave}$;

Reflection = $\dfrac{energy\ flux\ of\ trans.\ wave}{energy\ flux\ of\ inc.\ wave}$;

Energy flux $\equiv j = Re[\overrightarrow{S}] = \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\,|E_0|^2\,\vec{n}$; where $\vec{n} = \vec{z}$

$$T = \frac{j'}{j} \qquad\qquad R = \frac{j''}{j}$$

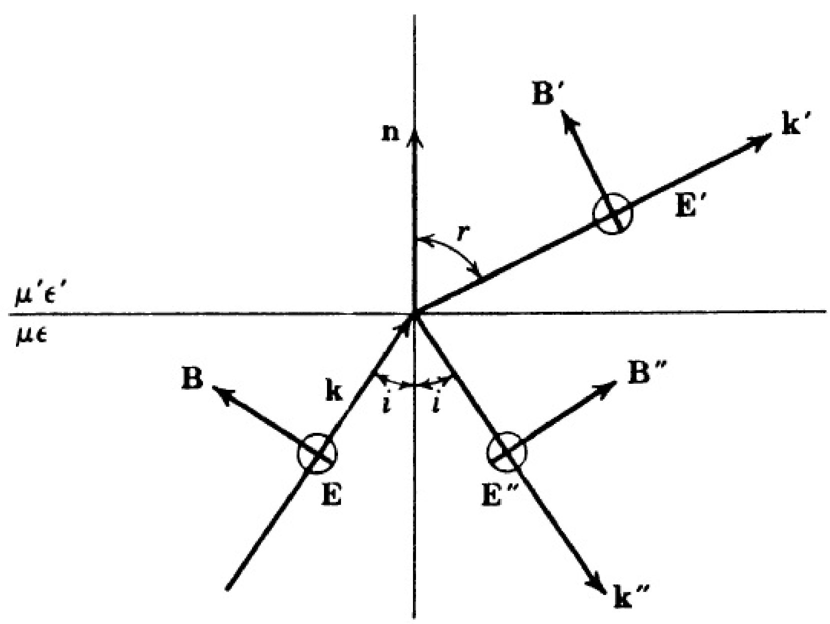

Figure 1: E-field is normal to the plane of inc.

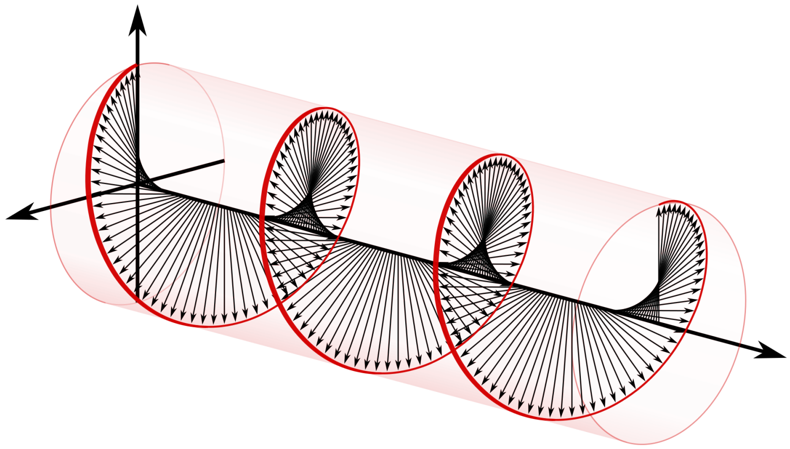

Figure 2: Propagation in z-direction

Right hand circular polarized Electric field :

$$\overrightarrow{E_R} = E_0 \left(\widehat{x} - i\widehat{y}\right) e^{i\left(\vec{k}.\vec{z}-\omega t\right)}$$

Left hand circular polarized Electric field :

$$\overrightarrow{E_L} = E_0 \left(\widehat{x} + i\widehat{y}\right) e^{i\left(\vec{k}.\vec{z}-\omega t\right)}$$

Note : The reflected wave has to be circular but in the opposite polarization.

