$$(\frac{N}{2}-m)!\,(\frac{N}{2}+m)! = 2(\frac{N}{2}!)^2, \tag{2}$$

and hence, assuming $N \gg m \gg 1$:

$$\left(\frac{N}{2}\right)^m + [1+2+\cdots+m]\left(\frac{N}{2}\right)^{m-1} \approx 2\left[\left(\frac{N}{2}\right)^m - [1+2+\cdots+m]\left(\frac{N}{2}\right)^{m-1}\right]. \tag{3}$$

Noting that $1+2+\cdots m \approx m^2/2$, show this leads to

$$m \propto N^{1/2}, \tag{4}$$

and the fractional width therefore goes as $N^{-1/2}$. Comment on this result for $N \approx 10^{23}$.