MP350 Classical Mechanics

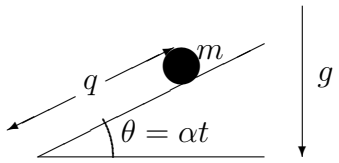
Problem set 1

1. The lagrangian of a one-dimensional mechanical system is given by

$$L(q, \dot{q}, t) = \frac{m}{2}\dot{q}^2(1 + \cos^2 q) - mg \sin q \,.$$

Determine the Euler–Lagrange equation of motion for the system.

2. A body with mass m is lying on a smooth, frictionless plane. The plane, which is originally horizontal, is lifted up at a constant rate such that the angle of the plane with the horizontal at time t is $\theta = \alpha t$ (see figure).

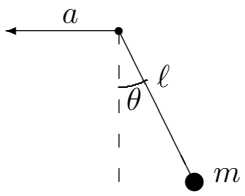


(a) Show that the lagrangian of the body, expressed in terms of the distance q from the base of the plane, is

$$L = \frac{1}{2}m\dot{q}^2 + \frac{1}{2}m\alpha^2 q^2 - mgq \sin \alpha t \,.$$

(b) Determine the Euler–Lagrange equations for the system.

3. A simple pendulum with mass m and length ℓ is suspended from a point which moves horizontally with constant acceleration a (see figure).



(a) Show that the lagrangian for the system can be written, in terms of the angle θ,

$$L(\theta, \dot{\theta}, t) = \frac{m}{2}\left(\ell^2\dot{\theta}^2 + a^2t^2 - 2a\ell t\dot{\theta}\cos\theta\right) + mg\ell\cos\theta \,.$$

(b) Determine the Euler–Lagrange equation for the system.