Note : when a circular polarized light enter a median, the transmitted wave does not have to be circular, in general it is elliptical :

$$\overrightarrow{E_e} = \left(E_x' \,\widehat{x} \pm E_y' \,\widehat{y}\right) e^{i\left(\vec{k}.\vec{z}-\omega t\right)}$$

Boundary Conditions (in term of $\overrightarrow{E_0}$) :

$$[1] : D_\perp \; continuous$$
$$[2] : B_\perp \; continuous$$
$$[3] : E_\parallel \; continuous$$
$$[4] : H_\parallel \; continuous$$

$$[\epsilon(\mathbf{E}_0 + \mathbf{E}_0'') - \epsilon'\mathbf{E}_0'] \cdot \mathbf{n} = 0$$

$$[\mathbf{k} \times \mathbf{E}_0 + \mathbf{k}'' \times \mathbf{E}_0'' - \mathbf{k}' \times \mathbf{E}_0'] \cdot \mathbf{n} = 0$$

$$(\mathbf{E}_0 + \mathbf{E}_0'' - \mathbf{E}_0') \times \mathbf{n} = 0$$

$$\left[\frac{1}{\mu}(\mathbf{k} \times \mathbf{E}_0 + \mathbf{k}'' \times \mathbf{E}_0'') - \frac{1}{\mu'}(\mathbf{k}' \times \mathbf{E}_0')\right] \times \mathbf{n} = 0$$

Figure 3: in order: [1] , [2] , [3] , and [4]

Applying the boundary conditions to E-field :

Note : on (z = 0) plane, $\vec{k} \, . \, \vec{z} = 0$ for all waves $\rightarrow e^{i\left(\vec{k}.\vec{z}-\omega t\right)}$ = 1

[1] :

$$\left[\epsilon\left(\overrightarrow{E_R} + \vec{E}_L{}''\right) - \epsilon'\,\vec{E}_e{}'\right] . \, \hat{z} = 0$$

$$\left[\epsilon\left(E_0\left(\hat{x} - i\hat{y}\right) + E_0{}''\left(\hat{x} + i\hat{y}\right)\right) - \epsilon'\left(E_x'\,\hat{x} \pm E_y'\,\hat{y}\right)\right]_{\perp} . \, \hat{z} = 0$$

$$\left[\epsilon\left(E_0\left(i\hat{y}\right)cos(i) + E_0{}''\left(i\hat{y}\right)cos(i)\right) - \epsilon'\left(E_y'\,\hat{y}\right)cos(r)\right] . \, \hat{z} = 0$$

$$\boxed{\epsilon\,(E_0 + E_0{}'')\,cos(i) - \epsilon'\,E_y'\,cos(r) = 0}$$

[2] :

$$\left[\vec{K} \times \overrightarrow{E_R} + \vec{K}'' \times \overrightarrow{E_L}'' - \vec{K}' \times \overrightarrow{E_e}''\right] . \, \hat{z} = 0$$

$$\left[k\,\hat{z} \times E_0\left(\hat{x} - i\hat{y}\right) + k''\,\hat{z} \times E_0{}''\left(\hat{x} + i\hat{y}\right) - k'\,\hat{z} \times \left(E_x'\,\hat{x} \pm E_y'\,\hat{y}\right)\right] . \, \hat{z} = 0$$

$$\left[k\,E_0\left(\hat{z} \times \hat{x} - i\hat{z} \times \hat{y}\right) + k''E_0{}''\left(\hat{z} \times \hat{x} + i\hat{z} \times \hat{y}\right) - k'\left(E_x'\,\hat{z} \times \hat{x} \pm E_y\,\hat{z} \times \hat{y}\right)\right] . \, \hat{z} = 0$$

$$\left[k\, E_0 \left(\hat{y} + i\hat{x}\right) + k'' E_0'' \left(\hat{y} + i\hat{x}\right) - k' \left(E_x' \,\hat{y} \pm E_y' \,\hat{x}\right)\right] . \,\hat{z} = 0 \checkmark$$

