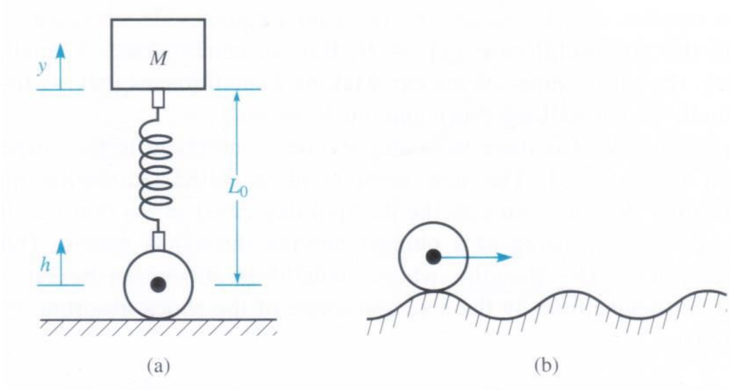
Part 1: Analysing a simple system (20 marks)

Consider a “unicycle” shown below, which consists simply of the chassis connected to the wheels through a spring:



(a) Simplified suspension system
(b) The driving frequency is determined by the spacing of the bumps and the speed of the vehicle

In this figure:

- *h* is the height of the wheel; the origin of the coordinates is such that when the wheel is rolling on flat, level ground, *h* = 0.
- The mass *M* represents the weight of the unicycle.
- The height of mass *M* above its reference level is called *y*. The reference level is chosen so that when the car is in level motion, *y* = 0.
- Under these conditions the length of the spring is L_0. The spring exerts forces in the *y*-direction, depending on the length of the spring and the spring constant *k*, according to Hooke’s law of elasticity.

Note: At equilibrium the spring is slightly compressed from its natural length due to the weight of the vehicle, making its length equal to L_0. Then the total force acting on *M* – gravity plus spring force – is zero, and *M* neither rises nor falls.

Section 1A: Mathematical Analysis

1. From the description and the laws of Physics, show that the motion of the unicycle can be described by the LCCDE (linear constant-coefficient differential equation) below:

$$\frac{d^2y(t)}{dt^2} + \frac{k}{M}y(t) = \frac{k}{M}[h(t)]$$

2. Determine the characteristic equation and eigenvalues for this system
3. Work out the frequency response of this system. Do you foresee any problems with this response?

Section 1B: Analysis using Matlab

In this section the system responses should be analysed using Matlab. Refer to the attached document *"A Brief MATLAB Guide"* in order to understand how to represent LTI systems in Matlab, and hence how to determine impulse response, step response and frequency response of systems. MATLAB is installed in the computer labs.

Using the commands given in the *Guide*, analyse the response of the system using the following parameters:

M = 200 kg

k = 1800 N/m

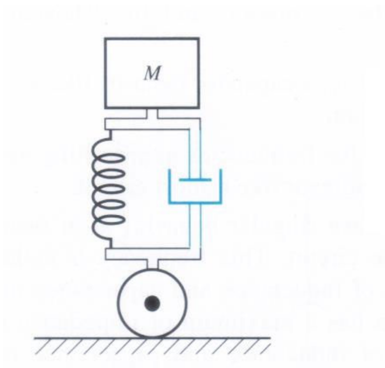
$L_0 = 0.25$ m

4. Determine the frequency response from 0.1 Hz to 20 Hz using the freqs command. Plot the magnitude and phase response over this frequency range.
5. Plot the impulse response and step response of the system (for 5 seconds duration) using the impulse and step functions. Include all plots (properly labelled) in your submission.
6. Discuss the response of the system. Does the frequency response match what was calculated in Part 1A? How does the response correspond to the systems physical parameters? Would this setup perform the required function as it is?

Note: The **function of the system** is to minimise the up and down motion of the vehicle (while still following contours of road).

Part 2: Improving the system (35 marks)

To improve the system, what is done is to add another component to the suspension system, as shown below:



Improved suspension system with "shock absorber".

The new component is called (somewhat misleadingly) a shock absorber. It is the same as a dashpot: a viscous-damping device consisting of a plunger moving through a viscous fluid. It adds an additional frictional force proportional to velocity and to a constant c describing the drag, determined by the size and shape of the shock absorber. This frictional force resists motion, i.e., acts in the opposite direction, slowing the motion and absorbing energy.

7. Work out the LCCDE for the system including the "shock absorber". Show all working.
8. Use Matlab to plot the frequency response (magnitude and phase), impulse response and step response of the system for the following values of *c*:
    a. *C* = 200
    b. *C* = 400
    c. *C* = 1800

   Hint 1: It would be more efficient to put all the necessary commands into a script file (a *.m* file) so you can edit the parameters and then run all the commands at one go.

   Hint 2: You can plot all 4 graphs at one go using a 2 x 2 matrix of plots using *subplot*(22*n*), where *n* determines which of the 4 subplots gets used.

9. From the plots determined above, describe the behaviour of the system as the value of *c* is changed. How has adding the "shock absorber" helped the system? What are the advantages and disadvantages of the different *c* values in terms of the primary function of the system?
10. Use Matlab to determine the optimal value of *c* in terms of the function of the system (in your opinion). Show all 4 response plots for this chosen value of *c*, and justify your selection (discuss and give reasoning) based on these plots.

<u>Part 3: Analysing system response for various conditions (25 marks)</u>

The step input can model a physical input such as hitting a kerb. As part of the design process, the system response to different driving and road conditions should be analysed.

Assume the vehicle is going to be driven over a slightly uneven road with some irregularities (e.g. cobblestones or potholes). The response of the system needs to be determined at 'low' speed and 'high' speed.

11. Model the road surface such that it can be put into Matlab. Clearly state all assumptions and mathematical basis of the model.
12. Use Matlab to determine the response of the system for the modelled road condition, for two different 'speeds'. Plot the response (y) of the system over time, with the plot of the input (road condition) above is as a comparison. Comment of the results.

Hint: You may want to use transforms to determine the response.