[3] :

$$\left[\, \overrightarrow{E_R} + \vec{E}_L{}'' - \vec{E}_e{}' \,\right] \times \hat{z} = 0$$

$$\left[\, E_0 \left(\hat{x} - i\hat{y}\right) + E''_0 \left(\hat{x} + i\hat{y}\right) - \left(E_x' \,\hat{x} \pm E_y' \,\hat{y}\right) \,\right] \times \hat{z} = 0$$

$$\left[\, E_0 \left(\hat{x} \times \hat{z} - i\hat{y} \times \hat{z}\right) + E''_0 \left(\hat{x} \times \hat{z} + i\hat{y} \times \hat{z}\right) - \left(E_x' \,\hat{x} \times \hat{z} \pm E_y' \,\hat{y} \times \hat{z}\right) \,\right] = 0$$

$$\left[\, E_0 \left(-\hat{y} - i\hat{x}\right) + E''_0 \left(-\hat{y} + i\hat{x}\right) - \left(-\, E_x' \,\hat{y} \pm E_y' \,\hat{x}\right) \,\right] = 0$$

$$\left[-\, E_0 \left(\hat{y} + i\hat{x}\right) + E''_0 \left(-\hat{y} + i\hat{x}\right) + \left(E_x' \,\hat{y} \mp E_y' \,\hat{x}\right) \,\right] = 0$$

$$-\, E_0 \left(\hat{y} + i\hat{x}\right) + E''_0 \left(-\hat{y} + i\hat{x}\right) = -\left(E_x' \,\hat{y} \mp E_y' \,\hat{x}\right)$$

$$E_0 \left(\hat{y} + i\hat{x}\right) - E''_0 \left(-\hat{y} + i\hat{x}\right) = \left(E_x' \,\hat{y} \mp E_y' \,\hat{x}\right)$$

$\perp$	$E_0 \left(\hat{x}\right) - E''_0 \left(\hat{x}\right) = E_y' \left(\hat{x}\right)$
$\parallel$	$E_0 \left(\hat{y}\right) + E''_0 \left(\hat{y}\right) = E_x' \left(\hat{y}\right)$

[4] :

$$\left[\frac{1}{\mu}\left(\vec{k}\times\overrightarrow{E_R}+\vec{k}''\times\overrightarrow{E_L}\right)-\frac{1}{\mu'}\left(\vec{k}'\times\overrightarrow{E_e'}\right)\right]\times\vec{n}=0$$

$$\left[\frac{1}{\mu}\left(k\,E_0\left(\hat{y}+i\hat{x}\right)+k''E_0''\left(\hat{y}+i\hat{x}\right)\right)-\frac{1}{\mu'}k'\left(E_x'\,\hat{y}\pm E_y'\,\hat{x}\right)\right]\times\vec{z}=0$$

$$\left[\frac{1}{\mu}\left(k\,E_0\left(\hat{x}-i\hat{y}\right)+k''E_0''\left(\hat{x}-i\hat{y}\right)\right)-\frac{1}{\mu'}k'\left(E_x'\,\hat{x}\mp E_y'\,\hat{y}\right)\right]=0$$

$$\frac{1}{\mu}\left(k\,E_0\left(\hat{x}-i\hat{y}\right)+k''E_0''\left(\hat{x}-i\hat{y}\right)\right)=\frac{1}{\mu'}k'\left(E_x'\,\hat{x}\mp E_y'\,\hat{y}\right)$$

$\perp$	$\frac{k}{\mu}E_0\left(\hat{x}\right)+\frac{k''}{\mu}E_0''\left(\hat{x}\right)=\frac{k'}{\mu'}E_x'\left(\hat{x}\right)$
$\parallel$	$\frac{k}{\mu}E_0\left(\hat{y}\right)+\frac{k''}{\mu}E_0''\left(\hat{y}\right)=\frac{k'}{\mu'}E_y'\left(\hat{y}\right)$

Using : Energy flux $\equiv j = Re\overrightarrow{[S]} = \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\,|E_0|^2\,\vec{n}$:

$$T = \frac{j'}{j} = \frac{|S'|}{|S|} = \sqrt{\frac{\epsilon'}{\mu'}}\sqrt{\frac{\mu}{\epsilon}}\left(\frac{E_0'}{E_0}\right)^2 sin^2(i)$$

$$R = \frac{j''}{j} = \frac{|S''|}{|S|} = \sqrt{\frac{\epsilon''}{\mu}}\sqrt{\frac{\mu}{\epsilon}}\left(\frac{E''_0}{E_0}\right)^2 sin^2(i) = \sqrt{\frac{\epsilon''}{\epsilon}}\left(\frac{E''_0}{E_0}\right)^2 sin^2(i)$$